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III-6-12. Showers Formed by Nuclei with Energy of $\gtrsim 5.10^{10}$ ev/Nucleon*

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The type of primary particles interaction with emulsion nuclei and the dependence of multiplicity, angular distribution and inelasticity upon energy have been investigated for showers, formed by nuclei ($z \geq 2$). The results of the present analysis have been compared with the well-known data on the nucleon-nucleon showers.

§1. Introduction

The study of the heavy nuclei ($z \geq 2$) of the cosmic-ray primary component interaction, in our opinion, is of a definite interest from the two standpoints. Firstly, there is nearly no possibility at present or in the nearest future to receive in the laboratory, for instance, α-particle with the energy of order of 50 Bev.

Secondly, the process of nucleus-nucleus interaction in the high-energies region represents, obviously, the other extremity in comparison with "clear" nucleon-nucleon or pion-nucleon collision. Indeed, with the collision of two nuclei, having atomic weight A, the nuclear substance density, designed by cross-section, will be of the order of value $A^{1/3}$ times as much as in the case when the primary particle is a nucleon.

That is why in showers formed by nuclei we obtain the best conditions for the investigation of the head-on or close to head-on collisions.

§2. The choice of the kind of interaction between the primary α-particle and the emulsion nuclei

1. In the course of theoretical investigation of the interaction between the nucleons and the nuclei it was supposed[1)] that with energies $\geq 10^{11} \sim 10^{12}$ ev the tunnel mechanism of interaction between the primary particle and the nucleus is to take place. The problem grows more complicated as soon as we pass to the analysis of collisions of primary nuclei with photographic emulsion nuclei. As it can be shown, all the evaluations of the degree of classicality and justifiability of the tunnel mechanism of interaction for the primary nucleus of atomic weight A increase $A \sim A^{4/3}$ times as much as in the case when the primary particle is a nucleon, none the less more sound foundations are necessary. Therefore, as well as in the case of the single-charged primary particles[2)], we need experiment for the final elucidation of the type of interaction.

2. Analogically to [3] let us examine value $\xi = N_1(l \geq 2)/N_1(l \geq 2) + N_2(l < 2)$, where $N_1(l \geq 2)$ is the number of showers with the tunnel length l greater or equal to the length of the maximal tunnel in the mean light nucleus of the emulsion. The geometrical computation with cross-section $\sigma_i = \pi(R_i + R_\alpha - 2\Delta R)^2$ leads to value $\xi_c = 0.35 \sim 0.39$ for the nucleus

Table I.

Type of showers	τ_{n_s, N_h}	$t_{0.01} \cdot \sigma_\tau$	b_{N_h/n_s}	$\tau_{N_h, E}$	$t_{0.01} \cdot \sigma_\tau$
nucleon-nucleon					
$l<2$	-0.09 ± 0.24	0.31	-0.03	-0.22 ± 0.23	0.30
$l \geq 2$	0.12 ± 0.32	0.42	0.03	-0.03 ± 0.32	0.42
All showers (except nucleon-nucleon)	0.35 ± 0.17	0.22	0.11	-0.17 ± 0.19	0.24

* This paper was read by G. B. Zhdanov.

of silver and correspondingly for the mean heavy nucleus of the emulsion. The experimental value is independent of energy and is equal to $\xi_e=0.27\pm0.04$ (the probability of agreement of ξ_e and ξ_c $P(\xi)\leq0.11$). Thus in the case of α-showers the tunnel model is found to be in better agreement with the experimental data than in the case of showers formed by nucleons[2,4].

The energy of the primary particle

$$E=2\gamma_c^2 l, \quad \gamma_c\gg1\,, \qquad (1)$$

γ_c was defined from the equation

$$\gamma_c=\varepsilon \operatorname{ctg}\theta_\Gamma \qquad (2)$$

where multiplier

$$\varepsilon=0.7$$

takes into account the constancy[4,6] of the transverse momentum and the energy spectrum of the primary particles.

3. As the next step let us examine distribution in terms of N_h. It appeared that $\bar{N}_h(l\leqslant2)$ do not depend on energy, the value of relation $\bar{N}_h(l\geq2)/\bar{N}_h(l<2)=2.48\pm0.44$. Thus the mean values of N_h reliably (at least with 99% probability) are divided into two groups corresponding to the type of primary particle-nucleus interaction, and we are led to the empirical formula

$$\bar{N}_h=(0.85\pm0.1)\cdot A_\alpha{}^*\cdot(l^*+1) \qquad (3)$$

This result is significant, as when the showers were divided according to the lengths of the tunnels, the value N_h was not considered at all, and the given now analysis in terms of N_h confirms the tunnel type of the interaction of the α-particles of high energy.

4. If the tunnel assumption is justified, n_s and N_h, N_h and E should not be correlated. The results of the calculation, given in Table I, confirm this assumption.

5. All the previous discussions depend principally upon the correct definition of energy of the primary particle. Therefore, now we shall investigate the question about the type of nucleus-nucleus interaction from somewhat another point of view. Let us study the dependence of n_s upon the atomic weight of the primary nucleus A, the character of this dependence being defined by the type of interaction only. Let us assume, that only the overlapping volumes of the two colliding nuclei interact. The results of the calculation of the collisions between the

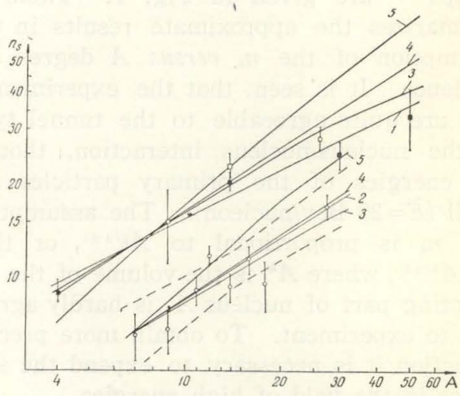

Fig. 1. The dependence of multiplicity upon the atomic weight of the primary nucleus A. 1.—shaded squares (using data [6]) $\bar{E}=20$ Bev/nucleon; the open squares refer to showers with $E>7$ Bev/nucleon [17] and were neglected in calculation. 2.—circles for $E>1.5$ Bev/nucleon [17]; The dotted lines correspond to the 95 per cent error limits. 3.—$n_s\sim A^*(l^*+1)$, where the size of the tunnel l^* is defined by the relation of masses of the overlapping volumes of the target nucleus A^*_{target} and the primary nucleus A^*: $l^*=A^*_{target}/A^*$. 4.—$n_s\sim A^{2/3}$; 5.—$n_s\sim A^*$.

$b_{N_h/E}$	$\tau_{n_s,E}$	$t_{0.01}\cdot\sigma_\tau$	$b_{n_s/E}$	$t_{0.01}\cdot\sigma_b$	c
	$0.42^{+0.36}_{-0.56}$	0.77	$0.10^{+0.08}_{-0.13}$		
-0.91	0.82 ± 0.08	0.10	0.15 ± 0.02	0.019	$0.74^{+0.08}_{-0.04}$
-0.30	0.94 ± 0.04	0.05	0.29 ± 0.01	0.015	$0.52^{+0.01}_{-0.02}$
-1.22	0.64 ± 0.11	0.14	0.17 ± 0.03	0.039	$0.70^{+0.05}_{-0.04}$

Table II.

Experiment	$\overline{E}=20$Bev/nucleon	$n_s \sim A^{0.52\pm0.08}$
	$E>1.5$Bev/nucleon	$n_s \sim A^{0.66\pm0.12}$
Calculation	$n_s \sim A^* \cdot (l^*+1)$	$n_s \sim A^{0.56}$
	$n_s \sim A^*$	$n_s \sim A^{0.86}$

primary nuclei with atomic weight A and the nuclei of photographic emulsion, and the experimental data, obtained by different groups[6,17] are given in Fig. 1. Table II summarizes the approximate results in the assumption of the n_s *versus* A degree dependence. It is seen, that the experimental data are quite agreeable to the tunnel type of the nucleus-nucleus interaction, though the energies of the primary particles are small ($\overline{E}=20$ Bev/nucleon). The assumption that n_s is proportional to $A^{2/3}$ [6], or that $n_s \sim A^*$ [17], where A^* is the volume of the interacting part of nucleus A, is hardly agreeable to experiment. To obtain more precise deduction it is necessary to expend the statistics in the field of high energies.

§ 3. Multiplicity *versus* energy of primary α-particle

Let us investigate the n_s dependence upon E of the type

$$n_s = aE^b \, . \qquad (4)$$

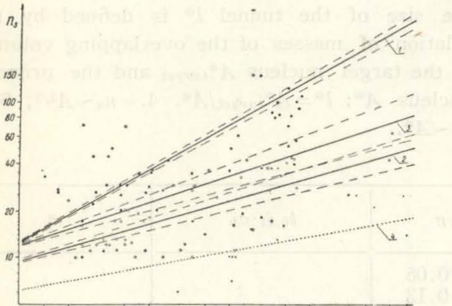

Fig. 2. The dependence of multiplicity upon primary particle energy for α-showers. 1.—dots for showers of $l \geq 2$ type. 2.—circles for showers of $l<2$ type. 3.—all the showers except quasi-nucleon-nucleon. 4.—crosses for quasi-nucleon-nucleon showers. The dotted lines correspond to the 95 per cent error limits.

The obtained result is quoted in columns 7–10 of Table I and in Fig. 2. It is evident, that the correlation between n_s and E for $l \gtrless 2$ types of interaction takes place, indeed, while it is not revealed for nucleon-nuclon showers. This result is agreeable to the data of Boos and Takibaev[8]. Showers of the $l \geq 2$ type which correspond to the head-on collision best of all give the most strong dependence of n_s upon E: $b=0.29\pm0.01$. With the exception of value b for quasi nucleon-nucleon showers, all the values of the exponents are valid with a high precision ($P=0.99$).

In the paper of Milehin[9] the multiplicity *versus* energy is defined by the equation of state. The calculated C values are also given in Table I.

As the difference in the exponents values obtained for $l \gtrless 2$ types of interaction is real (with a probability of $P=0.99$), the exponents values can not be united into one resultant value. Thus in the energy interval of $5\cdot(10^{10} \sim 10^{13})$ ev/nucleon

$n_s = 5.8(E/2M)^{0.1^{\pm 0.08}_{\pm 0.13}}$, quasi nucleon-nucleon showers

$$n_s = 9.0(E/2M)^{0.15\pm0.02}, \quad l<2$$
$$n_s = 11.5(E/2M)^{0.29\pm0.01}, \quad l \geq 2 \qquad (5)$$

$n_s = 11.4(E/2M)^{0.17\pm0.03}$, all the showers (nucleon-nucleon excluded),

where E is the α-particle energy per nucleon, M is the nucleon mass.

The Chicago group[6] in the energy interval 10–40 Bev/nucleon obtained $n_s \sim E^{0.40\pm0.08}$, and it was noted that with the higher energies the decrease of the exponent value is to be expected, which does not disagree with the results of the present work.

§ 4. The degree of the shower particles angular distribution anisotropy *versus* energy.

The results of the 76 nucleus shower analyses are given in Table III. The differential angular distributions for all of these showers have one maximum, and integral angular distributions in the coordinate system of Duller-Walker[10] represent straight lines.

The dependence of the degree of anisotropy σ upon the primary particle energy in nucleus showers is plotted in Fig. 3. The open squares refer to α-showers and shaded squares, to the showers formed by nuclei ($Z>2$)*. It is quite clear that in both cases σ is subject to linear increase with the energy logarithm. Therefore subsequently we shall study all the nucleus ($Z\geq 2$) showers without division.

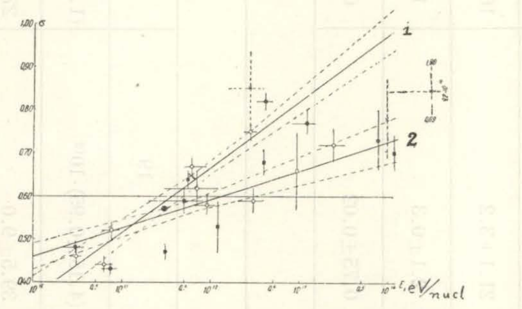

Fig. 3. The dependence of the degree of anisotropy σ upon the primary particle energy. 1.—dots and circles for nucleon-nucleon and nucleon-nucleus showers; 2.—shaded and open squares for α-nucleus and nucleus-nucleus showers. The dotted lines stand for the corresponding 95 per cent error limits. Crosses stand for quasi-nucleon-nucleon showers: it is seen, that as in the case of $n_s(E)$, $\sigma(E)$ for the above-mentioned showers is nearer to nucleon-nucleon than to nucleus-nucleus showers.

For the quantitative study of the relation between σ and E let us use the method of regression analysis. It would have been more correct to use the confluent analysis, but as in this case it may be led to the sequence of regression analyses[11] and $\sigma'=$ const, $\sigma''=0$, we shall confine ourselves to the first approximation:

$$\sigma=0.412+(0.066\pm 0.008)\lg(E/2M) \quad (6)$$

§ 5. The degree of anisotropy *versus* tunnel length

To solve this problem let us compare the angular distribution of the showers, formed by nucleons with those formed by nuclei. It is evident that, in showers that are considered to be nucleon-nucleon, the tunnel length l approximates 1, while in showers formed by nuclei $l>1$. The $\sigma(E)$ dependence for showers formed by nucleons may be obtained by using data [12].

* Such a division is due to the different statistics for these two groups of showers.

The results of the calculation are also given in Table II and in Fig. 3. As it has been already noted[12),13)] the nucleon-nucleon and the nucleon-nucleus showers give $\sigma(E)$ dependence coinciding within the limits of errors, that can be presented as follows:

$$\sigma=0.268+(0.146\pm 0.009)\lg(E/2M)\,. \quad (7)$$

It is obvious that with $E>10^{12}$ ev/nucleon the 95 per cent error limits of $\sigma(E)$ for nucleon and nucleus showers do not overlap. At lower energies there is practically no difference.

§ 6. Nucleus showers and angular distribution with two maxima

The received dependence of the degree of angular distribution anisotropy upon the primary particle energy for the nucleon and nucleus showers allows to elucidate why up to now only individual cases of nucleus showers with two-maximum angular distribution[14)] have been registered. As it was noted by Polish physicists[12)], the two maximums may be revealed only if $\sigma>0.6$. Showers formed by nucleons get into this region at $E\gtrsim 5.10^{11}$ ev/nucleon, and the nucleus showers at $E\gtrsim 5.10^{12}$ ev/nucleon, wherein the number of processed showers in general is very small.

§ 7. The inelasticity of showers formed by α-particles

As the direct energy measurement is possible only for a limited number of showers, we use the transverse momentum constancy to estimate K. In our laboratory* and by authors [14,15] the distribution and the average $P_\perp$ value in showers formed by nuclei ($Z\geq 2$) are the same as in the nucleon-nucleon showers. This allows to use already elaborated methods for the nucleus shower analysis. Let us define, as usual, the inelasticity coefficient (in the center-of-mass-system)[8)]

$$K=0.75 \times\frac{P_\perp\mu}{M(\gamma_c-1)}\sum_{i=1}^{n_s}(\gamma_c\sqrt{Z^2+\mathrm{ctg}^2\theta_i}-\sqrt{\gamma_c^2-1}\,\mathrm{ctg}\,\theta_i)\,. \quad (8)$$

The results of the analysis for the various types of α-showers are given in Fig. 4. The data on nucleon-nucleon showers deduced

* Unpublished.

Table III. The energetic relation of the observed characteristics of nucleon-nucleon, nucleon-nucleus and nucleus-nueleus showers. The dispersion of angular distribution

$$\sigma^2 = 1/n_s \sum_{i=1}^{n_s} (\lg \operatorname{tg} \theta_i - \overline{\lg \operatorname{tg} \theta_i})^2$$

φ—the angle of inclination of the integral angular distribution line;

$[\lg (\gamma_c \operatorname{tg} \theta), \lg F(\theta)/1-F(\theta), F(\theta)$—the part of tracks with angles smaller than $\theta]$.

$P(\chi^2 > \chi_q^2)$—the probability of the alignment of the studied differential angular distributions with those of Gaussian type; the measure of two maximums

$$\mathfrak{D} = (N_e - N_i)/\sum n_s$$

N_i—the number of particles contained in the inner part of the differential angular distribution in the range of $\pm\ 0.674\sigma$

N_e—the number of particles in the outer part.

Type of showers	E ev/nucleon	10^{10} — 10^{11}		10^{11} — 10^{12}	10^{12} — 10^{13}	10^{13} — 10^{14}	
nucleon-nucleon	Number of showers	38	22	22	43	9	
	$\overline{E}$ ev	$(3.0\pm0.5)\cdot10^{10}$	$(6.5\pm1.4)\cdot10^{10}$	$(6.02\pm1.28)\cdot10^{11}$	$(2.82\pm0.43)\cdot10^{12}$	$(2.48\pm0.83)\cdot10^{13}$	
	$\overline{n_s}$	9.8 ± 1.6	11.6	21.2 ± 4.5	21.1 ± 3.2	16.2 ± 5.4	
	$\overline{N_h}$	5	1.3	2.4 ± 0.5	2.1 ± 0.3	1.7 ± 0.6	
	σ	0.46 ± 0.02	0.44 ± 0.02	0.67 ± 0.02	0.75 ± 0.02	0.72 ± 0.04	
nucleon-nucleon	$\overline{E}$ ev			3.10^{11}			
nucleon-nucleus	σ			0.57			
nucleon-nucleus	Number of showers	29		3	19	11	1
	$\overline{E}$ ev	$(7.4\pm1.4)\cdot10^{10}$		$(6.95\pm4.02)\cdot10^{11}$	$(4.15\pm0.95)\cdot10^{12}$	$(1.23\pm0.37)\cdot10^{13}$	8.10^{13}
	$\overline{n_s}$	15.9 ± 2.9		33.3 ± 19.1	39.5 ± 9.0	27.3 ± 8.2	44

$\overline{N}_h$	8	24.0±13.8	16.3±3.7	14.4±4.3	6
σ	0.43±0.01	0.62±0.04	0.82±0.02	0.77±0.03	0.73±0.07

α-nucleus	Number of showers	26	17	9	1
	$\overline{E}$ ev	$(7.5\pm1.5)\cdot10^{10}$	$(9.0\pm2.2)\cdot10^{11}$	$(3.05\pm1.0)\cdot10^{12}$	$9.45\cdot10^{12}$
	$\overline{n}_s$	18.4±3.6	17.3±4.2	23.9±8.0	29
	$\overline{N}_h$	10.5±2.1	5.5±1.4	4.7±1.7	3
	σ	0.52±0.02	0.58±0.02	0.59±0.03	0.66±0.09
	$\mathrm{tg}\,\varphi$	1.53	1.37	1.34	
	$P(\chi^2>\chi^2_q)$	0.13	0.16	0.02	
	$\mathfrak{D}$	−0.06±0.05	−0.14±0.06	−0.08±0.07	

nucleus-nucleus	Number of showers	15	3	1	1	1	1	1
	$\overline{E}$ ev	$(3.0\pm0.8)\cdot10^{10}$	$5\cdot10^{11}$	$5.5\cdot10^{11}$	$3.1\cdot10^{11}$	$1.2\cdot10^{12}$	$4.0\cdot10^{12}$	$1.2\cdot10^{14}$
	$\overline{n}_s$	48.2±12.4	120	515	327	37	210	160
	$\overline{N}_h$	19.5±5.0	17	15	15	1		18
	σ	0.48±0.01	0.59±0.08	0.64±0.02	0.47±0.02	0.53±0.06	0.68±0.03	0.70±0.04
	$P(\chi^2>\chi^2_q)$	0.001	0.02					
	$\mathfrak{D}$	−0.26±0.04	+0.07±0.04					

from the observations of Boos and Takibaev[8] are quoted for comparison.

It is seen that for $l<2$ showers the inelasticity coefficient decreases with the energy much more less than in the case of nucleon-nucleon showers.

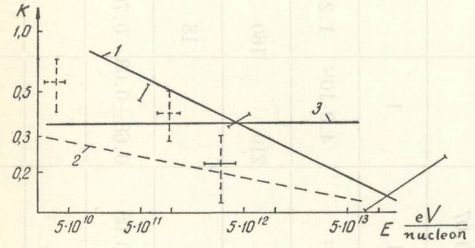

Fig. 4. The dependence of inelasticity upon the primary particle energy. 1.—nucleon-nucleon showers (according to data [8]); 2.—$l<2$-showers; 3.—$l\geq 2$-showers.

In the showers of $l\geq 2$ type the K value does not depend on the energy and in the terms of given definition is 0.4~0.6.

The obtained dependences $n_s(E)$ and $\sigma(E)$ contain significant information. As for the greater part of the showers the angular distribution in the laboratory system is of a Gaussian type in coordinates $\lg \operatorname{tg} \theta$, using method [16] we may write equation[8] as follows:

$$K=f[n_s(E), \sigma(E), l] \, . \tag{9}$$

Thus we obtain dependences $K(E)$ for nucleon-nucleon, $l<2$ and $l\geq 2$ showers that are also plotted in Fig. 4. It is seen that they reflect clearly the dependences of K upon E, obtained only on the basis of assumption that $P_\perp=$const..

Conclusions

Comparing the results obtained for the nucleus showers with the data obtained for the nucleon-nucleon showers we are led to the following deductions:

1. When colliding with the emulsion nuclei, high-energy nuclei interact with the tunnel. In the case of primary nucleons the tunnel model is applicable only at extremely high energies[2].

2. The multiplicity dependence upon energy in showers, formed by nucleons and nuclei are different: for α-showers in the energy region $5(10^{10}\sim 10^{13})$ ev/nucleon

$$n_s\sim(E^{0.15\pm0.03}\sim E^{0.29\pm0.01}), \quad \tau_{n_s,E}=0.65\sim0.95$$

while for nucleon-nucleon showers[8]

$$n_s\sim E^{0.05}, \qquad \tau_{n_s,\gamma_c}\sim0.3$$

3. For nucleus showers $\sigma\sim(0.066\pm0.008)\times\lg(E/2M)$, while in showers formed by nucleons $\sigma\sim(0.146\pm0.009)\lg(E/2M)$. The difference obtained can not be explained by the angular distribution dependence upon the tunnel length.

4. The angular distribution with two maximums for nucleus showers is available only at energies $\gtrsim 5.10^{12}$ ev/nucleon.

5. The inelasticity coefficient in the cases of head-on collisions of α-article with the emulsion nuclei does not depend upon the energy and is equal to $K=0.4\sim0.6$.

6. The N_h average value that may serve as a good measure for target nucleus excitation is defined by the tunnel volume and does not depend upon the primary particle energy:

$$\bar{N}_h=(0.85\pm0.1)A_\alpha{}^*(l^*+1),$$
$$\text{at } E>5.10^{10} \text{ ev/nucleon.}$$

The differences in the characteristics of the showers formed by the nucleons and the nuclei lead to the conclusion that the nucleus-nucleus interactions do not come to the superposition of the individual nucleon-nucleon collisions.

The results of the present investigation allow to suggest that the nucleus-nucleus collisions involve mostly the head-on nucleon-nucleon collisions.

In conclusion it must be emphasized that the dependences obtained for α-showers tend toward the certain dependences for nucleon-nucleon showers, when we pass from $l\geq 2$ to $l<2$ and further to quasi nucleon-nucleon interactions. This fact allows to assume that systematic errors are not of main importance.

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