Numerical simulation of primordial black hole formation

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If the amplitude of the density perturbation is sufficiently large in some region in the early universe dominated by radiation, this region starts to contract against pressure gradients after the horizon reentry and collapses to form a black hole (Primordial Black Hole, PBH). In order to analyze formation process of these PBHs, in our previous paper we developed an asymptotic expansion approach to calculate time evolution of a spherical perturbed region outside the horizon. Our formalism is valid even when the amplitude of the perturbation is sufficiently large to form a PBH in the end.

In this paper results of numerical computation of time evolution of the perturbed region after the horizon reentry are presented. The initial conditions for this numerical computation are given using our asymptotic expansion technique. By calculating time evolution of various initial perturbations, the condition for PBH formation is investigated. We found this condition is well described and understood in terms of two quantities. The first one is the density perturbation at the time of horizon crossing averaged over the overdense region, which is denoted by $\delta_{hc}$ and measures how strong gravity is. The second one is the absolute value of the maximum value of radial derivatives of the perturbation profile, represented by $\Omega$. We found the condition for PBH formation is expressed as $\delta_{\text{min}}(\Omega) < \delta_{hc}$ and $\delta_{\text{min}}(\Omega)$ is an increasing function of $\Omega$ and spans $0.45 < \delta_{hc} < 0.6$. That is, when $\Omega$ is large $\delta_{hc}$ has to be larger to form a PBH. This implies a larger value of $\Omega$ makes pressure gradients more effective so that it is even more difficult for gravity to dominate and create a PBH. This condition will be useful in computing abundance of PBHs precisely.