Momentum exchange of a plasma with its surroundings

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A plasma has been applied to accelerate a space vehicle and/or drive a fluid by exchanging momentum with its surroundings. The amount of the exchanged momentum must be related with its spatial profile, because a momentum is represented by a vector and the degree of asymmetry of the plasma profile is also represented by a vector, such as

\[ \mathbf{q} = \int_V \mathbf{r} n(\mathbf{r}) \, dV \]

using a plasma density \( n(\mathbf{r}) \) at a position \( \mathbf{r} \). Here, the integral is made over the plasma volume \( V \) and the origin of the coordinate might be given to be \( \mathbf{q} = 0 \) in case of no momentum exchange.

Here, let us consider a steady-state one-dimensional plasma with a particle generation rate, \( G \), and a momentum generation rate, \( F \), using a fluid model [1, 2]. Accordingly, in the bounded space of \(-a \leq x \leq a\), we have

\[
\frac{d}{dx}(nv) = G
\]

and

\[
\frac{d}{dx}(nmv^2) = -ne\frac{d\phi}{dx} - \frac{dp}{dx} + F,
\]

where \( n, v, p, \) and \( m \) are the ion density, the ion velocity, the ion pressure, and the ion mass, respectively, and \( \phi \) is the electrostatic potential. Assuming the uniform electron temperature of \( T_e = \text{const.} \), from the negligible small electron mass, we also have the Boltzmann equation

\[ n = n_0 e^{\phi/k_B T_e} \].

Then, after specifying \( G \) and \( F \) and assuming \( p \propto n^\kappa \) \((1 \leq \kappa \leq 3)\), we can obtain the profiles of \( n, v, p, \) and \( \phi \), under the boundary condition that

\[ v^2 = (k_B T_e + \kappa k_B T_i)/m \]

at the ends \( x = \pm a \).

The results show that some of the momentum derived into a plasma from its surroundings, \( \int_a^{-a} F \, dx \), is ejected from the ends; and the rest of the momentum derived is consumed to form the asymmetric plasma profile and changes to the net force due to the electron and ion pressure at the ends. In other words, the momentum transported from the plasma to the surroundings is compensated by the net momentum from the ends and the net force due to the electron and ion pressure at the ends. The ratio of the rate of the momentum ejected from the ends to the net force at the ends is 1 to 1 when \( T_i = 0 \) or \( \kappa = 1 \), although the ratio is involved in general.