Asymptotic properties of energy spectra for high-energy cascade electron and photon in strong magnetic fields


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Energy densities, \( \pi(E, t) \) and \( \gamma(E, t) \), for high-energy cascade electron and photon show the ratio of \( D \) to \( C(1) \) after enough penetration through strong magnetic fields, thus we have \( \pi(E, t) = \frac{D}{D+C(1)} \frac{\nu(v, t)}{E_0} \) and \( \gamma(E, t) = \frac{C(1)}{D+C(1)} \frac{\nu(v, t)}{E_0} \), where \( \nu(v, t) \) denotes the sum of \( \pi(E, t) \) and \( \gamma(E, t) \) and \( v \) the ratio of \( E \) to the incident energy, \( E_0 \). Then \( \nu(v, t) \) is determined from the diffusion equation

\[
\frac{\partial \nu(v, t)}{\partial t} = -\frac{\nu(v, t)}{v^{1/3}} \int_0^1 \eta(u) du + 2 \int_0^1 \frac{\nu(v/u, t)}{(v/u)^{1/3}} \eta(u) \frac{du}{u},
\]

with \( \eta(u) = \frac{D}{D+C(1)} \psi(u) + \frac{C(1)}{D+C(1)} \phi(1-u) / u \). We can solve the equation by asymptotic expansion of \( \nu(v, t) \sim v^{-5/3} \sum_{k=0}^{\infty} \frac{\nu_k(x)}{k!} v^{k/3} \), where \( x = \eta_0 t \) and \( \eta_0 = \tilde{\eta}(0) \) with \( \tilde{\eta}(s) \equiv \int_0^1 u^s \eta(u) du \). From the limiting properties at \( v \to 0 \) and the decrease of total particle energy, we have

\[
\nu_0'(x) = \left\{ 2\tilde{\eta}(1/3)/\eta_0 - 1 \right\} \nu_1(x), \quad \nu_1'(x) = \left\{ 2\tilde{\eta}(1/3)/\eta_0 - 1 \right\} \nu_2(x)/2, \quad \nu_2'(x) = \nu_3(x)/3,
\]

where \( e^{-\eta_0 t} \) denotes survival energy of the incident particle. Applying Laplace transforms, we get the solution for \( \nu_k(x) \)'s, so that for \( \pi(E, t) \) and \( \gamma(E, t) \). Energy density and transition curve of electron for photon incident shower, so obtained, are indicated in Figs. 1 and 2, compared with the results derived by the numerical integration method [2]. Thus we find the asymptotic expansion well explain the properties of cascade in strong magnetic fields of enough penetration about \( t > 1 \).

Figure 1: Energy spectra derived by asymptotic expansion (dots) and by numerical integration method (lines).

Figure 2: Transition curves derived by asymptotic expansion (dots) and by numerical integration method (lines).


1Retired now.