Numerical analysis of quantum phase transitions with dynamic control of anisotropy

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A phase transition is one of the most important topics in the statistical mechanics. Especially, in a second-order phase transition, many kinds of physical quantities show the power-law behavior at the phase transition point, that is also referred to as the critical point. This behavior is called as the critical phenomenon. The set of the exponents (critical exponents) enables us to classify the phase transitions into the universality classes. Critical exponents take the universal value that is independent of the detail of the system but depend only on the dimensionality and symmetry property.

This is also the case for the quantum phase transitions (QPTs). A QPT is the phase transition between two different ground states that occurs at zero temperature by controlling the quantum fluctuations. In general, by the path-integral, one can treat a $d$-dimensional quantum system in the same way as a corresponding $(d + 1)$-dimensional classical system, resulting in the $(d + 1)$-dimensional counterpart of the universality class. However, there exists some exceptions that shows critical phenomena peculiar to quantum systems. There are researches that aim to find such novel critical phenomena.

In this situation, it is reported that a two-dimensional Heisenberg model on the square lattice with some specific kind of the nearest neighbor interaction pattern, which is referred to as the staggered dimer model, shows the QPT with apparently unconventional critical exponents [1]. However, we consider this result an artifact resulting from the anisotropy of the system. We employ the Robbins-Monro algorithm [2] from the field of machine-learning in order to control the system size dynamically for carrying out the Monte Carlo simulations of the system with effectively isotropic geometry.

We applied this method to the following two kinds of anisotropic models: the columnar and staggered dimer model. The difference of these models is only the spatial interaction pattern. The former is known to show the conventional critical behavior. By using this method, as the former shows conventional behavior, we revealed that the latter also belongs to the $O(3)$ universality class, which is not unconventional, and that the optimal aspect ratio of the staggered dimer model shows non-monotonic and non-trivial behavior that might result in the apparently unconventional critical exponents.

References