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III-5-10. The Momentum Spectrum of Muons Underground

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A determination has been made of the absolute underground muon spectrum at a depth of 7.2 kg cm^{-2} in the range 250 MeV to 100 GeV. Good agreement is obtained with the spectrum calculated for this depth from the sea level spectrum by making reasonable assumptions about energy loss.

Introdu ction

The momentum spectrum of mesons at a given depth underground may be predicted from the sea-level spectrum (or more precisely from the production spectrum deduced from the observed sea-level spectrum) provided the mechanism of momentum-loss in the intervening rock is known. By a comparison of the underground spectrum with the sea-level spectrum it is therefore possible to verify theoretical predictions of momentum-loss as a function of momentum. Furthermore, observations below ground do not discriminate against mesons closely associated with other air shower particles, since these are absorbed in the rock. The rejection of these events in sea-level measurements could cause a bias in the spectrum. Measurements underground with a spectrometer capable of resolving momenta appreciably greater than the momentum-loss in the rock can therefore also contribute useful information on the sea-level spectrum.

Muons observed underground are, however, frequently accompanied by secondaries of interactions in the rock. In order to resolve a majority of such multiple events our spectrometer uses Geiger counter trays at five levels to determine the muon trajectories. The apparatus was described and preliminary results reported at the 1959 Moscow Conference on Cosmic Rays[1]. The experiment was carried out under 7.2 kg cm^{-2} of sandstone rock, the depth being determined by weigh-

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ing the rock in a core drilled to the depth of the apparatus at the conclusion of the experiment.

Experimental Results

We have rejected all events for which no trajectory could be deduced consistent with the observed record of counters fired. These are attributed mainly to side-showers, *i.e.* showers which trigger the apparatus but which have no complete trajectory within its aperture. These constitute about 5% of all penetrating events recorded. With these events rejected good agreement is obtained between the observed zero field distribution and the predicted distribution which takes into account the "noise-level" deflection in the apparatus. The r.m.s. "noise" deflection corresponds to a momentum of 180 Gev/c. We have obtained 1882 acceptable events at a field of 12,730 Gauss and 2,490 at 930 Gauss.

Following Berrett *et al*, we assume a production spectrum of the form

$$M(p)dp = k\frac{\delta}{p+\delta}p^{-(\gamma+1)}dp$$

and calculate numerically the underground spectrum for various values of γ and δ, allowing for momentum-loss as discussed below, and allowing also for scattering in the rock above the apparatus, and for μ-e decay in the atmosphere. The deflection distribution is then predicted for each assumed spectrum by a Monte Carlo calculation on the computer SILLIAC which simulates the spectrometer.

For $\delta=90$ Gev/c, we obtain for the high field results, $\gamma=2.62\pm0.04$ in the range 2 Gev/c to 200 Gev/c. An equally good fit is obtained for $\delta=150$ Gev/c if γ is increased by 0.1. The experimental results are shown in compressed form in Table I. They are consistent with the spectra of Pine *et al*[3] and Ashton *et al*[4].

We then predict the deflection distribution at 930 Gauss for $\delta=90$ Gev/c and $\gamma=2.62$. This gives the spectrum down to 250 Mev/c and the results are shown in compressed form in Table II. The χ^2 probabilities are respectively 0.5 and 0.3 and the high and low field results, indicating a good fit in each case.

The spectrum which best fits the experimental results is shown in Fig. 1 (curve a), together with the deduced spectra assuming ionization-loss only (curve c) and constant lost of 2.14 Mev/c gm^{-1} cm^2 (curve b). This value is chosen to give the correct range for our depth. The sea-level spectrum deduced from our results is also given together with the experimental results of Pine *et al* and Ashton *et al* for comparison.

Table I. High Field Deflection Distribution.

Deflection (c.u.)	Observed	Predicted
0— 1	222	225
3— 4	223	254
5— 6	234	221
7— 9	241	262
10—13	244	241
14—18	212	203
19—29	262	248
30—91	244	228
	1882	1882

Table II. Low Field Deflection Distribution

Deflection (c.u.)	Observed	Predicted
0	659	647
1	855	890
2	363	360
3	171	181
4	106	92
5	80	67
6— 7	71	86
8— 9	70	54
10—14	53	60
15—19	35	30
20—91	27	23
	2490	2490

Momentum-Loss

For ionization-loss we use the Bethe-Block formula[5] with the Fermi[6] correction for the density effect. For bremsstrahlung we use the formula of Hayakawa and Tomonaga[7]. For pair production we use the formula of Mando and Ronchi[8] which is based on the Racah[9] differential cross section since Block *et al*[10] have shown that this is more accurate than the Bhabba[11] cross section from which

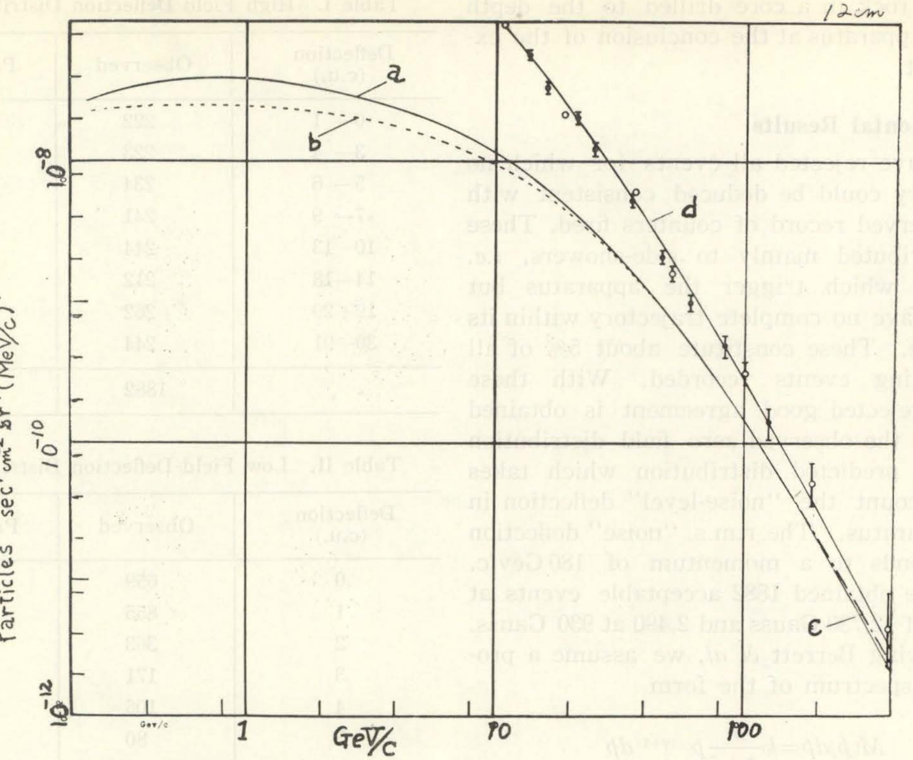

Fig. 1. Momentum Spectrum:
a) Observed underground spectrum.
b) Predicted underground spectrum for momentum-loss constant at 2.14 Mev/$c\,g^{-1}$ cm^2.
c) Predicted underground spectrum for ionization-loss only.
d) Deduced sea-level spectrum: Full circles-Pine *et al*. Open circles-Ashton *et al*.
Curves b) and c) have been drawn only where they are distinguishable from a).

the formula of Hayakawa and Tomonaga is calculated. For nuclear-loss we use the following expression derived from the differential cross section of Kessler and Kessler[12]:

$$-\frac{dE}{dX}=\frac{2a}{\pi}\sigma NE\left(\frac{2}{3}\ln\frac{E}{\mu}-\frac{29}{36}\right),$$

Where σ is the photo-nuclear cross section.

Curve (a) of Fig. 2 is based on the above expressions, curves (b) and (c) are those of Pine *et al*[3] and Ashton[13] respectively, and curve (d) is our own expression except that for nuclear-loss we use the formula[14]

$$-\frac{dE}{dX}=\frac{2a}{\pi}\sigma NE$$

based on the Weizacker-Williams method. Below 10 Gev/c these curves are virtually identical.

The intensity at a depth of 200 kg cm^{-2} calculated for $\gamma=2.62$ differs by about 15% for curves (a) and (d). On the other hand, a change of γ from 2.62 to 2.55 represents a difference of about 25% in the intensity at this depth. The momentum necessary to penetrate this depth is about 700 Gev/c.

Discussion

Good agreement is obtained between the sea-level spectrum corresponding to our observed underground spectrum and the spectra of Pine *et al*, and Ashton *et al*, over the whole range of our observed momentum from 250 Mev/c to 200 Gev/c. The intensity scale on the Pine spectrum and on our spectrum are independent absolute measurements. The good agreement gives confidence in both the magnitude and form of the momentum-loss expressions. The poor fit of constant momentum-loss is evident, but we are unable to distinguish between the other forms of momentum-loss used. The small difference in predicted intensity at 200 kg cm^{-2} for cur-

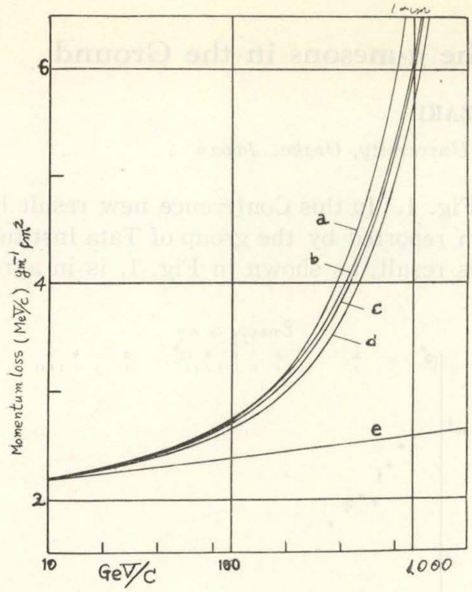

Fig. 2. Momentum-Loss:
a) Our formulae.
b) Pine *et al.*
c) Ashton.
d) Our formulae with Weizacker-Williams nuclear-loss.
e) Ionization-loss only.

ves (a) and (d) shows that it is not possible to deduce precise information about nuclear-loss by comparison with depth-intensity measurements and currently available results on the momentum spectrum. The difficulty is of course, enhanced by possible errors in the measurements of both depth and vertical intensity. At higher momenta, however, the nuclear-loss becomes relatively more important and thus any deduction of the momentum spectrum beyond 1,000 Gev/c from depth-intensity measurements deep underground is very dubious.

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Discussion

Wolfendale, A. W.: Did the 41 double μ-events go through the whole spectrograph? If so, the rate seems high.

Oglivie, K. W.: 15 of these were completely unambiguous, but the others were probably cases in which one or other did not pass properly through the field.

Stoker, P. H.: I suppose that the distance of separation between the two particles must be also taken into account for the observed number of 41 accompanied *mu*-mesons.

Oglivie: In the case of no ambiguity there is no problem. The other cases are so uncertain as to origin that they are not profitable to discuss.