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INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part II

II-2-7. Störmer's Inner Allowed Regions and the Radiation Belts*

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§ 1. Introduction

This work was undertaken in the hope that the study of the inner allowed regions,[1] deduced from Störmer's theory, by now classical, can shed some light on the properties on the trapping mechanism that operates within the radiation belts. It is generally accepted that the radiation belt particles follow trapped orbits which lie entirely within the inner allowed regions. It is essential to recall that these zones are characterized by Störmer's constant of motion gamma. It will be seen that in the radiation belts this constant provides a criterion for the validity domain of the adiabatic condition, as far as the mirror latitude is concerned.**

§ 2. Gammas and Distances

Using Van Allen's energy spectrum of both protons and electrons trapped at the hearts of each of the radiation belts, the gammas corresponding to the inner allowed regions centered at the hearts of each radiation belt were calculated (Fig. 1).

The values of γ corresponding to the trapped particles are very high and of the order of one hundred for the trapped electrons.*** For large values, γ is approximately equal to $1/2r$; where r is the distance in Störmer units from the dipole to the trapping line of force. Consequently γ decreases with the increase of energy and distance.

The distances in Störmer vary between .002 and .08 for the trapped electrons and .08 and .33 for the trapped protons.

Following Alfvén[2] the perturbation method and the adiabatic condition hold for $r<0.1$. It will be shown that even for the larger values of r that occur for protons the particles will reflect at the adiabatic mirror latitude λ_r.

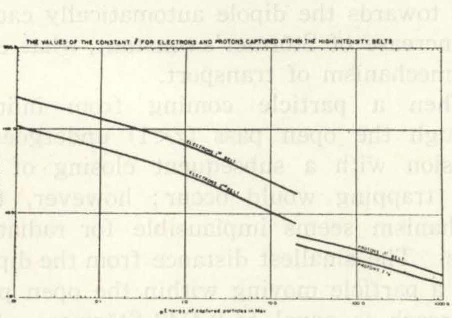


Fig. 1. Values of the constant gamma corresponding to particles trapped at the center of the first and second radiation belts.

§ 3. Width of the Zone. Homogeneity

The width of the inner allowed regions at the equator, in Störmer units, was calculated (Table I). The variation of the field across the zone at the equatorial plane was studied by comparing the radii of curvature corresponding to the opposite end points of the inner allowed region (Fig. 2).

For large values of γ the inner allowed regions are narrow and of many orders of

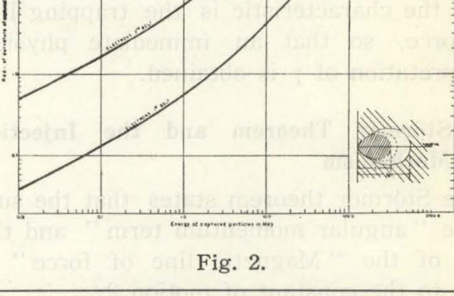


Fig. 2.

* Work supported by the Mexican National Commission of Nuclear Energy.

** The adiabatic mirror latitude λ_r is given by $(\sin^2\alpha/B)=$cst. where α is the pitch angle, B the geomagnetic field.

*** The geomagnetic theory of cosmic rays deals with values of gamma that differ only slightly from unity.

Table I. Width of the Inner Allowed Regions and the Alfvén Discriminant.

Electrons					Protons		
	1st Belt		2nd Belt			1st Belt	
ε Mev	(r_2-r_1) km	$\Delta r/2r$	(r_2-r_1) km	$\Delta r/2r$	ε Mev	(r_2-r_1) km	$\Delta r/2r$
.01	.08	3.9×10^{-6}	.94	2.1×10^{-5}	20	147	.008
.02	.11	5.6×10^{-6}	1.34	3.0×10^{-5}	30	180	.009
.05	.18	9.1×10^{-6}	2.16	4.8×10^{-5}	50	233	.012
.10	.25	1.3×10^{-5}	3.10	6.9×10^{-5}	75	288	.015
.20	.37	1.9×10^{-5}	4.57	1.0×10^{-4}	100	335	.017
.60	.74	3.8×10^{-5}	9.20	2.0×10^{-4}	500	819	.042
1.00	1.07	5.5×10^{-5}	13.12	2.9×10^{-4}	1000	1270	.065
10.00	7.88	4.1×10^{-4}	98.21	2.2×10^{-3}			
20.00	15.52	8.0×10^{-2}	188.50	4.2×10^{-3}			

$\Delta r=r_2-r_1$ is the width of the inner region at the equator. For large values of gamma $\Delta r/2=\rho$ where ρ is the radius of curvature; hence $3(\Delta r/2r)$ is the Alfvén discriminant.

magnitude smaller than the distance from the centre of the zone to the dipole. The width Δr of the inner allowed region at the equator is approximately equal to $1/4\gamma^3$ and Alfvén's[2)] discriminant to $3(\Delta r/2r)$, since for small energy of trapped particles (large γ) the radii of curvature are the same across the zone and equal to the half width which is $1/8\gamma^3$.

§ 4. The Characteristic Line of Force

A line of force, given by the equation

$$\cos^2\lambda/r=2\gamma \tag{1}$$

lies entirely within the inner allowed zone; we shall call this line "the characteristic line of force."

The reciprocal of γ is then equal twice the distance from the dipole to the characteristic line of force at the equator, and this may be used as a definition of Störmer's constant for the cases of $\gamma>1$. In these cases the characteristic is the trapping line of force, so that an immediate physical interpretation of γ is obtained.

§ 5. Störmer Theorem and the Injection Mechanism

The Störmer theorem states that the sum of the "angular momentum term" and the term of the "Magnetic line of force" is equal to the constant of motion 2γ:

$$r\cos\lambda v_\varphi+\cos^2\lambda/r=2\gamma$$

for $r<1$ the angular momentum term becomes negligible with respect to the magnetic line term; consequently, *the particle remains in the vicinity of the characteristic line of force.*

The injection mechanism that brings the particle from the source to the radiation belts must *deposit* the particle in the vicinity of the trapping line.

The neutron albedo source, automatically, fulfils this condition as it "deposits" the particles where the decay into protons and electrons takes place.

A transport of the particles across the line of force could render the solar plasma cource effective for the radiation belts. The transport towards the dipole automatically causes an increase of Störmer's constant, what ever the mechanism of transport.

When a particle coming from infinity through the open pass ($\gamma<1$) undergoes a collision with a subsequent closing of the pass trapping would occur; however, this mechanism seems implausible for radiation belts. The smallest distance from the dipole that a particle moving within the open pass can reach is equal to 0.4142 Störmers. For the energies of the particles trapped within the radiation belts, this corresponds to several hundred thousand kilometers. Moreover, the collision would have to change the value of gamma in order to raise it to the values that prevail within radiation belts, which is very improbable. These arguments seem to exclude such a mechanism of trapping, and if the solar plasma is to contribute significantly to the radiation belts, some form of transport mechanism must be postulated.

§ 6. Orbits within the Inner Allowed Region

Some representative orbits of charged particles within the inner allowed regions and their dependence on gamma are shown in Figs. 3, 4, and 5. The orbits were calculated by numerical integration of Störmer's differential equations of motion of a charged particle within a dipole magnetic field. In Fig. 3 a magnetic reflection type orbit is

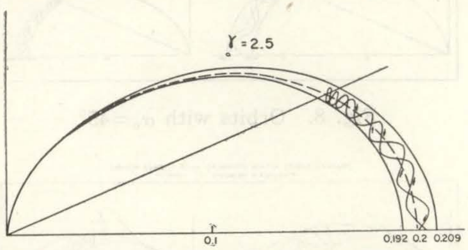
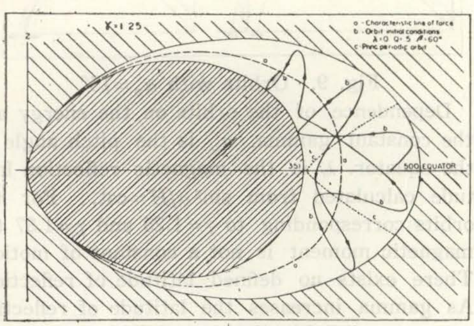


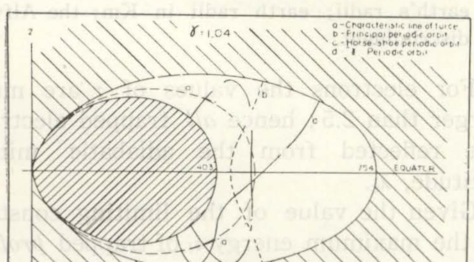
Fig. 3. Orbits within the inner allowed regions: $\gamma=2.5$. A typical magnetic-reflection type orbit for which the adiabatic condition holds. The dotted line is the characteristic line of force.



THE ORBITS WITHIN THE INNER ALLOWED ZONES

Fig. 4. $\gamma=1.25$

a) The characteristic line of force.
b) An orbit for which $\mu=$cte. It leaves the equator at Q=0.5 and pitch angle=60°.
c) A principal periodic orbit.



THE ORBITS WITHIN THE INNER ALLOWED ZONES

Fig. 5: $\gamma=1.04$

a) The characteristic line of force.
b) Principal periodic orbit.
c) Horse-shoe periodic orbit.
d) Gamma periodic orbit.

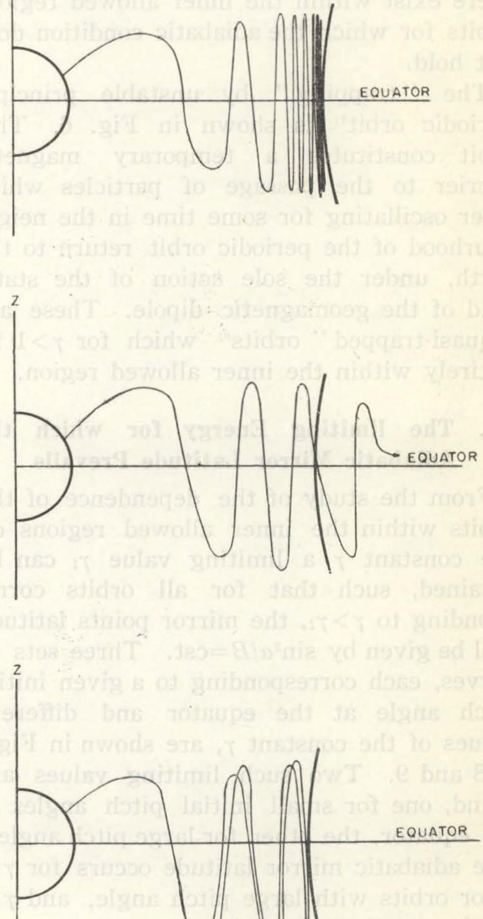


Fig. 6. Magnetic barrier: The effect of an unstable periodic orbit. In this schematic drawing the upper curve represents an orbit which leaves the earth and approaches asymptotically the unstable periodic orbit. The middle curve and the one below are orbits nearly asymptotic to the periodic orbit; after oscillating some time in the vicinity of the periodic orbit, the middle curve leaves to infinity ($\gamma<1$) and the other returns to earth ($\gamma\leqslant1$).

shown for $\gamma=2.5$; it is a typical orbit of radiation belt particles; it bounces back and forth between the narrow boundaries of the inner region with a radius of curvature equal approximately to the half-width of the region. It reflects at the latitude λ_r.

For smaller values of the constant gamma (see Figs. 4 and 5) different types of periodic orbits[3) 4) 5) 6)] appear that are not trapped by the characteristic line of force. Consequently, there exist within the inner allowed regions orbits for which the adiabatic condition does not hold.

The "trapping" by unstable principal periodic orbit[7)] is shown in Fig. 6. This orbit constitutes a temporary magnetic barrier to the passage of particles which after oscillating for some time in the neighbourhood of the periodic orbit return to the earth, under the sole action of the static field of the geomagnetic dipole. These are "quasi-trapped" orbits[8)] which for $\gamma>1$ lie entirely within the inner allowed region.

§ 7. The limiting Energy for which the Adiabatic Mirror Latitude Prevails

From the study of the dependence of the orbits within the inner allowed regions on the constant γ a limiting value γ_l can be obtained, such that for all orbits corresponding to $\gamma>\gamma_l$, the mirror points latitude will be given by $\sin^2\alpha/B=\text{cst}$. Three sets of curves, each corresponding to a given initial pitch angle at the equator and different values of the constant γ, are shown in Figs. 7, 8 and 9. Two such limiting values are found, one for small initial pitch angles at the equator, the other for large pitch angles. The adiabatic mirror latitude occurs for $\gamma\geq 2$ for orbits with large pitch angle, and $\gamma\geq 2.5$ for small pitch angles.

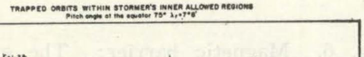


Fig. 7. Orbits with $\alpha_o=75°$

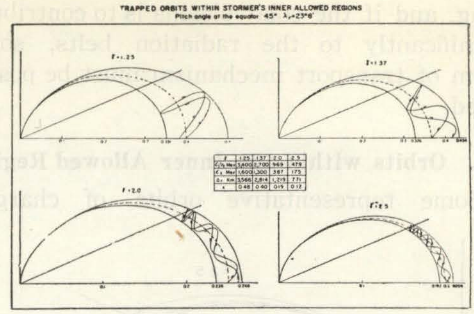


Fig. 8. Orbits with $\alpha_o=45°$

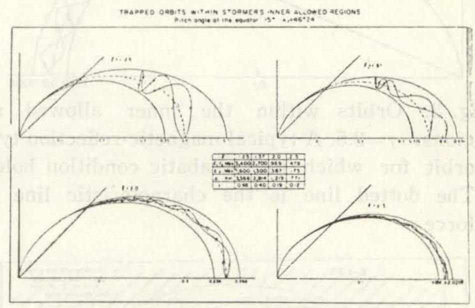


Fig. 9. Orbits with $\alpha_o=15°$

Dependence of the orbits on the energy and the constant gamma. α_o is the pitch angle at the equator. λ_r is the magnetic reflection latitude calculated from $\sin^2\alpha/H=\text{cst.}$. For the orbits corresponding to $\gamma=1.25$ and $\gamma=1.37$ the magnetic moment is not a constant of motion. There exists no defined latitude of reflection. As gamma increases the latitude of reflection increases until it reaches λ_r. The critical value of the constant gamma for which $\mu=\text{cte}$ is equal to 2 for large initial pitch angles and 2.5 for small initial pitch angles. The inset table contains the values of: energy of protons, trapped at one and a half ($\varepsilon 1\frac{1}{2}$) and two (ε_2) earth's radii; earth radii in Km; the Alfvén discriminant.

For electrons the values of γ are much larger than 2.5; hence *all* trapped electrons are reflected from the adiabatic mirror latitude, λ_r.

Given the value of the limiting constant γ_l the maximum energy ε_l of trapped *protons* which reflect at λ_r has been calculated (Fig. 10). This maximum energy varies inversely with the second power of the distance from the dipole. For $\gamma=2$, ε_l is of the order of 969 Mev at the centre of the first belt and 387 Mev at 2 earth's radii while for $\gamma=2.5$ the corresponding values are: 480 and 175 Mev.

Discussion

In this paper a criterion has been found for determining the limiting energies ε_l, of particles trapped in a dipole field, which reflect at the adiabatic mirror latitude λ_r given by $\sin^2\alpha/B=\text{cst}$.

A comparison with previously used criteria will be found in the following Table II: here r is the distance of the trapping line in Störmers; x is the limiting Alfvén discriminant; $\varepsilon 1\frac{1}{2}$, ε_2 are the maximum energies in Mev at one and a half and two earth's radii, for which the λ_r holds.

For particles of $\varepsilon>\varepsilon_l$ no well defined reflection zone exists, the magnetic reflection occurs at latitudes smaller than λ_r. Consequently trapped particles of larger energies will remain *closer to the equator* than the particles for which the adiabatic condition holds.

It is important to emphasize that particles with energies too large for the adiabatic condition to hold, *will still be trapped*.*

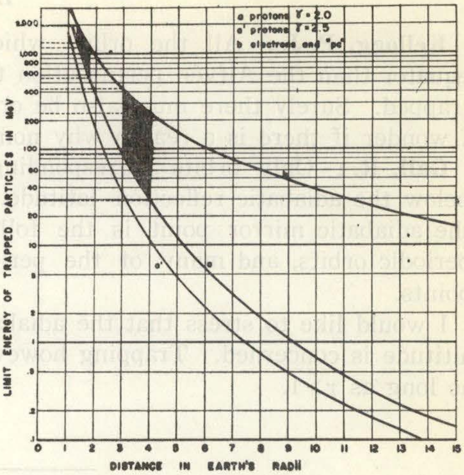


Fig. 10. Maximum energy of trapped particles for which the magnetic moment is a constant of motion is plotted *vs.* distance.
Curve (a) Maximum energy of trapped protons corresponding to $\gamma=2$.
Curve (a′) Maximum energy of trapped protons for $\gamma=2.5$.
Curve (b) Maximum pc and maximum energy of trapped electrons for $\gamma=2$.

Table II.

	Alfvén	Singer[8]	Present paper	
			$\gamma=2$	$\gamma=2.5$
r	.10	.16	.25	.20
x	.03	.076	.19	.12
$\varepsilon 1\frac{1}{2}$	37	221	969	480
ε_2	12	75	385	175

Following the Störmer theory, finite inner allowed regions exist, and consequently trapping will take place for all particles with energies corresponding to $\gamma\leq 1$.

For all trapped particles due to the finite width of the inner allowed regions, the radius of curvature at both sides of the inner allowed regions will not be the same, the orbits will not be symmetric with respect to the equator; changing of the latitude of the mirror points will result.

A study of the *long-term stability* of the orbit must be made before we can be sure that the particles will finally end up in the lower altitudes where collisions will remove them from the trapping regions.

* These particles will however be more easily dispersed by magnetohydrodynamic perturbation.

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Discussion

Kellogg, P. J.: All the orbits which you showed turned around nearer to the equator than the Alfvén perturbation theory would predict, that is, they were better trapped. Surely there must also be orbits which turn farther from the equator, and I wonder if there is a reason why none of these were found.

Gall, R.: Only orbits corresponding to particles of higher energy ($\gamma<2$) reflect below the adiabatic reflection latitude. The reason why these orbits reflect below the adiabatic mirror point is the following: these orbits are nearly asymptotic to periodic orbits, and many of the periodic orbits never reach the adiabatic mirror points.

I would like to stress that the adiabatic conditions guides us as far as the mirror altitude is concerned. Trapping however occurs also for energies for which $\mu\neq$cst, as long as $\gamma>1$.

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II-2-8. Mechanism of the Mirror Point Loss

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In cosmic ray physics various problems are concerned with the behavior of charged particles in a magnetic field, *i.e.*, formation and stability of the radiation belts, the observed anisotropy of the heavy primaries[1], and so on.

We have made a simple model experiment[2] in order to get characteristic information about the loss mechanism of electrons from a magnetic bottle. There may be several causes to make a trapped electron escape, for example, (i) the scattering with the residual gas atoms[3], (ii) the energy loss due to ionization (iii) the break-down of the adiabatic invariance[4,5]. The purpose of our experiment is to distinguish them from one another with varying conditions and to find the explanation of the cosmic ray phenomena. Here we give some of the preliminary results so far obtained.

As shown schematically in Fig. 1, a pair of air core coils are connected coaxially by a cylindrical brass vacuum chamber, an electron gun is set near the center of the chamber, and a collimated electron beam of 1.5

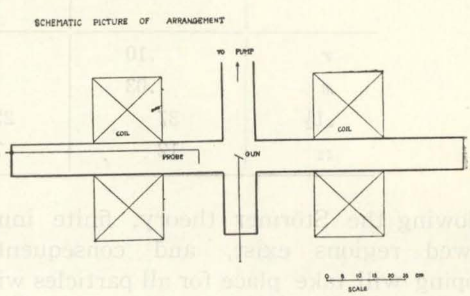

Fig. 1. Schematic picture of arrangement.

Kev is injected with a pitch angle $\alpha_0=30°\pm5°$. It is very important to make the electron gun as small as possible for having definite injection conditions, for minimizing the shadow effect, and for avoiding the effect of the magnetic field in an accelerating section. The gun used is of $\sim$1 cm$\times$1 cm ϕ and gives a current of the order of mA. Effects of space charge or plasma motion in the chamber are negligible. Though both of the stationary and pulsive injections are possible, most parts of observations have been done with the pulse of 5 μsec duration. Currents