III-5-9. Hyperon Effect in the High Energy μ-meson Spectrum at Sea Level*

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Production ratios of strange particles in collisions of 25 Gev protons with light nuclei indicate that in 30% of the collisions the baryon carrying most of the energy may be a hyperon. The fraction of hyperons probably increases with energy in proportion to the multiplicity of the particles produced, till it reaches a saturation value of about 0.8 at 10^{13} ev. As discussed by Peters[1], among other effect, this will lead to the consequence that most of the π-mesons above an energy of a few hundred Gev found in the atmosphere will result from hyperon decay rather than through the "pionization" process. In this paper, the sea level flux and energy spectrum of muons arising via the hyperon process have been calculated. Starting from the known flux and energy spectrum of primary cosmic rays, it is shown that above 10^{12} ev the flux and energy spectrum of muons agrees very well with the also absolute flux and the spectrum measure directly in the experiment of Barton[13] and of Miyake *et al*[16] and as inferred by Duthie *et al*[7] from their measurement of high energy γ-ray spectrum in the atmosphere. Below 10^{12} ev the hyperon contribution to the μ-meson flux decreases progressively, as expected, till, at about 100 Gev, it becomes nearly half of the total flux of μ-mesons as measured by various investigators.

§1. Introduction

Recently Peters[1] has discussed the importance of "hyperon effects" in various cosmic ray phenomena in the atmosphere and underground. He has shown that on the average, in 0.3 of the collisions of protons of 25 Gev, the bulk of the energy is carried away by a hyperon. The hyperon probability, denoted by p_y, will increase with energy, most probably, in the same ratio as the multiplicity of the secondary particles, because it has been shown that the *ratio* of the number of pions to non-pions produced in collisions is almost independent of energy[2),3)]. Therefore, as shown elsewhere[4] we can write

$$p_y=0.18E^{0.15} \qquad (E \text{ in Gev})$$

upto energies of the order of 10^{13} ev where $p_y \approx 0.8$. It can be assumed that this ratio remains constant at 0.8 beyond this energy; this is what one would expect if a statistical equilibrium is achieved between two nucleons and 4 hyperons of strangeness -1, and if the production of strangeness -2 hyperons is negligible in comparison. As pointed out by Peters[1], on this model, we expect that most of the μ-mesons of energy above a few hundred Gev observed at sea level will arise from the decay of hyperons. The exact energy region where the μ-mesons from hyperon decay become comparable in number to the μ-mesons arising from π-meson production (or "pionization") will depend on energy distribution of π-mesons in high energy pionization collisions. Further, one would expect that when the energy of the hyperons increases to a point where their decay length and interaction mean free path become comparable, the hyperon contribution to μ-meson spectrum would begin to drop till, for primary energies beyond 10^{15} ev the μ-mesons will again arise predominantly through the pionization process. In view of these possibilities a calculation has been made, using the known primary spectrum and flux, to obtain a quantitative estimate of the hyperon contribution to the μ-meson flux at various energies.

The masses and life-times used for the π-meson and the various hyperons are taken from the tables compiled by Barkas and Rosenfeld[6]. The primary flux introduced is

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that of Kaplon et al[7] at an energy of 4500 Gev. A slope of 1.7 for the integral energy spectrum has been used on the lower energy side of this point and a slope of 1.8 has been used for the higher energy side. Since the measured flux is close to the region of primary energies investigated, the results are not very sensitive to exact slope assumed. The ratio of the number of nucleons brought in by heavy primaries to protons is taken as 0.43 (Ginsburg[8]). From an analysis of the attenuation of high energy nucleons in the atmosphere and in corporating the hyperon hypothesis, one of us has concluded that the total elasticity of collisions increases as 0.5 $E^{.05}$ upto an energy of 10,000 Gev[9] where the elasticity becomes 0.8. What happens to the elasticity beyond this energy is not known. For calculating the highest energy point at 10,000 Gev muon energy, we go upto primary energies of the order of 100,000 Gev. We assume that the elasticity stays around 0.8 in the energy region of 10,000 to 100,000 Gev.

§2. Outline of the calculation

All nuclear collisions which ultimately contribute to the high energy muon flux at sea level, must occur at very high altitude. This is independent of whether the muons are produced through the hyperon process or through ordinary pion production process. It can be shown that, on the average, the first interaction is responsible for over 80% of the μ-mesons, above a certain energy, arising from hyperons of energy less than 10^{13} ev. Above this energy the hyperons produced in the second or higher collisions penetrate into the dense layers of the atmosphere and are lost through interaction before they have a chance to decay; so these contribution is neglected above 10^{13} ev. Therefore, only the flux from the first generation hyperons will be calculated and correction for second generation of hyperons will be applied subsequently for muon energies of upto 1000 Gev. For primary energies of upto a few thousand Gev, the decay length of the hyperon is small, and, therefore, we will assume that it always decays within a short distance from production. The effect of increased decay length for hyperons of energy greater than a few thousand Gev will be estimated separately.

The probability that a cosmic ray particle incident on top of the atmosphere produce a μ-meson at sea level through the hyperon process is given by

$$\mathscr{P}=\frac{p_y C_y}{2\lambda_N\sqrt{\gamma'^2-1}}\frac{h_0}{c\tau_\pi\Gamma_y}\int_{\gamma_{\pi\min}}^{\gamma_{\pi\max}}\frac{d\gamma_\pi}{\gamma_\pi}\times\int_0^{x_0}e^{-x((1/\lambda_N)-(1/\lambda))}x^u\left[\int_x^{x_0}e^{-y/\lambda}y^{-(1+u)}dy\right]dx$$

where

- p_y: the probability that an energetic hyperon be produced in a nuclear collision;
- C_y: the branching ratio for decaying into a charged π-meson.
- h_0: scale height. This is taken as 7 kilometers.
- λ_N: interaction mean free path of the incident particles
- τ_π: proper life-time of π-meson
- c: velocity of light
- γ_π: energy, in rest units, of the decay π-meson in the laboratory system
- Γ_y: energy, in rest mass units, of the hyperon
- γ': energy of the π-meson, in its rest units, in the hyperon rest frame
- γ: interaction mean free path for π-mesons

$$u\equiv\frac{h_0}{c\tau_\pi\gamma_\pi}$$

$$\gamma_{\pi\min}=\Gamma_\gamma[\gamma'-\sqrt{\gamma'^2-1}]$$

$$\gamma_{\pi\max}=\Gamma_y[\gamma'+\sqrt{\gamma'^2-1}]$$

- x_0: total thickness of the atmosphere = 1030 gm/cm^2

It can be shown that

$$\mathscr{P}=p_y C_y\frac{K}{\Gamma_y}\ln\frac{1+\Gamma_y/B}{1+\Gamma_y/A}$$

where

$$B=\frac{h_0}{c\tau_\pi(\gamma_\pi'+\sqrt{\gamma_\pi'^2-1})}$$

and

$$A=\frac{h_0}{c\tau_\pi(\gamma_\pi-\sqrt{\gamma_\pi'^2-1})}$$

Now

$$\Gamma_y=E_y/M_y$$

and

$$E_y=\eta(E)E$$

where E is the proton energy, and $\eta(E)$ is taken as $0.5E^{.05}$ upto 10,000 Gev and 0.8 after that; and $p_y=0.18E^{0.15}$ upto $E=10,000$ and

0.8 for higher energies. Integrating $\mathscr{P}$ into the differential primary energy spectrum, we get, for the flux $F(E_\mu)$, of μ-mesons of energy greater than E_μ:

$$F(E_\mu)=KC_yF_0M_y$$

$$\times\int_{E_{\min}}^{\infty}\frac{p_y(E)}{E^{\gamma+2}\eta(E)}\ln\frac{1+\dfrac{\eta(E)E}{BM_y}}{1+\dfrac{\eta(E)E}{AM_y}}dE$$

E_μ is related to $E_{\min}$ as

$$E_\mu=\frac{m_\mu}{m_\pi}[\alpha\Gamma_{y_{\min}}m_\pi]$$

where

$$\Gamma_{y_{\min}}=\frac{E_{\min}\eta(E)}{M_y}$$

and α is the value of

$$\gamma_\pi/\bar{\Gamma}_y$$

where $\bar{\Gamma}_y$ is the weighted average value of Γ_y over the primary energy spectrum which will lead to a given γ_π. Value of α is slightly different for different hyperons.

F_0 is taken by normalising the differential spectrum to the flux given by Kaplon *et al*[9] for energies larger than 4,500 Gev. γ is taken as 1.7 for $E<4500$ Gev and 1.8 for $E>4500$ Gev.

Contribution for the heavy primaries is taken in proportion to the number of nucleons brought in by heavy nuclei. This will be very nearly the case because for each heavy nucleus the effective area of the measuring apparatus will be increased in proportion to the number of nucleons brought in.

It is assumed that all four hyperons of strangeness -1 are produced in equal proportion. Appropriate values of C_y and constant α are used for each hyperon.

Correction for second generation hyperons and for the increased decay length of hyperons at high energies are applied as discussed earlier.

§ 3. Results and Discussion

The calculated flux of μ-mesons arising via the hyperon process has been plotted in Fig. 1. In the same figure we show the fluxes at different energies as measured by various experiments using magnetic spectrometers and range energy relation. Also indicated on the plot are the points derived by Duthie *et al*[5] on the basis of their measurements on the γ-ray energy spectrum in the atmosphere. It is seen that the calculated μ-meson contribution from the hyperon process accounts for nearly all the μ-mesons between 1,000 Gev and 5,000 Gev. Below 1,000 Gev the hyperon contribution is progressively smaller than the observed μ-meson flux. As explained by Peters[1], this is exactly what one expects: that the pionization process should be the predominant process at lower energies. The difference of the two curves in Fig. 1 must represent the contribution of pionization to the μ-meson flux at sea level. It is seen that the hyperon contribution will tail off at energies of the order of 10,000 Gev.

The μ-mesons arising from pionization should occur with about equal frequency in negative and positive charges. On the other hand, assuming that only strangeness -1

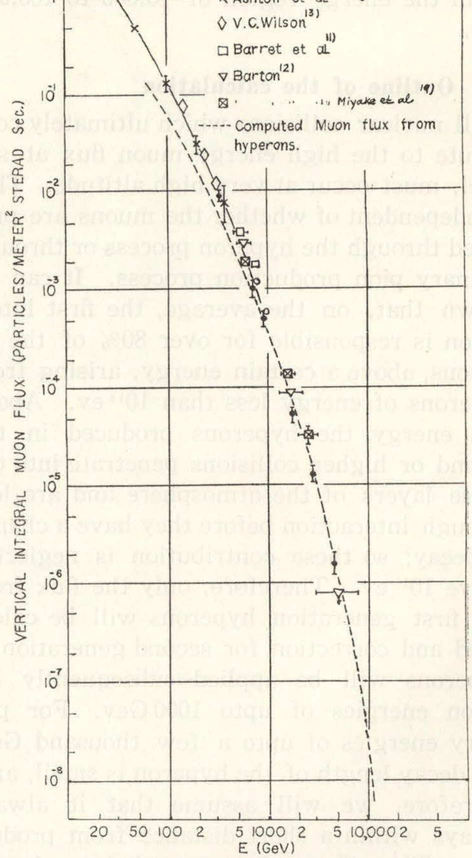

Fig. 1. Vertical integral muon spectrum at sea level as measured by various experiments and as calculated from the hyperon process.

hyperons, the negative to positive ratio of μ-mesons arising from hyperon process should be about 4.7. Therefore, by comparing the relative contributions of the two processes we can obtain the expected *negative* excess, $(\mu_- - \mu_+)/(\mu_- + \mu_+)$, for the μ-mesons as a function of energy. It is seen that the negative excess will be $\sim$.31 at 100 Gev, .39 at 500 Gev and .64 between 1,000 Gev and 5,000 Gev. These numbers are, of course, subject to alterations because of the errors in the measured muon flux and the primary flux and uncertainties in the hyperon probability p_y. However, since the incident particles are predominantly positive, it is likely that in first collision, which leads to nearly all the muons observed at sea level, Σ^+ may be produced more frequently than other hyperons, particularly with respect to Σ^-. In that case the negative excess of muons will be drastically reduced, and may even disappear.

As already mentioned the coincidence of the hyperon μ-meson flux and spectrum with the one calculated by Duthie *et al*[5] from their observations on the γ-ray spectrum in the atmosphere supports the hypothesis that the π-mesons in the atmosphere in the energy range 2,000 Gev to 10,000 Gev arise predominantly through the hyperon process. One has, here, a very natural explanation of the peculiar steepness of the spectrum observed for γ-rays of energy $>1{,}000$ Gev and for μ-mesons of energy above 2000 Gev. All the peculiar effects are ascribable to the "pulse" of π-mesons due to the hyperon process which gets superimpose on the π-mesons from the pionization process over a limited energy range (also see the appendix).

Acknowledgment

We are indebted to Professor B. Peters for many stimulating discussions on this and related topics.

Appendix

It is worthwhile to discuss the connection between the calculated muon spectrum and observations of γ-ray spectra in the atmosphere in greater detail. Bristol group[7] find that the slope of the balloon altitude spectrum for γ-rays is 2.0 ± 0.5 over the energy interval 500–1,500 Gev, while for measurements made at air-plane altitudes the slope is 2.5 ± 0.2 over the same energy interval and 2.8 ± 0.3 in the energy range of 1,500–5,000 Gev. The Japanese group[15] have reported a slope of 1.8 ± 0.2 for γ-rays of energy less than 1000 Gev, and a slope of 2.8 ± 0.2 for γ-rays of energy $>1{,}000$ Gev when measurements are made at mountain altitudes. On the other hand for their measurements at balloon altitude, extending upto an energy of 4,000 GeV there is no indication of a change of slope at 1,000 Gev, the slope remaining 2.0 ± 0.2 all through.

In the hyperon hypothesis the steepening of the spectrum is attributed to the finite lifetime of hyperons. If one measures, predominantly, the effect of hyperons produced at a depth x gm/cm^2 below the top of the atmosphere, the steepening will set in at those hyperon energies for which the hyperon decay length is equal to hyperon interaction length *at the depth* x *gm/cm*^{2} in the atmosphere. For μ-mesons underground the relevant depth x is about 70 gm/cm^2, for observations on γ-rays, at air-plane altitudes, x will be about 150 gm/cm^2 and for γ-rays observed at balloon altitudes it will be about 10 gm/cm^2. Therefore, because of the exponential nature of the atmosphere, the steepening in the γ-ray spectrum at air-plane altitudes should set in first, the steepening in muon spectrum at sea level at a slightly higher energy and steepening in the γ-ray spectrum at balloon altitudes at far higher energies. All this is nicely consistent with the observation that, while there is an indication of steepening of the muon spectrum and the γ-ray spectrum at air-plane altitudes at similar energies, the γ-ray spectrum at balloon altitudes stays the same over the energy region upto which the measurements have been made.*

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III-5-10. The Momentum Spectrum of Muons Underground

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A determination has been made of the absolute underground muon spectrum at a depth of 7.2 kg cm^{-2} in the range 250 MeV to 100 GeV. Good agreement is obtained with the spectrum calculated for this depth from the sea level spectrum by making reasonable assumptions about energy loss.

Introdu ction

The momentum spectrum of mesons at a given depth underground may be predicted from the sea-level spectrum (or more precisely from the production spectrum deduced from the observed sea-level spectrum) provided the mechanism of momentum-loss in the intervening rock is known. By a comparison of the underground spectrum with the sea-level spectrum it is therefore possible to verify theoretical predictions of momentum-loss as a function of momentum. Furthermore, observations below ground do not discriminate against mesons closely associated with other air shower particles, since these are absorbed in the rock. The rejection of these events in sea-level measurements could cause a bias in the spectrum. Measurements underground with a spectrometer capable of resolving momenta appreciably greater than the momentum-loss in the rock can therefore also contribute useful information on the sea-level spectrum.

Muons observed underground are, however, frequently accompanied by secondaries of interactions in the rock. In order to resolve a majority of such multiple events our spectrometer uses Geiger counter trays at five levels to determine the muon trajectories. The apparatus was described and preliminary results reported at the 1959 Moscow Conference on Cosmic Rays[1]. The experiment was carried out under 7.2 kg cm^{-2} of sandstone rock, the depth being determined by weigh-

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