Similarity solutions of Fokker-Planck equation with
time-dependent coefficients and fixed/moving boundaries

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The Fokker-Planck equation (FPE) is one of the basic tools which is widely used for studying the effect of fluctuations in macroscopic systems. It has been employed in many areas: Physics, chemistry, hydrology, biology, even finance, and others. Because of its broad applicability, it is therefore of great interest to obtain solutions of the FPE for various physical situations.

Generally, it is not easy to find analytic solutions of the FPE. Exact analytical solutions of the FPE are known for only a few cases. In most cases, one can only solve the equation approximately, or numerically. Most of these methods, however, are concerned only with FPE's with time-independent diffusion and drift coefficients. Solving the FPE's with time-dependent drift and/or diffusion coefficient is in general an even more difficult task.

Here we would like to present a general way to construct exact similarity solutions of the FPE with time-dependent drift and diffusion coefficients. Such similarity solutions exist when the FPE possesses proper scaling behavior. By the introduction of the similarity variable, the FPE can be reduced to an ordinary differential equation. It is interesting to find, by the natural requirement that the probability current density vanishes at the boundary, that the resulted ordinary differential equation is integrable, and the probability density function can be given in closed form. Exactly solvable FPE’s with time-dependent coefficients can then be obtained whenever the drift and diffusion coefficients are such that certain integral corresponding to the ordinary differential equation can be exactly integrated. Our work thus extends the number of exactly solvable FPE’s.