Simple and intuitive mathematics for learning elementary physics

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Mathematics is a useful tool for describing concepts in physics [1]. However, students fail to notice physical meaning when they are focused on mathematical calculations. Simple and intuitive mathematics is effective for learning elementary physics, and therefore diagrams for visual thinking for mathematics are effective for avoiding the above situation [2, 3]. Geometrical viewpoints inspire ideas in physics, as seen from the fact that kinematics, referred to as the geometry of motion, describes the motion of a particle. In this presentation, some problems on the motion of a particle in a uniform gravitational field are illustrated using simple diagrams.

The particle is projected from an initial position with an initial velocity at an elevation angle $\theta$ under the influence of a uniform gravitational field. At what angle should the particle be projected to obtain the maximum horizontal range? Can the horizontal range be the same when a particle is projected in two different angles? The horizontal range of the particle is $2v_0^2 \sin \theta \cos \theta / g$, where $v_0$ is an initial velocity and $g$ is the gravitational acceleration. The particle should be projected at an angle of $45^\circ$ in order to make the distance at which the particle returns to the launching elevation a maximum. The horizontal range is the same when a particle is projected at each of two angles with a sum of $90^\circ$. These problems can be solved by physical consideration and simple arithmetic without reliance on differential calculus and integration calculus, trigonometric identities, and analytical geometry. Students can be convinced at a glance by drawing the diagrams for illustrating the maximum of $\sin \theta \cos \theta$ at $\theta = 45^\circ$ and $\sin \theta \cos \theta = \sin \phi \cos \phi$ at $\theta + \phi = 90^\circ$.

From the pedagogical viewpoint of physics education, improving students’ understanding of physics is more important than the ability to perform basic mathematical operations. Mathematical intuition cannot be cultivated only by gaining and improving the ability to calculate. As Nelsen [2] has emphasized, elegant visual demonstrations of mathematical ideas stimulate and encourage the understanding of the meanings behind mathematical theorems. The popularization of visual thinking using pictures and diagrams will activate mathematical education. A visual understanding of mathematical theorems is also advantageous for physics education, because students cannot perceive the essence of physical phenomena, being inundated by a flood of equations. The combination of mathematics and physics in the classroom produces good results. Visual thinking for mathematics is also effective for elementary physics, although Nelsen [1] did not describe the application of visual thinking for mathematics to elementary physics. However, the area of application of visual thinking for mathematics can be expanded, if diagrams used in proofs without words are utilized for other fields. We can expect the educational effect that training in geometry can develop academic ability in physics.