On the hardness of practical instances for time-dependent MPS simulation of typical quantum algorithms

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Simulability of quantum circuits using the time-dependent matrix-product-state (TDMPS) method \cite{vidal2003,kawaguchi2004} strongly depends on the problem instance. For hard instances, the floating-point precision and/or the threshold \( m_{\text{trunc}} \) for the Schmidt rank of a linear-nearest-neighbour bipartite splitting should be set to large numbers in order for an accurate simulation \cite{saitoh2012}. Accumulations of rounding errors due to precision shortage and truncation errors are quite significant in TDMPS simulations for the quantum circuit model in contrast to those in common TDMPS simulations \cite{banuls2006} for Hamiltonian models. Thus we employ multiprecision computing as we reported in Refs. \cite{saitoh2012,saitoh2013}.

The hardness under TDMPS simulation is usually evaluated by the largest Schmidt rank \( r_{\text{max}} \) and the distribution of Schmidt coefficients. The simulation cost of a quantum circuit is \( O(nN_{g}r_{\text{max}}^{3}) \) (floating-point operations) where \( n \) is the number of qubits and \( N_{g} \) is the number of elementary quantum gates in the circuit \cite{vidal2003}. This becomes \( O(nN_{g}m_{\text{trunc}}^{3}) \) when a truncation of nonzero Schmidt coefficients is possible. In order to keep \( m_{\text{trunc}} \) small, a dominant number of nonzero Schmidt coefficients should be negligibly small. So far, we found that all the nonzero Schmidt coefficients were nonnegligible for the quantum circuits tried in Refs. \cite{saitoh2012,saitoh2013}. In addition, it may happen that a significantly large floating-point precision is required for an exceptional case \cite{saitoh2012}, which is another factor of the hardness.

In this contribution, we evaluate the hardness of problem instances of typical quantum algorithms for the TDMPS simulation method under realistic settings. We consider a Grover’s quantum search with oracle circuits constructed for some instances of the satisfiability problem and a Shor’s quantum factoring for nontrivial composite numbers with the semi-classical quantum Fourier transform.

It does not take significantly long time to simulate a quantum circuit with less than 100 qubits using the TDMPS method. For example, it takes only approximately seven minutes \cite{saitoh2013} to simulate a 65-qubit quantum circuit for the Deutsch-Jozsa algorithm under a certain setup, using our open-source simulation library named ZKCM_QC on a single CPU core of a typical CPU. Therefore, we can evaluate the dynamics of Schmidt coefficients and accumulating numerical errors for relatively large quantum circuits which are intractable for brute-force simulation methods.

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