Electromagnetic showers in strong magnetic fields derived analytically assuming simplified cross-sections


Laboratory of Information Science, Okayama Shoka University, Japan
Department of Fundamental Science, Okayama University of Science, Okayama 700-0005, Japan
Department of Natural Sciences, Kawasaki Medical School, Kurashiki 701-0192, Japan

nakatuka@olive.plala.or.jp

Diffusion equation of energy spectra for high-energy cascade electron and photon, \( \pi(E, t) \) and \( \gamma(E, t) \), in strong magnetic fields [1] can easily be solved if we simplify the cross-sections of radiation and pair production, \( q\phi(W/E) dW/E^{4/3} \) and \( q\psi(E/W) dE/W^{4/3} \), by taking \( \phi(W/E) = \psi(E/W) \approx a \). Mellin transforms, diagonal matrix, and residues were applied. Then \( \pi(E, t) \) is expressed as

\[
E\pi(E, t) = \frac{2}{3}(W_0/E)^{-1}\left(720 - 1800x + 1200x^2 - 300x^3 + 30x^4 - x^5\right)e^{-x}/360 \\
+ (2/3)(W_0/E)^{-2/3}x^2(360 - 480x + 180x^2 - 24x^3 + x^4)e^{-x}/72 \\
+ (2/3)(W_0/E)^{-1/3}x^3(120 - 90x + 18x^2 - x^3)e^{-x}/36 \\
+ (2/3)x^4(30 - 12x + x^2)e^{-x}/36 + (2/3)(W_0/E)^{1/3}x^5(6 - x)e^{-x}/72 \\
+ (2/3)(W_0/E)^{2/3}x^6e^{-x}/360 + (2/3)(W_0/E)^{-1}x^3e^{-y}/6 \\
+ (2/3)(W_0/E)^{-1/3}x^2(3 - y)e^{-y}/3 + (2/3)(W_0/E)^{-1}x(6 - 6y + y^2)e^{-y}/6
\]

for gamma-incident shower of energy \( W_0 \), where \( x = at \) and \( y = at(W_0/E)^{1/3} \). Derived spectra and transition curves are indicated in Figs. 1 and 2. The solution will be helpful to investigate properties of the shower in strong magnetic fields, as \( E^{-5/3}dE \) spectra at low energy limit and/or \( (W_0/E)^{2/3} \)-dependence of the peak value of transition curve are confirmed in the solution.

Figure 1: The analytical (lines) and the numerical integration (dots) results for energy spectra of electron at \( t \) of 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1, 2, 5, from bottom to top.

Figure 2: Analytical results of transition curve of electron for gamma-initiated cascades. Four curves show \( W_0/E \) of \( 10^2, 10^4, 10^6, 10^8 \), from bottom to top.


1Retired now.