Quantum fluctuations of particles and fields in smooth path integrals

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Evaluation of the Feynman path integrals is usually done in discretized time-space, in which derivations and integrations are replaced with finite differences and summations, respectively, because the discretization can simplify evaluation of path integrals especially in numerical calculations. However, the discretization explicitly breaks continuous symmetries of time-space such as the translational symmetry down to discretized symmetries and it sometimes leads to qualitative discrepancies such as doublers in the Dirac fields and magnetic monopoles in the lattice QED. Therefore, from viewpoint of complementarity it is desired to perform the path integrals in continuous or smooth time-space without discretization.

In this presentation we develop an approach to approximate evaluation of the smooth path integrals for particles and fields in Euclidean time-space. The paths are described by sum of smooth functions and are weighted with $\exp(-S)$ by appropriate methods such as the Metropolis method. One possibility for the smooth functions is to use the Gauss function $[1]$. In this case, the path of particles, $q(\tau)$, as a function of the Euclidean time $\tau$ is expressed as,

$$q(\tau) = \sum_{i=1}^{N_{\text{sum}}} q_i \exp \left[ -\frac{(\tau - \tau_i)^2}{\xi_i^2} \right],$$

where $q_i$, $\tau_i$, and $\xi_i$ are height, position, and scale of the Gauss functions, respectively, and in the numerical simulations $q_i$ is changed according to the weight $\exp(-S)$ while $\tau_i$ and $\xi_i$ are fixed. We discuss how accurate the weighted smooth paths reproduce quantum behaviors of particles and fields.

Figure 1: (Left) Examples of quantum fluctuations for the harmonic oscillator in the simulation conditions A, B, and C $[1]$. (Right) Distributions of the coordinate for the harmonic oscillator together with the squared wave function of the ground state denoted by dashed line.