International Conference on Cosmic Rays and the Earth Storm

II-1. Hydromagnetic, Geomagnetic Rapid Variation, Whistlar and VLF Emission

A. Hydromagnetic

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Date	Time	Paper Numbers
Sept. 4	15 : 30–17 : 30	from II–1A–1 to II–1A–3
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INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part II

II-1A-1. Hydromagnetic Waves in the Exosphere

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We study the reflection and refraction of hydromagnetic waves, under the physical condition in the exosphere, at a plane boundary between two semi-infinite homogeneous media under a uniform (main) magnetic field, and apply the results to the propagation of hydromagnetic waves in the exosphere. The transverse (Alfvén) wave propagates in a complicated way, depending on the direction of the main field with an anisotropic velocity, but the isotropic (modified Alfvén) wave propagates in a simple way, independent of the direction of the main field, with an isotropic velocity. In the equatorial region, waves of both type propagate along the main field and can not propagate directly to the lower atmosphere.

§1. Introduction

The previous work on the reflection and refraction of hydromagnetic waves (Namikawa, 1961)[1] is extended and applied to the propagation of hydromagnetic waves in the exosphere. The impact of solar wind on the earth's dipole field may produce disturbances on the surface of magnetic cavity. The disturbances may propagate as hydromagnetic waves through the earth's atmosphere under geomagnetic dipole field. Therefore it is interesting to investigate the propagation of hydromagnetic waves in a partially ionized gas under a dipole field. But as it is mathematically difficult, we study here, as a first step, on the reflection and refraction of weak hydromagnetic waves at a plane boundary between two semi-infinite homogeneous media under a uniform magnetic field and discuss the propagation of hydromagnetic waves in the exosphere.

§ 2. Transmission Equations

Above several hundreds *Km*, (this height depends on the period of waves), the Joule dissipation and the Hall effect terms in the equation of motion may be neglected. As the gas pressure in the exosphere is small compared with the magnetic pressure, the gas pressure gradient term in the equation of motion is also neglected. Therefore the linearized hydromagnetic equations in the exosphere are as follows (Piddington, 1959)[2]

$$\frac{\partial \boldsymbol{h}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{H}_0)\,, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t}=\nabla\times\boldsymbol{h}\times\frac{\boldsymbol{H}_0}{4\pi\rho_0}\,, \tag{2}$$

$$\rho_0\nabla\cdot\boldsymbol{v}+\frac{\partial\rho}{\partial t}=0\,, \tag{3}$$

where $\boldsymbol{h}, \boldsymbol{v}, \boldsymbol{H}_0, \rho_0$ and ρ are the perturbed magnetic field, velocity, a dipole field, (we take it here as a uniform field for the simplicity of computation), density and perturbed density. Plane solutions of equations (1)—(3) are sought of the form $\exp i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})$, where $\boldsymbol{k}$ is the wave normal. From the equations (1)—(2), we have

$$\frac{\omega^2}{k^2}=V^2S^2\,, \tag{4}$$

where

$$S=\cos\psi \quad \text{or} \quad 1\,, \tag{5}$$

and

$$V=\frac{H_0}{\sqrt{4\pi\rho_0}}$$

is the Alfvén wave velocity.

There are two types of solutions of the equations (1)—(2), having quite different properties. One type of hydromagnetic wave has the perturbation magnetic field and the gas velocity vector in the plane containing the uniform field $\boldsymbol{H}_0$ and wave normal $\boldsymbol{k}$. This wave is called isotropic wave (or called extraordinary wave by Piddington) and propagates with the Alfvén wave velocity

$$V=\frac{H_0}{\sqrt{4\pi\rho_0}}\,.$$

The energy flow is along the wave normal. The second type of wave is purely transverse (called ordinary wave by Piddington) and propagates with velocity $V\cos\psi$ where ψ is the

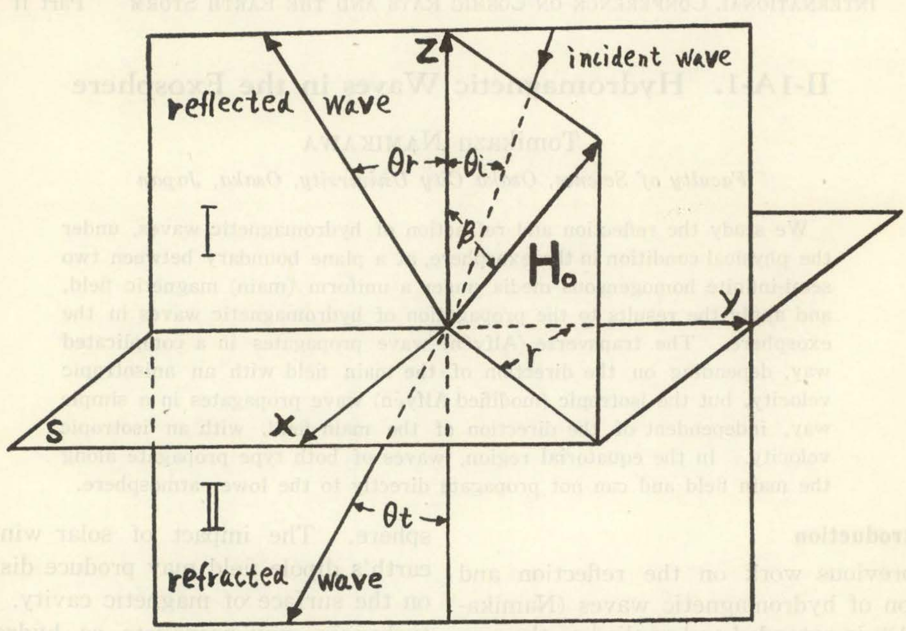

Fig. 1. Illustrating the reflection and refraction of Alfvén waves at the plane surface S.

angle between the uniform field $\boldsymbol{H}_0$ and wave normal $\boldsymbol{k}$. Energy flow is directed along the uniform field $\boldsymbol{H}_0$.

§ 3. Reflection and Refraction of Plane Waves

Let axes O_x, O_y and O_z be chosen in the way shown in Fig. 1. Let $\boldsymbol{H}_0$ make an angle of β with O_z and its projection on O_{xy} make an angle of γ with O_y. Denote the angle of incidence, reflection and refraction by θ_i, θ_r, and θ_t, respectively. Without loss of generality, β and γ may be taken to lie in the range 0 to $\pi/2$.

We denote Alfvén wave velocity V_{I} and V_{II} in the region I and II in Fig. 1. We have the laws of reflection and refraction and the amplitude relation between the reflected or refracted wave and the incident wave as a consequence of the boundary conditions (continuity of the perturbed magnetic field, tangential electric field and the normal component of the velocity at the boundary surface) which must be satisfied at any time and at any point of the boundary surface. The laws of reflection and refraction are found in the previous paper (Namikawa, 1961)[1] and reproduced here.

a) Incident transverse wave and reflected or refracted transverse wave.

$$\tan\theta_r=\tan\theta_i(1+2\tan\beta\cos\gamma\tan\theta_i)^{-1}, \tag{6}$$

$$\tan\theta_t=\tan\theta_i\left\{\frac{1}{R}+\left(\frac{1}{R}-1\right)\cos\gamma\tan\beta\tan\theta_i\right\}^{-1}, \tag{7}$$

where

$$R=\frac{V_{\mathrm{II}}}{V_{\mathrm{I}}}.$$

b) Incident isotropic wave and reflected or refracted isotropic wave.

$$\theta_r=\theta_i \tag{8}$$

$$\sin\theta_t=R\sin\theta_i \tag{9}$$

c) Incident transverse wave and reflected or refracted isotropic wave.

$$\sin\theta_r=\tan\theta_i(\sin\beta\cos\gamma\tan\theta_i+\cos\beta)^{-1} \tag{10}$$

$$\sin\theta_t=R\tan\theta_i(\sin\beta\cos\gamma\tan\theta_i+\cos\beta)^{-1} \tag{11}$$

d) Incident isotropic wave and reflected or refracted transverse wave.

$$\tan\theta_r=\cos\beta\sin\theta_i(1+\sin\beta\cos\gamma\sin\theta_i)^{-1} \tag{12}$$

$$\tan\theta_t=R\cos\beta\sin\theta_i(1-R\sin\beta\cos\gamma\sin\theta_i)^{-1} \tag{13}$$

The perturbed magnetic field, velocity and electric field are given as follows:

$$\begin{aligned} h_y&=\frac{S^2-(\cos^2\phi+\sin^2\beta\sin^2\gamma)h_x}{\sin\beta\sin\gamma(\sin\beta\cos\gamma-\cos\beta\tan\theta)}\\ &=\frac{-\sin\beta\sin\gamma(\cos\phi\sin\theta-\sin\beta\cos\gamma)h_x}{S^2-\{(\cos\phi\cos\beta/\cos\theta)-\sin\beta\sin\gamma(\cos\beta\tan\theta-\sin\beta\cos\gamma)\}}, \end{aligned} \tag{14}$$

$$\left.\begin{aligned} v_x&=\mp\frac{V\cos\phi}{H_0S}h_x, \quad v_y=\mp\frac{V\cos\beta}{H_0S\cos\theta}h_y\pm\frac{V}{H_0S}\sin\beta\sin\gamma\sin\theta h_x,\\ v_z&=\pm\frac{V}{H_0S}\left(\sin\beta\sin\gamma\cos\theta h_x+\frac{\sin\beta\cos\gamma}{\cos\theta}h_y\right), \end{aligned}\right\} \tag{15}$$

$$\left.\begin{aligned} E_x&=\pm\frac{V}{S}\left\{\sin\beta\sin\gamma(\sin\beta\cos\gamma\cos\theta-\cos\beta\sin\theta)h_x+\frac{\cos^2\beta+\sin^2\beta\cos^2\gamma}{\cos\theta}h_y\right\},\\ E_y&=\mp\frac{V}{S}\left\{(\cos\beta\cos\phi+\sin^2\beta\sin^2\gamma\cos\theta)h_x+\frac{\sin^2\beta\sin\gamma\cos\gamma}{\cos\theta}h_y\right\}, \end{aligned}\right\} \tag{16}$$

where $S=\cos\psi$ for the transverse wave, and $S=1$ for the isotropic wave. The reflected waves take the upper sign, while the incident or the refracted waves take the lower sign.

When the main field H_0, wave normal and the normal at the boundary are not in the same plane, the law of reflection and the refraction and the expressions of amplitude of reflected or refracted waves are complicated. An incident wave of one type is reflected and refracted in the sum of waves of two mode. This case will be discussed in another paper.

When the uniform magnetic field is perpendicular to the boundary plane, an incident wave is reflected or refracted in the same mode. The transverse wave has the components h_x, v_x, and E_y. The boundary conditions give the following equations:

$$h_{xr}=\frac{1-R}{1+R}h_{xi}\,, \tag{17}$$

and

$$h_{xt}=\frac{2}{1+R}h_{xi}\,, \tag{18}$$

where i, r and t denote incident, reflected and refracted wave respectively. From these equations, we get

$$\left|\frac{h_{xt}}{h_{xr}}\right|>1\,, \quad \text{when} \quad 3\geq R\,, \tag{19}$$

and

$$\left|\frac{h_{xt}}{h_{xr}}\right|<1\,, \quad \text{when} \quad R>3\,. \tag{20}$$

The isotropic wave has the components h_y, v_y and E_x. We have from the boundary conditions,

$$h_{yr}=\frac{(\cos\theta_t/\cos\theta_i)-R}{(\cos\theta_t/\cos\theta_i)+R}h_{yi}\,, \tag{21}$$

and

$$h_{yt}=\frac{2(\cos\theta_t/\cos\theta_i)}{(\cos\theta_t/\cos\theta_i)+R}h_{yi}\,. \tag{22}$$

From these equations, we get the same relations (19) and (20).

When the uniform magnetic field vector lies in the plane of incidence, an incident wave is reflected or refracted in the same mode as in the previous case. The equations (17)—(22) hold also in this case. Therefore the amplitudes of reflected or refracted transverse wave are independent on the directions of the uniform field and of the incident wave, but they depend only on the ratio R of the Alfvén wave velocities of both region.

When the uniform magnetic field vector is parallel to the boundary surface, an incident wave of both type propagates along the boundary surface when $R>1$.

§ 4. Propagation of Hydromagnetic Wave in the Exosphere

We discuss the reflection and refraction of hydromagnetic waves in the upper atmospheric region (Piddington 1959)[2], where the disturbances travel as hydromagnetic waves in the ion plasma alone. Losses are small in this region. In the lower atmospheric region, the medium behaves for waves of all periods between 1 sec and 10^4 sec as a rigid conductor and as a dispersive medium. As this model studied here is simple, these results obtained above can not be applied directly to the propagation of hydromagnetic waves through the exosphere. But we may say following several remarks from the results obtained above. In the exosphere, the variation of V (Alfvén wave velocity) with altitudes (Dessler and others 1960)[3] is appreciable within the wave length of hydromagnetic waves with the period of several tens of second. Therefore, application of the results to the propagation of hydromagnetic waves in the exosphere is possible only for the waves which have smaller period than that of several tens of second. The result is mainly applicable to propagation of hydromagnetic waves in a meridian plane. In the polar region, the main field will be nearly perpendicular to the boundary surface. The laws of reflection and refraction of transverse wave are simple (Namikawa, 1961)[1]. But in the middle latitudes, where the main field makes an angle with the boundary surface, the transverse wave propagates in a complicated way. The isotropic wave propagates in a simple way in both region. But an incident isotropic wave which has a larger incident angle than that, determined by the equations $|\sin\theta_i|=1/R$ can not be refracted. In the exosphere the magnitude of Alfvén wave velocity has a maximum at the altitude of about 2000 km. Therefore above 2000 km, waves of both type may be reflected intensely (*cf.Eqs.*(19) and (20)). In the equatorial region, the main field is nearly parallel to the boundary surface. An incident wave of both type propagates along the boundary and go into higher latitudes. Therefore the waves can not propagate directly to the lower atmosphere. In the polar region, both type of waves can propagate directly to the lower region, so that we may observe large geomagnetic disturbances in the polar region. Isotropic wave propagates in a simple way, independent on the direction of the main field, with an isotropic velocity, so that this waves may be responsible for the disturbance of world wide character, while the transverse wave propagates in a complicated way, dependent on the direction of the main field, with an anisotropic velocity, so that this wave may be mainly responsible for the geomagnetic disturbances of local character.

References

1) T. Namikawa: J. Geomag. Geoelect. **12** (1961) 117.
2) J. H. Piddington: Mon. Not. R. Astr. Soc. **119** (1959) 173.
3) A. J. Dessler, W. E. Francis and E. N. Parker: J. Geophys. Res. **65** (1960) 2715.

Discussion

Cole K.D.: The names extraordinary and ordinary have been applied by Astrom to hydromagnetic waves in one way and by other authors in the opposite way. An analysis I have made of the transition from hydromagnetic to electromagnetic waves in a fully ionised gas show that Astrom's labelling is consistent with the well established labelling for radio waves. For consistency authors should use Astrom's system of labelling.

Namikawa, T.: I agree with you.