Dynamics of skew tent map in nonlinear Frobenius-Perron equation

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A globally coupled map lattice

\[ x_{n+1}^{(j)} = (1 - \epsilon) f(x_n^{(j)}) + \epsilon h_n \quad (j = 1, 2, \ldots, N), \]  

is one of the representative models to express chaos network. Here \( n \) represents a discrete time step, \( j \) the index of the elements, \( f \) denotes the chaotic single-site map, and \( \epsilon \) the coupling constant. The coupling is brought about by the mean field

\[ h_n = \frac{1}{N} \sum_{i=1}^{N} f(x_n^{(j)}). \]  

In the limit of large system size, i.e. \( N \to \infty \), the ensemble governed by Eq.(1) can be characterized by its probability distribution density \( \rho_n(x) \), whose evolution obeys the nonlinear Frobenius-Perron equation

\[ \rho_{n+1}(x) = \int \delta(x - F_n(y)) \rho_n(y) dy, \]  

where the mean-field map \( F_n \) is given by

\[ F_n(x) = (1 - \epsilon) f(x) + \epsilon h_n, \]  

and the mean field \( h_n \) is calculated from

\[ h_n = \int f(x) \rho_n(x) dx. \]  

As chaotic map \( f \), we adopt skew tent map

\[ f(x) = \begin{cases} 
  x/p & (0 \leq x \leq p) \\
  (1-x)/(1-p) & (p < x \leq 1)
\end{cases} \]  

Because this map has two regions where the slope of absolute value is different, it possesses more complicated dynamical system than the tent map. We compare the return maps of the mean field obtained from Eq.(3) with those from Eq.(1). The latter is noisy state of former (Fig.1(a) and (a')), but there are the cases which is not slightly so (Fig.1(b) and (b')). It is found that there are domains of the return map depending on the initial conditions for some coupling strength.

![Fig. 1: The return map of the mean field](image-url)