Nonlinear Sigma Model with General Potential: Local Existence and Smoothness Properties

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In this paper, we prove the local existence and smoothness properties of the nonlinear sigma model with general potential. We start from general Lagrangian of nonlinear sigma model,

\[ \mathcal{L} = \frac{1}{2} g_{ab} \partial_{\mu} \phi^a \partial^\mu \phi^b - V(\phi), \]  

where the real scalar field \( \phi \) is a mapping from the \( d + 1 \) Minkowski space, \( M^{d+1} \approx \mathbb{R} \times \mathbb{R}^d \) to general \( n \) dimensional Riemannian manifold \( N^n \) with metric \( g_{ab} \) and \( V(\phi) \) is the scalar potential.

We follow the method in [1] by embedding isometrically \( N^n \) into \( \mathbb{R}^q \) for large enough \( q \) and construct the equivalent equation in \( \mathbb{R}^q \). Using method in [2], we prove the local existence and smoothness properties of the equation in \( \mathbb{R}^q \) and show that the local existence of the equation in \( \mathbb{R}^q \) imply the local existence of the original equation.