JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 17, SUPPLEMENT A-III, 1962
INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part III

III-3-9. A General Approach to the Electromagnetic Origin of Cosmic Ray Energy

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§ 1. Introduction

Since no ordinary nuclear processes can give charged particles energies comparable with the energies of cosmic rays, it is natural to suppose that, in the last analysis, cosmic ray energies must, somehow or another, find their origin in the ordinary motions of neutral matter in bulk. In developing this view, however, we are immediately confronted with the following problem.

Suppose that any large mass of matter is moving so that its volume elements attain velocities even comparable with the velocity of light, but not so nearly equal to that velocity that the masses involved become relativistically affected. Then the energy per nucleon cannot become greater than m_0c^2, which amounts to about 10^9 ev per nucleon, if m_0 is the rest mass of the nucleon.

In order to account for cosmic ray energies, we must thus envisage a process in which a small amount of matter can steal, from a much larger amount of matter, energy which it stores up for the subsequent purpose of converting it into cosmic ray energies for its nucleons.

Now how can matter steal energy from other matter and store it up? The answer is to be found in processes by which the energy of motion or of compression resulting from motion can be responsible for the production of an electric current which then stores the energy in the form of a magnetic field. The process is of course illustrated in its most elementary form by a dynamo, run from the energy of a large fly wheel, the energy lost by the fly wheel being stored in an electromagnet, for example.

In what follows, I shall use the word Sun-spot. It is to be distinctly understood, however, that what I have to say is not necessarily bound up with actual Sun-spots or Star-spots. I simply use the word Sun-spot as a convenient means of referring to a place where a magnetic field is to be found.

It is not my present purpose to enter into the problem involved in the *process* by which the magnetic energy of the spot has been stolen from the kinetic or compressional energy of the matter around it. While I have some ideas on this matter, it is not my purpose to expound them now. I propose to rely simply upon the experimental fact that such magnetic fields exist, but I do wish to discuss the details of a second theft, in which a new thief in the form of a small amount of matter in the spot runs off carrying with it, in the form of a magnetic field, an amount of energy which is, as it were, out of all proportion to its fair share, and indeed, of such amount that when shared with its nucleons, it can provide for them, cosmic ray energies.

§ 2. Development of the Mechanism

We are familiar with the oft-quoted statement to the effect that a mass of gas shot out from the magnetic field of a spot carries with it a magnetic field, stolen, as it were from the spot. I wish to discuss the circumstances of this matter in some detail, and to that end shall, for simplicity consider a conducting ring of gas, Fig. 1 which is at present at O, Fig. 2 and is in motion upwards with its plane perpendicular to the lines of force of the Sun spot. This motion results from external causes.

In the first place I shall discuss matters as though this ring can be treated as an ordinary ring of high ohmic conductivity,—a copper ring for example, and shall apply the laws of electrodynamic to it in elementary form. I am well aware of the vulnerability of this procedure. The kind of ring we are considering is a ring of particle density 10^3 particles per c.c. or less. The mean free path of such particles will be of the order of 10^{12} cms. Collisions will not occur, and the whole notion of ohmic conductivity *as applied to our particular problem* evaporates. A rigorous treatment demands that we attack the problem as one of free particle motion under the proper electrodynamic laws applicable in such cases. However, I shall later show that the end results following from the more rigorous treatment are essentially the same as those following from the more elementary treatment; and, for this reason, I shall follow the elementary procedure, at first in any case, on account of its greater intuitive appeal.

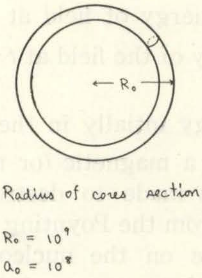

Fig. 1.

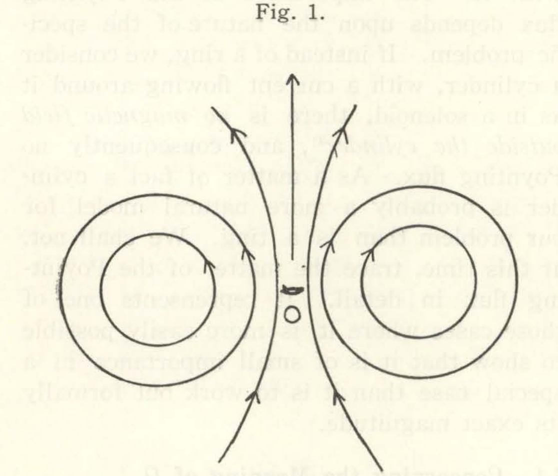

Fig. 2.

If the ring is in motion in the upward direction, as seen in Fig. 2, an induced current will be sit up of such amount that the change of flux through the ring resulting from the external field is just compensated by the flux sit up by the induced current, so that the ring is able to defend itself against change of flux. Presently we shall discuss some limitations of this power of the ring to defend itself, but for the moment we shall regret it as complete, so that by the time the ring has reached a region well outside of the primary magnetic field, it will have, in virtue of its own induced current the same total flux—say N_0, which it enjoyed when at O, where however the flux was due entirely to the external magnetic field. The induced current I, in electrostatic units will be given at this stage by

$$\frac{LI}{c}=N_0 \qquad (1)$$

where L is the self induction of the ring.

At this stage it is of interest to consider the energy in the ring, and to calculate how much an individual nucleon could possess if all the energy could be divided equally among the nucleons and if they could be set free from one another.

The energy W in the right is given by

$$W=\frac{1}{2c^2}LI^2 \qquad (2)$$

So that, from (1)

$$W=\frac{(LI)^2}{2c^2L}=\frac{N_0^2}{2L}\,.$$

If the field at O due to the spot is H_0, and R_0 is the radius of the ring when at O, then

$$N_0=\pi R_0^2 H_0$$

and

$$W=\frac{\pi^2 R_0^4 H_0^2}{2L}\,.$$

For the dimensions concerned, L is of the order of $10\,R$ where R is the radius of the ring at the point where the flux due to the external field is negligible. Thus

$$W=\frac{\pi^2 R_0^4 H_0^2}{20\,R}\,. \qquad (3)$$

The energy per nucleon will be ω, where

$$\omega=\frac{W}{2\pi R_0(\pi a_0^2)n_0}\,. \qquad (4)$$

Where subscript zero applies to the ring when at O. Thus, from (3) and (4)

$$\omega=\frac{R_0^3 H_0^2}{40 a_0^2 n_0 R}=\frac{300\,R_0^3 H_0^2}{40 a_0^2 e n_0 R}\ \text{ev.} \qquad (5)$$

Putting $R_0=10^9\,$cm, $a_0=10^8\,$cm, $H_0=1000\,$gauss,

$$\omega=\frac{1.5\times 10^{18}}{n_0}\left(\frac{R_0}{R}\right)\text{ev.}$$

Now the ring expands by self repulsion of its current elements. It cannot expand with a velocity greater than c, so that if the distance from O to a point where the primary field is negligible is $10\,R_0$ and if the average velocity of the ring is $0.1\ c$, the time to travel $10\,R_0$ is $100\,R_0/c$ and $R<100\,R_0$, so that

$$\omega>\frac{1.5\times10^{16}}{n_0}\ \text{ev}$$

if $\quad n_0=1000,\ \omega>1.5\times10^{13}$ ev. (6)

The smaller n_0, the greater ω. However, in order to justify our discussion in its present form we must not have n_0 so small as to cause the current I to exceed the saturation current. The existence of saturation current follows from the fact that we cannot obtain a current density greater than n_0ce. As a matter of fact, with $n_0=200$, and with the dimensions assigned as above, I could just attain the saturation value[1], so that we conclude that using this least permissible value for n_0, we obtain

$$\omega=7.5\times10^{13}\ \text{ev per nucleon.}$$

Naturally, there is considerable latitude in the choice of various dimensions assumed, and those we have taken are chosen only for illustration. Moreover, it is to be observed that our result represents a lower limit for the problem treated since energy is also transferred to the nucleons during their journey of the primary field.

§ 3. Transfer of the Energy to the Nucleons

So far we have discussed only the energy stored in the magnetic field of the ring, and while we have calculated the amount per nucleon, we have not actually provided for its being shared among the nucleons, so that each processes its share independently of its fellows. We now appeal to the well known theorem which, as applied to a volume V bounded by a surfaces S, is

$$\iiint E\rho u d\tau=-\frac{1}{8\pi}\frac{d}{dt}\iiint(E^2+H^2)d\tau-\frac{c}{4\pi}\iint[EH]_n\,ds \quad (7)$$

where u is the velocity of the density ρ.

Neglecting for the moment, the Poynting flux represented by the surface integral, and integrating from $t=0$ when the magnetic field has a finite value to $t=\infty$, we have, if Ω represents the total work done by the field on the nucleons over the period $t=0$ to ∞,

$$\Omega=-[\text{Energy of field at } t]_0^\infty.$$

Since the energy of the field at $t=\infty$ is zero[2], we have

$$\Omega=\text{Energy intially in the field.}$$

Thus if ever a magnetic (or magnetic plus electric) field is made to destroy itself, its energy, apart from the Poynting flux all goes into work done on the nucleons associated with it. The importance of the Poynting flux depends upon the nature of the specific problem. If instead of a ring, we consider a cylinder, with a current flowing around it as in a solenoid, there is *no magnetic field outside the cylinder*[3], and consequently no Poynting flux. As a matter of fact a cylinder is probably a more natural model for our problem than is a ting. We shall not, at this time, trace the matter of the Poyinting flux in detail. It reprensents one of those cases where it is more easily possible to show that it is of small importance in a special case than it is to work out formally its exact magnitude.

§ 4. Concerning the Meaning of Ω

It is to be noted that the theorem (7) is a direct algebraical consequence of the Maxwell Lorentz field equations *without the force equation of motion of a particle* which, indeed

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1) From (1)

$$I=\frac{Nc}{L}=\frac{\pi R_0^2H_0c}{10R}$$

The total number of ions per unit length, per unit cross section is given by $\pi a_0^2n_0$, so that the saturation current I_s is given by

$$I_s=\pi a_0^2n_0ce$$

Thus

$$\frac{I}{I_s}=\left(\frac{R_0}{R}\right)\frac{H_0R_0}{10a_0^2n_0(5\times10^{-10})}$$

so that with $R/R_0=100$ and with the values of H_0 and R_0 already assumed we find $n_0=200$ as the minimum value permissible for n_0, in accordance with the statement made above.

2) In our problem, the ring expands without limit on amount of the mutual repulsion of its current elements,

$$\text{Energy of magnetic field}=1/2\ LI^2$$
$$=1/2\ (LI)^2/L.$$

Since LI is constant as the ring expands, the energy is zero at $t=\infty$ since $L=\infty$ at $t=\infty$

3) When the current in the cylinder is *changing* there is a magnetic field, but one of small order.

is not contained in or deduce from those equations. Thus, without some further statement, the work done on the particles and represented by Ω, is simply a *name* for the space and time integral of $\rho uEd\tau$. However, suppose we assume an equation of motion of a charged nucleon of the form

$$m_0\frac{d}{dt}\frac{w}{(1-\beta^2)^{\frac{1}{2}}}=\left(E_0+\frac{[wH_0]}{c}\right)e \quad (8)$$

where w is the average velocity for the nucleon as a whole, E_0 and H_0 are the fields excluding the field of the nucleon, and assumed uniform over the nucleon, and n_0 is the rest mass of the nucleon. Then we have as follows:

The integrated value J of $E\rho ud\tau$ over the nucleon is given by

$$J=E_0\iiint\rho ud\tau+\iiint E_i\rho ud\tau\ ,$$

where E_i is the field due to the nucleon itself

$$J=E_0we+\iiint E_i\rho(w+\delta w)d\tau \quad (9)$$

where δw is the departure of u from the average w.

$$J=E_0we+w\iiint E_i\rho d\tau+\iiint E_i\rho\delta wd\tau\ .$$

It is well known that the first of the above integrals leads to the value

$$\iiint E_i\rho d\tau=-m_e\frac{d}{dt}\frac{w}{(1-w^2/c^2)^{\frac{1}{2}}} \quad (10)$$

where m_e is the electromagnetic mass $2e^2/3\alpha c^2$, where α is radius of the nucleon.

Again, since the scalar product of w and $[wH_0]/c$ is zero, equation (8) when incorporated in the first term of (9) causes that equation, with the further use of (10) to lead to

$$J=m_0w\frac{d}{dt}\frac{w}{(1-w^2/c^2)^{\frac{1}{2}}}-m_ew\frac{d}{dt}\frac{w}{(1-w^2/c^2)^{\frac{1}{2}}}+\iiint E_i\rho\delta wd\tau \quad (11)$$

$$=(m_0-m_e)w\frac{d}{dt}\frac{w}{(1-w^2/c^2)^{\frac{1}{2}}}+\iiint E_i\rho\delta wd\tau\ . \quad (12)$$

From this it readily follows that

$$J=(m_0-m_e)c^2\left\{\frac{1}{(1-w^2/c^2)^{\frac{1}{2}}}-1\right\}+\iiint E_i\rho\delta wd\tau \quad (13)$$

The last integral represents a phenomena which involved serious consideration in the early days of electrodynamics, and had to do with what was then regarded as a violation of the conservation of energy by the electron of Lorentz[4]. This matter was bound up with Lorentz's pure electromagnetic theory of mass. The said integral, and the quantity m_e occurring in (13) cease to give trouble if we relieve electromagnetic theory of the responsibility of providing a meaning for mass. The term m_e cannot be omitted even though we cease to regard electromagnetic theory as the sole origin of mass, but it can be made play only a subordinate role if $m_0\gg m_e$. It is regretable that we have become involved in such a lengthy discussion of a matter which is primarily a concern of electrodynamics rather than cosmic ray theory, but it has been felt necessary to deal with the matter to the end of providing a basis for regarding the time integral of the left hand side of (7) as giving for *such nucleon* the increase of kinetic energy for that nucleon as ordinarily understood.

§ 5. A Difficulty concerning the Development of Induced Current in the Gas Ring

We have started with the gas ring in the position O, Fig. 1 under conditions in which it has through it a certain flux N_0 resulting from the primary field, strictly speaking, however, it is not fair to start in this manner, for the gas had to be *brought* into this position from a place where there was no flux through it; or, the primary magnetic field had to be created while it was at O. In either case, if the gas has the properties of a perfect conductor induced currents will be generated during the processes of arriving at the condition in which the gas finds itself; and, by the fundamental principle which we have invoked the gas ring will find itself in a condition with no flux, and will maintain that condition throughout, even to the point where it has been ejected from the primary field, so that the ring will arrive in this position with no magnetic energy, and our primary purpose will have been defected. We can avoid this calamity by the following considerations.

As already stated, the ring cannot defend itself against change of flux to an extent

4) See the writers. This has yet to be supplied.

which calls for it to acquire a current greater than the saturation current, which we shall call I_s. Let I represents a general value of the ring current, so that we must have $|I|\leq|I_s|$.

Suppose that the horizontal axis, Fig. 3 represents the path of the ring from $-\infty$ to the left to $+\infty$ to the right. Let the ordinates in Fig. 3 represent N, the *flux through*

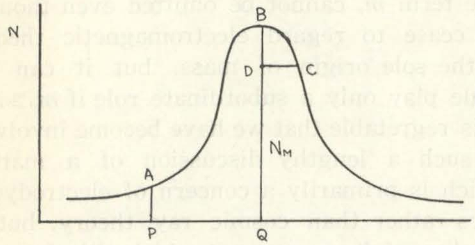

Fig. 3.

the ring resulting from the external field alone. On passing from left to right I goes from zero according to the law $LI=Nc$ by which procedure it keeps the total flux through the ring zero. This continues until we reach a value N_s of N, such that $cN_s/L=I_s$. At this stage represented by the point P, the power of the ring to defend itself against change of total flux ceases, because the saturation current has been reached. As the ring continues to move to the right the total flux starts to increase until, at the point Q, the resulting flux is N_M-LI_s/c. A further movement to the right now results in a decreases in N, so that the ring is once more able to defend itself against *change* of total flux by experiencing a *decrease* in the current from the value I_s. This decrease continues until a point C is reached such that BD=AP=LI_s/c, at which point the actual current in the ring becomes zero. We now have a ring with zero current and a flux equal to N_M-LI_s/c resulting entirely from the external field. A further decrease of the flux due to the external field as the ring continues to journey towards the right calls forth a growth of the current I in a direction opposite to the original direction; and by the time the ring has passed out of the external field, it will have acquired a current I_s, or a current given by N_M-LI_s/c which ever is the smaller.

In the light of the foregoing considerations we see how it is possible to realize a ring current when ring has left the magnetic field in spite of the argument presented at the beginning of this section.

Another procedure consists in starting with the gas ring as part of the system resposible for the primary magnetic field, in which case it carries a current in the opposite direction to the induced current in the discussion already given. It then, of course, became necessary to have a mechanism which will hurl the ring to outerspace. However, if the ring is hurled out, it will carry with it whatever total flux it possesed when at O.

§ 6. The Problem from the Standpoint of Rigorous Electrodynamics

The Lagrangian function L, for a charged particle in a field of vector potential U and and scalar potential φ is given by

$$L=-m_0c^2(1-\beta^2)^{\frac{1}{2}}+e/c(Uv)-e\varphi \qquad (14)$$

where v is velocity, and $\beta=v/c$.

The Lagrangian equations yield the equations of motion[5]. For axial symmetry with $\varphi=0$, and in e.s.u.,

$$m_0R^2\dot{\theta}(1-\beta^2)^{-\frac{1}{2}}+\frac{ReU}{c}=\text{const.} \qquad (15)$$

Now

$2\pi RU$=Total flux through ring=$N+LI/c$.

Writing $R\dot{\theta}=v_\theta$ where v_θ is the tangential velocity,

$$\frac{m_0Rv_\theta}{(1-\beta^2)^{\frac{1}{2}}}+\frac{e}{2\pi c}\left(N+\frac{LI}{c}\right)=\text{const.}$$

Since, at $t=0$, $v_\theta=0$, $I=0$ and $N=N_0$, the constant is $eN_0/2\pi c$. Hence

$$\frac{m_0Rv_\theta}{(1-\beta^2)^{\frac{1}{2}}}+\frac{eLI}{2\pi c^2}=\frac{e}{2\pi c}(N_0-N)\,. \qquad (16)$$

If $\bar{n}$ is total number of ions in the ring,

$$I=\bar{n}ev_\theta\,.$$

Hence

$$\frac{m_0R}{(1-\beta^2)^{\frac{1}{2}}}v_\theta+\frac{eL\bar{n}e}{2\pi c^2}v_\theta=\frac{e}{2\pi c}(N_0-N)\,.$$

The term, involving m_0 is negligible compared with the term involving $\bar{n}$ provided that

5) The equations of motion have been developed for the case of axial symmetry, by the writer in a paper "The Acquirement of Cosmic Ray Energies by Electromagnetic Induction in Galaxies; Journ. Frank. Inst. **258** (1954) 383.

$$L\bar{n} \gg \frac{2\pi c^2}{e^2} \frac{m_0 R}{(1-\beta^2)^{\frac{1}{2}}} .$$

Since $L \sim 10\, R$, this calls for

$$\bar{n} \gg \frac{2\pi c^2 m_0}{10 e^2 (1-\beta^2)^{\frac{1}{2}}} . \quad (17)$$

Now

$$\bar{n} = 2\pi^2 R_0 a_0^2 n_0 .$$

Hence, (17) becomes

$$\pi R_0 a_0^2 n_0 \gg \frac{m_0 c^2}{10 e^2 (1-\beta^2)^{\frac{1}{2}}} .$$

Putting $R_0 = 10^9$, $a_0 = 10^8$, $n_0 = 1000$, this gives

$$0.6 \times 10^{28} \gg \frac{6 \times 6 \times 10^{-24} \times 10^{21}}{10 \times 25 \times 10^{-20} (1-\beta^2)^{\frac{1}{2}}}$$

or

$$10^{28} \gg \frac{20 \times 10^{15}}{(1-\beta^2)^{\frac{1}{2}}} .$$

In other words, not until $(1-\beta^2)^{-\frac{1}{2}}$ became of the order 10^{12}, and the energy became 10^{12} times the rest energy, would the first term on the left hand side of (16) become important. Thus (16) may, for practical purposes,

$$\frac{LI}{c} = N_0 - N$$

which is the form we have used in our previous discussion, leading to

$$\frac{LI}{c} = N_0$$

when the ring has proceed out of the primary field.

JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN Vol. 17, SUPPLEMENT A-III, 1962
INTERNATIONAL CONFERENCE ON COSMIC RAYS AND THE EARTH STORM Part III

III-3-10. On the Origin of Cosmic Rays*

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Among divergent opinions on the origin of cosmic rays, one could perhaps find agreement in the following points. Two of them have already been noticed by Fermi in his epoch making theory and the third one has been emphasized by Hoyle, Ginzburg, Shklovskij and Oort, based on a number of evidences.

(I) Magnetic storage of cosmic ray particles in the Galaxy.

(II) Slow acceleration of particles by varying magnetic fields.

(III) Supernova origin as one of the most powerful sources.

If one wishes to go farther in each of the above points, there arise a number of different views which would not always make everybody happy. They may be listed up as follows.

(I-1) The stored region extends to the Galactic halo and particles in the halo and the Galactic plane are well mixed[1]. This may be accepted by the majority, but a small fraction of cosmic rays we are observing may not be in equilibrium but could be affected by near-by sources, as will be discussed later[2].

(I-2) The mean lifetime of stored cosmic rays is determined mainly by the escape from the Galaxy, but not by the nuclear absorption[3]. This is in essential agreement with the observed abundances of the light group and electrons which are regarded as the products of nuclear interactions with the interstellar matter whose thickness traversed by cosmic rays is estimated to be rather small[3),4)]. However, there seem to be at least two experimental results which could be regarded as evidences against the above view. One is the isotopic ratio of helium

* This review is prepared for introducing the papers presented in the ordinary session of "Origin" and for giving a general picture on the origin of cosmic rays.