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INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part II

II-4-18. Cosmic Ray Threshold Rigidities

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§1. Computation of Threshold Rigidities

In this report we present some calculations of threshold rigidities embodying recent work on the effect of the non-dipole field which represents, we believe, an improvement over the original Quenby and Webber[1] approximation. A more detailed account of the methods of deriving these values will be given later. No attempt is made to include the effect of fields of external origin in the calculations.

At high latitudes, $\lambda > 40°$, we have made use of an integration by Hultqvist[2] of the paths of magnetic field lines in a field given by the first five spherical harmonic terms based on the 1945 geomagnetic survey. Hultqvist gives the position at the earth's surface of both the actual field line and the dipole field line from which it was perturbed by the non-dipole part of the field. Now close to the earth, the trajectories of low rigidity particles spiral around the lines of magnetic force. As indicated by Quenby and Webber, once we know the paths of the field lines for the actual magnetic field we can use this property of the trajectories to give us a modified Störmer threshold rigidity. Briefly, we assume that the threshold rigidity, P_m, at the intersection point of the actual field line with the surface is the Störmer value corresponding to the geomagnetic latitude of arrival, $\bar{\lambda}$, of the original dipole line in the unperturbed field.

$$P_m = \frac{M}{4r_e^2} \cos^4 \bar{\lambda} \tag{1}$$

At low latitudes, Quenby and Webber gave an approximate correction to the Störmer threshold rigidities but neglected possible penumbral effects. To determine the penumbral correction in the actual field of the earth, we have used the survey of the nucleonic component intensity carried out in the equatorial region at an altitude of 18,000 ft by the Chicago group (Katz, Meyer, Simpson[3]). Since there is no penumbra in the vertical direction at the dipole equator, we can get the true relation between the cosmic ray intensity and the vertical threshold rigidity from the longitude effect at $\lambda=0$ using the modified Störmer thresholds of Quenby and Webber. Then by comparing these modified Störmer thresholds at other latitudes with those at positions of equal cosmic ray intensity on the dipole equator, we obtain the penumbral correction as a function of latitude. To a first approximation these corrections agree with the dipole calculations of Vallarta and others (*e.g.* Schwartz[4]). There are, however, significant deviations, due presumably to the non-dipole field. We have used the nucleonic component measurements as being the most accurate means at our disposal to finally determine the threshold rigidities up to $\lambda=25°$. A fuller account of this work in the equatorial region will be published jointly with the Chicago group.

Schwartz has found a penumbral correction in the dipole field up to $\lambda=55°$. We have therefore added this dipole penumbra to the modified threshold rigidities, based on Hultqvist's work in the region above $\lambda=40°$ to obtain the final values.

In the intermediate region, $\lambda=25°$ to $\lambda=40°$, we have no easy way of allowing for the non-dipole field. We simply plot, as a function of latitude, the corrections to the dipole Störmer thresholds calculated by the high and low latitude methods and then we interpolate the corrections in the intermediate region.

Tables have been prepared giving the values threshold rigidity found in the manner described above at 2.5° intervals of geographic latitude, θ, up to 70° geographic and at 5° intervals of geographic longitude, φ. In the calculations, the geomagnetic coordinate system based on the 1945 magnetic survey has been used (Jory[5]) and the value taken for $M/4r_e^2$ was 14.9 GV.

§2. Investigation into Threshold Rigidities in the Geomagnetic Field Using an Analogue Computer

The world-wide surveys by many workers

* This paper was read by H. Elliot.

(Simpson[3] Sandström[6] Rothwell and Quenby[7]) have shown that the centred dipole, or even the eccentric dipole approximation to the geomagnetic field, is insufficient in the calculation of threshold rigidities. Quenby and Webber[1] have developed an approximate treatment for the calculation of threshold rigidities, in which they estimate the effects of higher order terms in the expansion of the geomagnetic potential. The values they obtain have been successful in explaining, for example, the latitude dependence of flare increases (Carmichael and Steljes[8]) and the position of the cosmic ray equator. Recently Quenby and Wenk (§ 1.) have improved on the original approximations by including corrections for the penumbra and by use of the computations by Hultqvist at high latitudes which allow a better determination of the effective latitudes $\overline{\lambda}$ (defined in the paper by Quenby and Webber[1]).

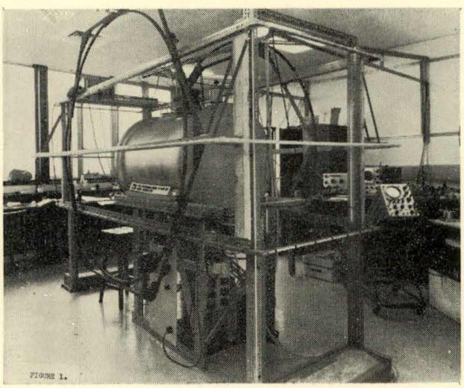

Fig. 1.

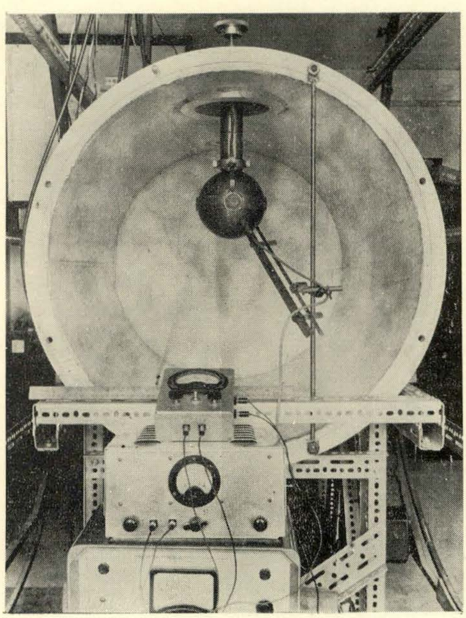

Fig. 2.

The penumbral corrections used by most workers in this field are based on the work of Schwartz[4] although Quenby, Wenk, Katz, Meyer, and Simpson[9] have, by interpolation, made estimates of the penumbra from airborne surveys (Katz, Meyer and Simpson[3]) in the region $\overline{\lambda}=\pm 30°$ (were λ is the geomagnetic latitude). Whilst no significant difference in the average behaviour of these penumbral corrections and Schwartz's predicted values is observed, there are large differences at individual points. The question arises whether these differences are due to the fact that the penumbral corrections are estimated by subtracting possibly incorrect "Quenby-Webber" threshold rigidities from the observed threshold rigidities, or whether they are due to the use of penumbral corrections calculated for a dipold field, when we know that the geomagnetic field deviates substantially from that of a dipole. The investigation of this latter problem, as well as the quantitative checking of the threshold rigidities predicted by Quenby and Webber, was the object of the experimental work to be reported here.

The problem of finding threshold rigidities is amenable to solution by means of an analogue computer. Such an analogue computer consists of a vacuum chamber in which is placed a small model of the geomagnetic field called a terrella. The cosmic radiation is simulated by a stream of electrons emitted from an electron gun mounted close to the surface of the terrella. The electron gun is fixed so that movement of the terrella is used to change its effective position.

The threshold rigidity was found when electrons were prevented from leaving the near environment of the terrella. This was seen as an increase in current from the gun to the surface of the terrella, or conversely a decrease in the current from the gun to the vacuum tank. When either of these current is plotted against the equivalent (or 'scaled') rigidity of the electrons a graph is obtained of the transparency of the geomagnetic field against rigidity of the incident radiation.

§ 3. Results Obtained with the Experiment

The preliminary experiments were all made using a centred dipole to represent the geomagnetic field. Fig. 3 shows a series of threshold rigidities obtained. It may be seen that, whereas the equatorial threshold is very sharp, above about latitude 15° there occurs a splitting of the threshold into two main decreases in percentage transparency, the region between these two main decreases exhibiting rather complex properties. This is fully in accord with the theory of the main cone and penumbra formulated by Lemaitre and Vallarta (Lemaitre and Vallarta[10]). Since the experiment averages over a finite spread in rigidities and solid angle, these pictures of the penumbra are probably nearer to the real case than that predicted by machine integrations of several particle orbits.

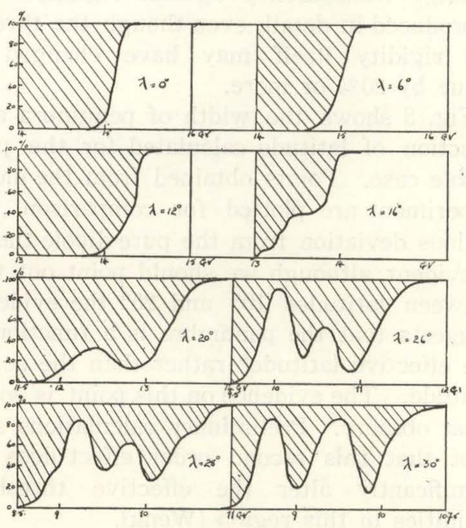

Fig. 3. Curves showing the percentage of primary cosmic radiation 'seen' at the surface of the earth assuming a centred dipole field, as a function of rigidity expressed in GV for several geomagnetic latitudes.

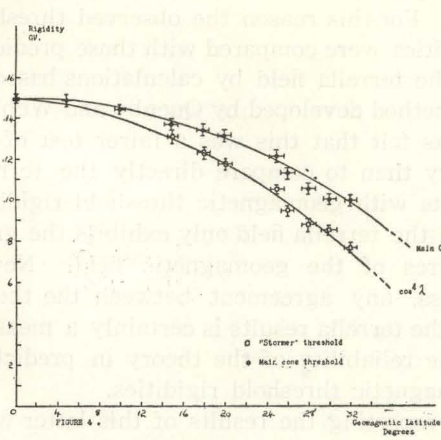

Fig. 4. Theoretical curves showing the expected variation of main cone and Störmer cone threshold rigidities with geomagnetic latitude, for a centred dipole field. Points obtained in the model experiment are plotted for comparison. Note: At low latitudes, where only one threshold is observed, it is assumed to be a "main cone" threshold.

Fig. 4 shows the variation of the main cone and Störmer cone threshold rigidities with latitude expected from theory together with the experimental points.

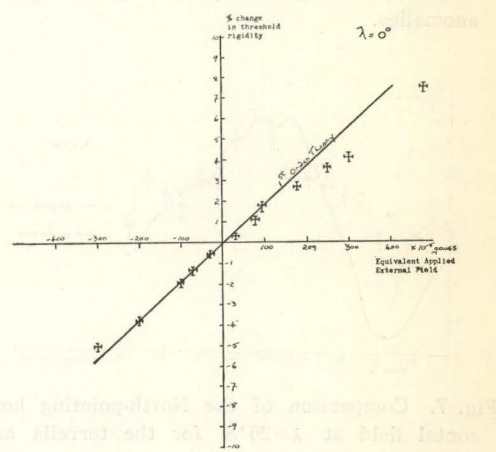

Fig. 5. Experimental points obtained from model experiment, using a centred dipole to represent the geomagnetic field. They show the change of threshold rigidity to be expected at the equator when an external uniform field is applied. The sense of the external field is taken to be positive when it is the same direction as the dipole. The straight line predicted by the first order theory is plotted for comparison.

Fig. 5 shows the change in the equatorial threshold rigidities observed upon the application of various external uniform fields parallel and antiparallel to the dipole axis. For comparison, the first order theory is plotted. The agreement is good up to fields equivalent to about $100\,\gamma$ in the terrestrial case.

Figs. 6 and 7 show typical samples of the

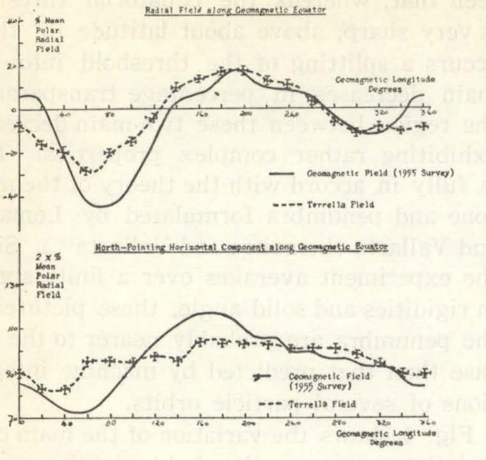

Fig. 6. Comparison of the model experiment approximation to the geomagnetic field generated by means of a centred dipole coil plus three radial dipole magnets representing the regional anomalies.

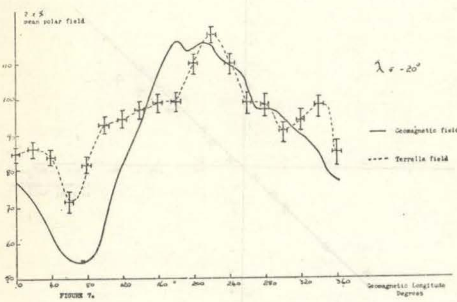

Fig. 7. Comparison of the North-pointing horizontal field at $\lambda=20°S$ for the terrella and the earth.

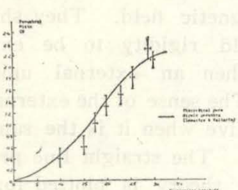

Fig. 8. The penumbral widths observed in the model experiment approximation to the geomagnetic field. Each point is the average of several longitudes. The deviation of the widths at each latitude does not exceed the s. d. of the measurements except at latitudes between $\lambda=10°$ and $\lambda=20°$. Points in this region are obtained from regions where the threshold is not significantly different from the centred dipole threshold rigidity.

degree of approximation to the geomagnetic field obtained with the terrella.

The modification of the terrella dipole field to include some of the more important anomaly fields did not substantially alter the penumbral features. In particular, the graphs obtained in the pure dipole case of the percentage transparency against rigidity were reproduced in detail, even though the threshold rigidity itself may have changed in value by 20% or more.

Fig. 8 shows the width of penumbra as a function of latitude calculated for the pure dipole case. Points obtained from the model experiment are plotted for comparison. No serious deviation from the pure dipole theory is evident, although we should point out that between latitudes 10° and 20° the evidence suggests that the penumbra is a function of the effective latitude* rather than the actual latitude. The evidence on this point is somewhat obscure. Preliminary calculations suggest that this second order effect does not significantly alter the effective threshold rigidities in this region (Wenk).

As regards the checking of the quantitative agreement between the calculations of Quenby and Webber and the observed threshold rigidities, the limitation of the model experiment in this context are the accuracy with which it is possible to "match" the geomagnetic field. For this reason the observed threshold rigidities were compared with those predicted for the terrella field by calculations based on the method developed by Quenby and Webber. It was felt that this was a fairer test of the theory than to compare directly the terrella results with geomagnetic threshold rigidities since the terrella field only exhibits the gross features of the geomagnetic field. Nevertheless, any agreement between the theory and the terrella results is certainly a measure of the reliability of the theory in predicting geomagnetic threshold rigidities.

In reporting the results of this latter work we shall use the term threshold rigidity to signify the lowest rigidity at which radiation is admitted, *i.e.* the Störmer cone threshold.

Fig. 9 shows the expected threshold rigidities along the terrella equator (analogous

* By effective latitude we mean that given by $p=14.9\cos^4\lambda'$ where p is the observed threshold rigidity and λ' is the effective latitude.

to the geomagnetic equator) together with the values found by experiment. It should be noted that the curve is perhaps 0.5 GV too low throughout, due to the fact that the equatorial dipole threshold was assumed to be 14.9 GV in the calculations whereas the experimental value obtained was 15.4±0.7 GV largely due to errors in measuring the radius.

Fig. 10 shows the threshold rigidities found plotted against the predicted threshold rigidities at several points on the terrella not along the equator. The line $y=x$ is plotted for comparison. Some of the points having low predicted values of threshold rigidity lie well off the line; this is probable due to the fact that the calculations in the particular region (λ=30°) where they were measured are subject to some of the worst approximations in the "Quenby-Webber" treatment and may be in error by ~10%.

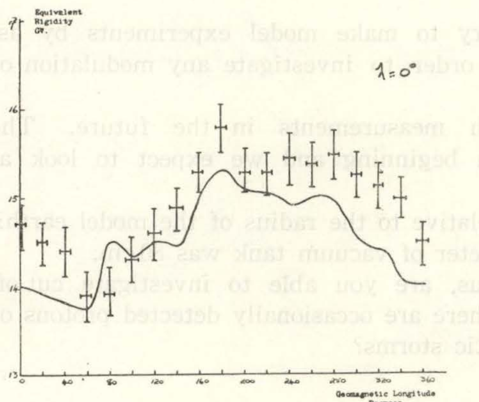

Fig. 9. Curve shows the expected equivalent threshold rigidities predicted by "Quenby-Webber" calculations for the terrella approximation to the geomagnetic field. Experimentally found values are plotted for comparison. The errors in the predicted thresholds probably amount to ~0.5 GV due to inaccuracies in the terrella field measurements and the errors inherent in the theoretical treatment.

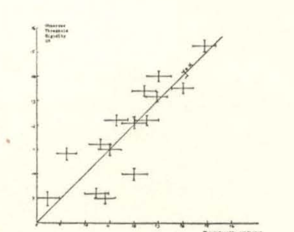

Fig. 10. The agreement between the thresholds predicted by "Quenby-Webber" calculations and those measured for the terrella field. The experimental points were obtained at many different latitudes and longitudes. For comparison the line $y=x$ is drawn.

§ 4. Conclusions

The three most important conclusions indicated by our experiments are as follow:-

1) The dipole field penumbra is still maintained in the geomagnetic field except between λ=10° and λ=20°. We suggest that this latter effect is due to the fact that here penumbral orbits are confined near to the equatorial plane and therefore pass over regional anomalies many times.
2) The threshold rigidities in the terrella field predicted by calculations using the treatment of Quenby and Webber do not significantly differ from those experimentally observed values. The r.m.s. difference observed was 6.2% between latitudes 10° and 30°. Along the equator r.m.s. difference was 2.1%.
3) The effect of an external uniform field is to increase the threshold rigidity at the equator if it is in the same sense as the dipole moment.

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Discussion

Swann, W. F. G.: I wish to warn against pitfalls resulting from expansions in spherical harmonics with the earth's centre as an origin. Taking an extreme case when a real dipole exists at the centre, and when another dipole exists 100 ft below the surface, a spherical harmonic analysis based on use of surface values of the field would give a first term representative of the centre dipole. Then there would follow about a million terms with coefficients essentially zero. Then the terms could be mixed up in such a manner as to reveal the second dipole and would finally form a convergent series.

It is possible to determine the field at any point in space by direct use of surface values without any spherical harmonic expansion in the case when there are no external currents.

Elliot, H.: Yes, I agree.

Kodama, M.: I would like to suggest to try to make model experiments by assuming ring current around the earth, in order to investigate any modulation of particle trajectories due to the ring current.

Elliot: It is intended to carry out such measurements in the future. The work reported in the present paper is only a beginning and we expect to look at many different field configurations.

Kondo, I.: What is the size of the tank relative to the radius of the model earth?

Elliot: Radius of terrella was 9 cm. Diameter of vacuum tank was 80 cm.

Simpson, J. A.: With your model apparatus, are you able to investigate cut-off at high latitudes when it is known that there are occasionally detected protons of sub-cut-off energies arising during geomagnetic storms?

Elliot: Yes. This will be done shortly.