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II-5-5. On the Streaming of a Plasma through the Geomagnetic Field*

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Alfvén's electric field theory of magnetic storms and aurorae is based on the assumption that the sun emits beams of ionized gas. The motion of such a beam through the interplanetary magnetic field is associated with an electric field across the beam. If the motion of such a beam, when it enters the geomagnetic field is analysed by means of the equations of motion for single particles, the results give a fairly good picture of magnetic storms and aurorae. As the motion of the particles in the equatorial plane produces space charges which are not taken into account these results are valid only if these space charges leave the equatorial plane along the magnetic lines of force. It is not evident, however, that the charged particles could leave the equatorial plane so fast that the electric field is influenced. Here we study the motion of charged particles in the equatorial plane under the assumption that no space charges leave this plane along the lines of force. The main results from this treatment agree qualitatively with Alfvén's results.

§ 1. Introduction

The electric field theory of magnetic storms and aurorae has been developed in a series of papers by Alfvén[1] and Block[2]. The theory is based on the assumption that the sun emits beams of ionized gas. The beam is thus emitted from a region where a magnetic field exists, and therefore should be magnetized. Such a beam can move across the magnetic field through the influence of the electric field from the polarization of the beam (Chapman and Ferraro[3]). If the motion of the charged particles in the equatorial plane is analysed by means of the equations of motion of solitary particles, it is found that there is a "forbidden region" around the dipole which the electrons can not reach. At the border of this region the electron density has a maximum. The motion perpendicular to the equatorial plane is oscillatory along the magnetic lines of force. If the amplitude of these oscillations is large enough, the electrons can hit the earth's surface. The line on which the largest number of electrons hit the surface of the earth is thus the one which is obtained if the border line of the forbidden region is projected along the magnetic lines of force upon the surface of the earth. This line is identified with the auroral zone. The variation of the polar distance of this curve is of the same type as the observed time variation of the magnetic polar disturbance and of the aurora.

However, for the above results to be true, it is necessary that there are no space charges in the equatorial plane large enough to give a significant change in the electric field. From the drift equation it is seen that the inertia term gives rise to large space charges, if the particle density is not very low. Thus it is necessary that the space charges leave the equatorial plane very quickly along the magnetic lines of force. This is hindered by the mirror effect of the magnetic field. Therefore, the assumption that the space charges do not affect the motion in the equatorial plane is not a very natural one. In fact, it has been argued that if the space charges are taken into account, the motion might be entirely different (Chapman[4] Cowling[5]).

Here, the particle motion in the equatorial plane is studied with the space changes taken into account, under the assumption that no space charges leave the equatorial plane along the lines of force. This, certainly, does not correspond very closely to reality. However, we thus get solutions for two

* This paper was read by H. Alfvén.

extreme cases, one, where all space charges leave the equatorial plane immediately (Alfvén's solution), and the other, where no space charges leave this plane (the solution presented here). The real situation should be somewhere between these two. Thus if the results for the two extreme situations do not differ too much, we will get quite a good picture of what happens in the real case.

Before presenting the theory we make a few remarks on the situation treated. The first one is that we treat only the stationary case, which means that we get information on the main phase of a magnetic storm only. The second remark is that we throughout neglect the induced currents, and treat the magnetic field as given. It turns out that, under these assumptions, it is essential for the existence of a solution that there is an interplanetary magnetic field. The motion for the case of no interplanetary magnetic field is completely different from the case of a small, but non-zero, magnetic field, and the neglect of this field is thus definitely not allowed. It is also essential that the particles in the beam should have a non-zero temperature.

The present investigation was started not to study magnetic storms, but to understand the behaviour of proposed thermonuclear device. Therefore, the magnetic field used is not a dipole field. The motion in a dipole field will be investigated in later paper*.

§ 2. Outline of the theory

We start with the assumption that the motion of a charged particle (or, more exactly, the motion of its guiding center) is given by the drift equation (Alfvén 1950)[1]:

$$\boldsymbol{u}=-\frac{\boldsymbol{B}}{eB^2}\times\left[e\boldsymbol{E}-\mu\nabla B-M\frac{d\boldsymbol{u}}{dt}\right] \qquad (1)$$

Here M is the mass of the particle, e its charge, and μ the magnetic moment. Now it is easily seen that if the last term (the inertia term) in (1) is neglected, then along a particle trajectory the particle density n will be proportional to the magnetic field strength B. If the inertia term is taken into account, the density will be different for electrons and ions. This means that we get space charges which modify the electric field. Therefore, we have to treat the problem in the following way. The electric field in (1) is left unspecified. We then assume that the inertia term gives only a small correction to the motion. This means that we can solve eq. (1) by making a formal expansion in powers of M/e. Here we take only the first and the second term into account. This gives a formal expression for the drift velocity, from which we can deduce an expression for the particle density. If we introduce the resulting charge density in the Poisson equation, we thus get a self-consistent solution.

* Preliminary results from this investigation show the same general character as for the field treated here.

§ 3. The particle density and the Poisson equation

In the equatorial plane the magnetic field is orthogonal to the plane. A beam of charged particles (ions and electrons of approximately the same density) is coming in from the sun, which is taken to be far away in the y-direction. The electric field in this beam is given by $E_x=-B_0U_y$, where U_y is the drift velocity, and B_0 is the interplanetary magnetic field. As the particles come from a region where a magnetic field exists, we assume that they have magnetic moments, μ_i for the ions, μ_e for the electrons.

In (1), we introduce a generalized potential ϕ, given by:

$$\left.\begin{aligned}\boldsymbol{E}&=-\nabla\varphi\\ \phi&=\varphi+\mu B/e\end{aligned}\right\} \qquad (2)$$

In the stationary case we have

$$\frac{d\boldsymbol{u}}{dt}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=\tfrac{1}{2}\nabla(u^2)-\boldsymbol{u}\times(\nabla\times\boldsymbol{u}) \qquad (3)$$

Introducing (2) and (3) in (1) we get

$$\boldsymbol{u}\left\{1+\frac{M}{eB^2}[\boldsymbol{B}\cdot(\nabla\times\boldsymbol{u})]\right\}=\frac{\boldsymbol{B}}{B^2}\times\nabla\left(\phi+\frac{Mu^2}{2e}\right) \qquad (4)$$

If you now put $\boldsymbol{u}=(\boldsymbol{B}/B^2)\times\nabla\phi$ in those terms of (4) that come from the inertia term we get, after some vector manipulation:

$$\boldsymbol{u}\left\{1+\frac{M}{eB^2}\left(\nabla^2\phi-\frac{1}{B}\frac{dB}{dr}\frac{\partial\phi}{\partial r}\right)\right\}=\frac{\boldsymbol{B}}{B^2}\times\nabla\left(\phi+\frac{M(\nabla\phi)^2}{2eB^2}\right) \qquad (5)$$

From (5) we see that $\boldsymbol{u}$ is orthogonal to the vector $\nabla\left(\psi+\frac{M(\nabla\psi)^2}{2eB^2}\right)$. Thus the charged particles move so that the quantity $\Psi=\psi+\frac{M(\nabla\psi)^2}{2eB^2}$ is a constant of the motion. If we now set up the expression for the number of particles streaming between two Ψ-lines, we get an expression for the particle density along a Ψ-line (for a detailed derivation see Karlson[6]), hereafter referred to as I):

$$n=n_0\frac{B(r)}{B_0}\left[1+\frac{M}{eB^2}\left(\nabla^2\psi-\frac{1}{B}\frac{dB}{dr}\frac{\partial\psi}{\partial r}\right)\right] \quad (6)$$

where n_0 is the particle density for $y=+\infty$. From (6) we get the charge density

$$\rho=e(n_i-n_e)=\frac{n_0}{B_0B}\left[M\left(\nabla^2\psi_i-\frac{1}{B}\frac{dB}{dr}\frac{\partial\psi_i}{\partial r}\right)-m\left(\nabla^2\psi_e-\frac{1}{B}\frac{dB}{dr}\frac{\partial\psi_e}{\partial r}\right)\right] \quad (7)$$

If we put this expression into the Poisson equation, if we neglect the electron mass, as compared to ion mass, and if we assume that the density is high in the sense that $n_0M/\varepsilon_0B_0B\gg1$, we get the equation

$$\nabla^2\psi_i-(1/B)\,(dB/dr)\,(\partial\psi_i/\partial r)=0 \quad (8)$$

This equation is separable, and its general solution is of the form

$$\psi_i=\sum_{m=0}^{\infty}R_m(r)\,[a_m\sin m\theta+b_m\cos m\theta] \quad (9)$$

Now it is easily seen that if there are no forbidden regions, so that $R_m(r)$ has to be regular at the origin and at infinity, then there is no solution for any value of m, except for $m=1$ (this solution is not regular at infinity, due to the existence of an electric field there). The equation for $m=1$ was solved numerically for a field of the type

$$\begin{cases}B(r)=B_0[1+(\alpha/2)(1+\cos\pi r/r_0)] & r\leq r_0\\ B(r)=B_0 & r>r_0\end{cases} \quad (10)$$

for different values of α (α=4, 9, 19 and 39). Figs. 1 and 2 show the resulting first order trajectories for ions and electrons, for the cases $\alpha=9$ and 39, respectively, with the value $\mu_iB_0/eE_0r_0=0.05$. In Figs. 3 and 4 we have plotted the electric field in the x-direction, and the resulting drift velocity on the y-axis.

When the drift velocities are known, it is possible to compute the resulting currents. Here we give only the expression for the current density. A detailed derivation is

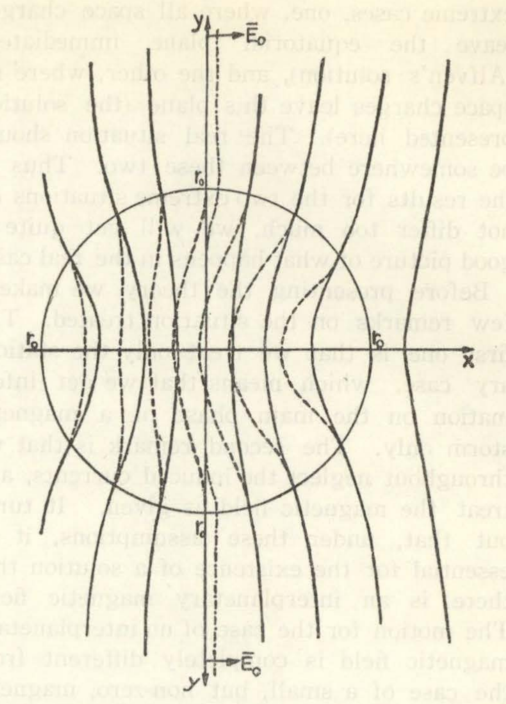

Fig. 1. Electron trajectories (broken lines) and first order ion trajectories (full lines) for the case $\alpha=9$. One second order ion trajectory is also shown.

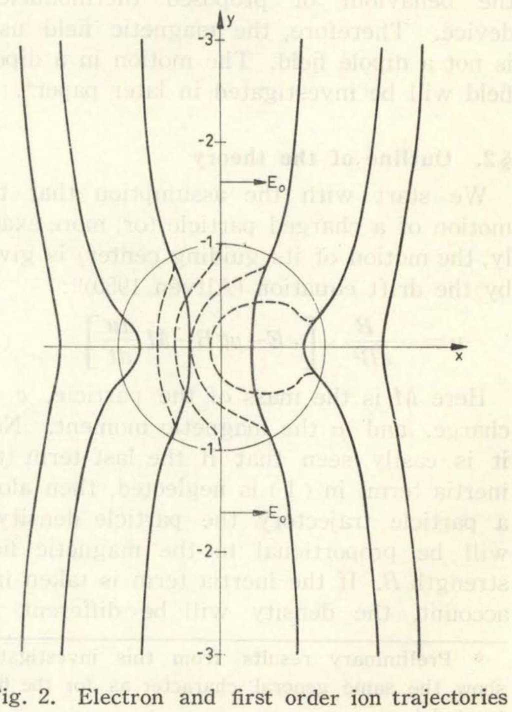

Fig. 2. Electron and first order ion trajectories for the case $\alpha=39$.

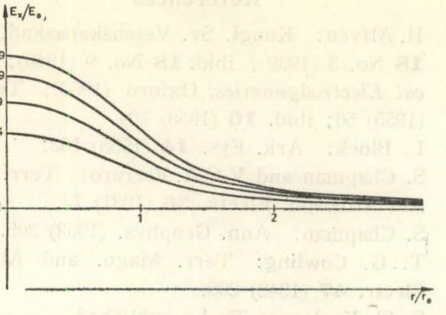

Fig. 3. The electric field in the x-direction on the y-axis for different values of α.

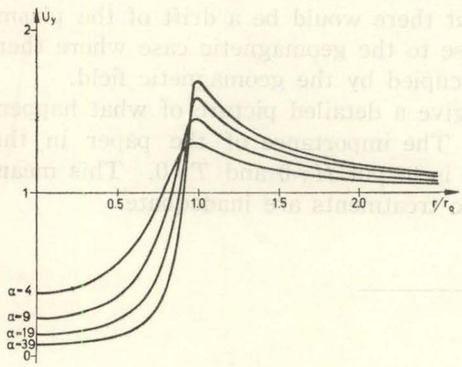

Fig. 4. The drift velocity in the y-direction on the y-axis for different values of α.

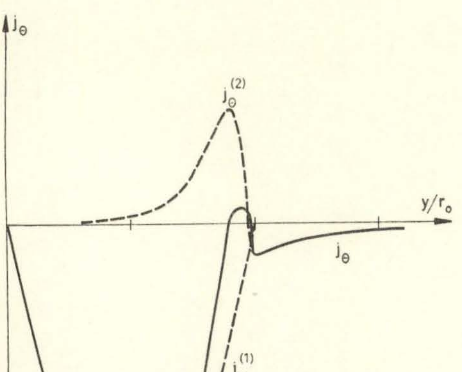

Fig. 5. The current distribution for the case $\alpha=9$ in units $Mn_0E_0^2/B_0^3r_0$.

given in I. We have

$$\boldsymbol{j}=\boldsymbol{j}^{(1)}+\boldsymbol{j}^{(2)} \tag{11a}$$

$$\boldsymbol{j}^{(1)}=\frac{2n_0(\mu_i+\mu_e)}{B_0}\,\frac{\boldsymbol{B}}{B}\times\nabla B \tag{11b}$$

$$\boldsymbol{j}^{(2)}=\frac{n_0M}{2B_0}\,\frac{\boldsymbol{B}}{B}\times\nabla\left(\frac{\nabla\psi_i}{B}\right)^2 \tag{11c}$$

In Fig. 5 we have plotted the θ-component of $\boldsymbol{j}^{(1)}$, $\boldsymbol{j}^{(2)}$ and $\boldsymbol{j}$ for $\theta=\pi/2$ in units $Mn_0E_0^2/B_0^3r_0$ for $\alpha=9$.

§4. Discussion

The problem treated here is certainly not a very good model of the phenomena associated with a magnetic storm. We have used a number of approximations, of which some, *e.g.* the use of the drift equation, the assumptions, of "high" density, and the neglect of collisions seem to be fairly good ones. However, we also neglected the influence of the induced currents, we used a field, which does not resemble the real field very much, and we have a solution only for the case that there are no forbidden regions. In a later paper we will treat the problem without all these bad approximations, but here we want to point out what conclusions can be drawn from our very crude model. The first one is that if the interplanetary magnetic field is taken to be zero, the electric field is zero at infinity, and *there is no solution at all.* Thus the existence of an interplanetary magnetic field is essential for the existence of a solution. If this field is taken into account, the particles can move into the stronger magnetic field even when the induced currents are not taken into account. If the interplanetary magnetic field is neglected, the beam can not move into the stronger field, but must push the field before it. Furthermore, it is necessary for the treatment here that the particles in the beam should have a non-zero temperature.

Thus we know that the polarized beam can penetrate into the strong magnetic field. We also know that there is a forbidden region if the inhomogeneity is strong enough (which is always the case for a dipole field, of course). The particle density is proportional to the magnetic field strength, and thus has a maximum at the border of the forbidden region. Furthermore, we get two different ring currents, one of which is caused by the density gradient, and tends to decrease the magnetic field. The other one is an inerita current, and its direction depends on the model used. All these facts agree with the result from the single particle treatment, and are essential for the explanation of magnetic storms and aurorae from the electric field theory.

References

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Discussion

Davis, Jr., L.: In your model I believe that there is an electric field at the center of the region of higher field. This means that there would be a drift of the plasma there. Thus this model is not particularly close to the geomagnetic case where there is very little drift over most of the region occupied by the geomagnetic field.

Alfvén, H.: The model is not meant to give a detailed picture of what happens in the earth's magnetic field during a storm. The importance of the paper in this connection is that it shows how essential it is to put $H \neq 0$ and $T \neq 0$. This means that the usual hydrodynamic or hydromagnetic treatments are inadequate.