Numerical Approach to Quark-Gluon World from Statistical QCD

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QCD (Quantum ChromoDynamics) is a standard theory of quarks and gluons. It is well established, and characters of nucleons and mesons are described by QCD, especially by the Wison’s lattice gauge theory. However, at non-zero temperature and density, still our knowledge is very limited, although such extreme conditions are realized at the center of compact stars and the early universe. At very high temperature and density, it is expected that the QCD shows phase transition(s), and the Relativistic Heavy Ion Collider (RHIC) was built to explore properties of QCD matter under such extreme conditions.

We propose and test a method by which Lee–Yang zeros of the QCD system at finite temperature and density can be determined without ambiguity; Lee-Yang zeros are the zeros of the grand partition function $\mathcal{Z}(\xi)$ to characterize the statistical nature of a system\cite{1} in the complex plane of fugacity, $\xi \equiv \exp(\mu/T)$; $\mathcal{Z}(\xi) = \prod (\xi - \alpha_l)$, where $T$ is temperature and $\mu$ is a chemical potential.

We construct the canonical partition function $Z_n$ from recently observed net-proton multiplicity in RHIC experiment and evaluate the grand partition function as $\mathcal{Z}(\xi, T) = \sum Z_n(T)\xi^n, (n = -N_{\text{max}}, \cdots, N_{\text{max}})$.

We first map the problem to a calculation of the residue of $\mathcal{Z}'/\mathcal{Z}$: $\mathcal{Z}'/\mathcal{Z} = \sum 1/(\xi - \alpha_l)$. For this equation, the left hand side is integrated along a contour, and the residues inside the contour are summed according to Cauchy’s theorem. As shown in Fig.1, starting from a contour the region is divided into four pieces, and the Cauchy integral is evaluated over each section. Each contour has the same of cut Baum-Kuchen(cBK). This divide-and-conquer process is iterated. When no residue is found in a section, no further divisions are applied to that section. Fig.2 is an example of Lee–Yang Zero for RHIC experimental data. All calculations were performed using the multiple-precision package, FMlib\cite{2}, and the number of significant digits was 50 - 100.

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\textbf{Figure 1:} Schematic of CBK contours in the divide-and-conquer search for residues.

\textbf{Figure 2:} LYZ diagram (STAR at $\sqrt{s} = 39$ GeV\cite{3}).

