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### *Discussion*

**Zatsepin, G. T.:** The idea of Emelyanov and Rosental is very nice, but the basic idea of Landau's hydrodynamical theory ($p/\varepsilon=1/3$) has no experimental foundation. Experiments show that $p/\varepsilon<1/3$.

**Zhdanov, G. B.:** Yes, I agree with you, but the main point of the authors was to show that there is no direct contradiction between the predictions of the Landau theory and the experimentally observed distributions.

**Namiki, M.:** I wonder that the constancy of $\zeta$ was assumed overall processes from very high temperature to very low temperature. Can we consider the results to have only a qualitative meaning?

**Zhdanov:** I suppose your point of view to be correct. The authors get only qualitative conclusions.

**Yamaguchi, Y.:** Are there any difference between $\pi$-$N$ and $N$-$N$ interactions in this theory? For example, do two fire-balls occur also in $\pi$-$N$ collisions with fair probability?

**Zhdanov:** I suppose that there must be no significant difference between $\pi$-$N$ and $N$-$N$ interactions in this respect.

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# III-7-11. Many-Body Theoretical Approach to the Fire-Ball Model in Multiple Production of Particles

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The "fire ball" observed in multiple production of particles is interpreted and analysed as a correlated assembly of produced particles like superfluid. It is shown that the momentum spectrum of produced mesons has a form $dk/k^2$ assuming $\lambda\phi^4$ for the final meson-meson interaction Hamiltonian. A possibility of the many fire-ball model is briefly discussed. Before discussing these problems a possible reduction of the S-matrix element or the cross section of the overall process is presented, in which its final interaction part may be separated from the part of high energy interaction in short collision time.

## §1. Introduction

About twenty years ago Heisenberg[1)] and others expected that the multiple production of particles in super-high energy phenomena would inform us about the interaction mechanism at a short distance among elementary particles or the fundamental principle as symbolized by the "universal length". Nevertheless, such a naive anticipation has not been achieved yet. It is true that the present accuracy and statistics of experimental data are not enough to derive some definite conclusions from the phenomena, but one may say that the essential reason of the above situation is the smearing effect of the troublesome final-state interactions among produced particles. It seems to us that the final-state interactions, being rather low energy interactions, smear the true aspect of the elementary interactions at super-high energy and then are responsible for the final distributions of produced particles. Another reason may be that considerable parts of the phenomena are fairly well explained by semi-kinematical arguments[2]. If all the phenomena were understood through the final-state interactions and semi-kinematics, we could derive few knowledges of the elementary interactions at super-high energy*. In order to see the characteristic feature of the elementary interactions at super high energy, we should separate the kinematical effects and clean up the smearing effects of the final interactions from the experimental data.

As an important effects of the final-state interactions, Fermi[3] and Landau[4] considered that they bring produced particles to be in equilibrium or in local equilibrium. Niu[2] also assumed in his model that mesons in each fire-ball are subject to Planck's distribution law. Following Landau, the meson cloud at the final stage may be considered to be in a frozen state with a very low temperature. This is consistent with the assumption in Niu's model. Now one may expect that the meson cloud (or the fire-ball) with a very low temperature is in a correlated state like the condensed one of superfluid. In the present note we shall study the correlation effects of Bogoliubov's type on the final distributions of produced particles. The purpose of this work is to increase the detailed knowledge of the final-state interaction effects.

Before entering into this work, we shall write down a possible reduction of the $S$-matrik element and the cross section of the overall process.

* It is noted that one extra meson, which has extremely high energy in the so-called "$d$-group" observed by Tokyo group of jet study, is evidently non-thermal and never smeared by the final state interactions.

## § 2. A possible reduction of cross section

Suppose that the Hamiltonian governing the overall process can be written as $H=H_0+H'$, in which $H_0$ is the Hamiltonian governing the asymptotic processes containing the final-state interactions and $H'$ stands for the super-high energy interaction in the collision time. We do not know whether the interactions in the collision time can be distinguished from others and described in the conventional Hamiltonian formalism. However, if the collision time is much shorter than the final interaction time, then it may be possible to make use of the above scheme of description at least in an asymptotic sense.

Now by denoting an eigenstate of $H$ belonging to its eigenvalue $E_i$ by $\Psi_i^{(\pm)}$, the $S$-matrix element of the process (I→F) is defined by $S_{\rm FI}=\langle\Psi_{\rm F}^{(-)}|\Psi_{\rm I}^{(+)}\rangle$ or by

$$S_{\rm FI}=-2\pi i\delta(E_{\rm F}-E_{\rm I})\langle\Phi_{\rm F}^{(-)}|H'|\Psi_{\rm I}^{(+)}\rangle\,, \qquad (1)$$

where $\Phi_i^{(\pm)}$ is the eigenstate of $H_0$ belonging to its eigenvalue $E_i$. The superscript $(\pm)$ corresponds to the well-known boundary conditions. Equation (1) can be modified as

$$S_{\rm FI}=-2\pi i\delta(E_{\rm F}-E_{\rm I})\langle\Phi_{\rm F}^{(-)}|T|\Phi_{\rm I}^{(+)}\rangle\,, \qquad (2)$$

where $T$ is the modified $T$-matrix. Assuming a separated motion of nucleons and meson clouds after collision, we can divide $H_0$ into two parts as $H_0=H_\pi+H_N$ and factorize $\Phi_i^{(+)}$ in the way $\Phi_i^{(\pm)}=\chi_i^{(\pm)}\varphi_i^{(\pm)}$. Here $H_\pi\varphi_i^{(\pm)}=E_{i(\pi)}\varphi_i^{(\pm)}$ and $H_N\chi_i^{(\pm)}=E_{i({\rm N})}\chi_i^{(\pm)}$, where $E_i=E_{i(\pi)}+E_{i({\rm N})}$. It is noted that $\varphi_i^{(+)}=U_\pi(0,-\infty)\phi_i$ and $\varphi_i^{(-)}=U_\pi^\dagger(\infty,0)\phi_i$, $U_\pi$ being the time evolution operator governed by $H_0$ and $\phi_i$ the free meson state. Now we make a plausible assumption about the form of $T$, that is, $T=\sum_l c_l A_l^{(\pi)} J_l^{({\rm N})}$, where $A_l^{(\pi)}$ is a combination of meson operators and $J_l^{({\rm N})}$ of nucleon operators, respectively, and $c_l$ is a $c$-number. Thus we can write the rate of transition probability as

$$w_{\rm FI}=2\pi\delta(E_{\rm F}-E_{\rm I})\times|\sum_l c_l\langle\phi_{\rm F}|U_\pi(\infty,0)A_l^{(\pi)}U_\pi(0,-\infty)|\phi_{\rm I}\rangle\times\langle\chi_{\rm F}^{(-)}|J_l^{(N)}|\chi_{\rm I}^{(+)}\rangle|^2 \qquad (3)$$

where $\phi_{\rm F}$ is a free meson state containing $N$ mesons and $\phi_{\rm I}$ the free vacuum state.

It is plausible that the final-state interactions bring each produced meson cloud into a model state, say $\varphi_{\text{mod}}$, insensitive to the initial state and the production mechanism. Therefore we can put

$$\langle\phi_{\text{F}}|U_\pi(\infty,0)A_{\text{I}}{}^{(\pi)}U_\pi(0,-\infty)|\phi_{\text{I}}\rangle$$
$$\simeq\langle\phi_{\text{F}}|U_\pi(\infty,t_0)|\varphi_{\text{mod}}\rangle$$
$$\times\langle\varphi_{\text{mod}}|U_\pi(t_0,0)A_l{}^{(\pi)}U_\pi(0,-\infty)|\phi_{\text{I}}\rangle \qquad (4)$$

Here the separation is to be insensitive to the choice of $t_0$ in a broad range, so that $t_0$ is put zero in the first factor and put infinity in the second factor in an asymptotic sense. Using the definition given by Gell-Mann and Goldberger[5] for $U_\pi(\infty, 0)$ and the random phase approximation for its matrix element, we can easily obtain the formula

$$\langle\phi_{\text{F}}|U_\pi(\infty,0)|\varphi_{\text{mod}}\rangle$$
$$\simeq\left(\frac{i}{\tau_{\text{mod}}}\right)\left(E_{\text{F}(\pi)}-E_{\text{mod}}+\frac{i}{2\tau_{\text{mod}}}\right)^{-1}\langle\phi_{\text{F}}|\varphi_{\text{mod}}\rangle\,,$$

where $E_{\text{mod}}$ and $\tau_{\text{mod}}$ are, respectively, the energy and the life time of the model state. Then $w_{\text{FI}}$ becomes

$$w_{\text{FI}}=2\pi\delta(E_{\text{F}}-E_{\text{I}})$$
$$\times\left[\frac{2\pi}{\tau_{\text{mod}}}\delta(E_{\text{F}(\pi)}-E_{\text{mod}})|\langle\phi_{\text{F}}|\varphi_{\text{mod}}\rangle|^2\right]$$
$$\times\left|\sum_l c_l\langle\varphi_{\text{mod}}|U_\pi(\infty,0)A_l{}^{(\pi)}U_\pi(0,-\infty)|\phi_{\text{I}}\rangle\right.$$
$$\left.\times\langle\chi_{\text{F}}{}^{(-)}|J_l{}^{(N)}|\chi_{\text{I}}{}^{(+)}\rangle\right|^2, \qquad (5)$$

where we have used the approximation of the long life time. The model state may be a mixed state. In such a case the corresponding modification should be required in Eq. (5). In the case of two or more fire-balls, Eq. (5) becomes a sum of the corresponding terms.

Now we can derive the final distributions of produced particles from Eq. (5). The momentum spectrum is defined by

$$n(\boldsymbol{k})=C\sum_N N\sum_{\boldsymbol{k}_2,\cdots,\boldsymbol{k}_N}\delta\left(\omega_{\boldsymbol{k}}+\sum_{j=2}^N\omega_{\boldsymbol{k}_j}-E_{\text{mod}}\right)$$
$$\times|\langle\boldsymbol{k},\boldsymbol{k}_2,\cdots,\boldsymbol{k}_N|\varphi_{\text{mod}}\rangle|^2\,, \qquad (6)$$

where $C$ is a normalization constant and $\omega_{\boldsymbol{k}}=$ the energy of a free meson with momentum $\boldsymbol{k}$, and we have written $\phi_{\text{F}}$ in a more detailed form $|\boldsymbol{k}_1,\boldsymbol{k}_2,\cdots,\boldsymbol{k}_N\rangle$. Discarding the conservation law of energy in Eq. (6), we obtain the spectrum

$$n_0(\boldsymbol{k})=\sum_{\boldsymbol{k}_2,\cdots,\boldsymbol{k}_N}|\langle\boldsymbol{k},\boldsymbol{k}_2,\cdots,\boldsymbol{k}_N|\varphi_{\text{mod}}\rangle|^2$$
$$=\langle\varphi_{\text{mod}}|a_{\boldsymbol{k}}{}^\dagger a_{\boldsymbol{k}}|\varphi_{\text{mod}}\rangle\,, \qquad (7)$$

where $a_{\boldsymbol{k}}$ is the annihilation operator of a free meson. Thus the final distributions of produced particles can be calculated only by the model state vector $\varphi_{\text{mod}}$ representing the fire-ball. The relative probability of finding a given multiplicity and inelasticity depends upon the last factor of Eq. (5), that is, the production mechanism of the fire-ball. It is, however, remarked that even in the last factor of Eq. (5) the elementary interactions are smeared to some extent by the final-state interactions.

## § 3. Distribution of produced particles.

We may consider a model state of the fire-ball to be an analogue of the compound nucleus state in the theory of nuclear reactions. Mesons in a fire-ball move in a limited volume, collide with each other and subsequently fly away as free particles. It is acceptable that the linear dimensions of the fire-ball may be of the order of $(1/\mu)N^{1/3}$, $N$ being the multiplicity. First let us consider an idealized problem in which both the volume and the number of mesons are infinite but the meson density remains finite. Since temperature of the system is so low that the number $N_0$ of mesons with zero momentum is very large, we can derive the model Hamiltonian from $H_\pi$ by keeping only its first two leading terms in the order of $N_0$. The model Hamiltonian has a structure similar to that in the problems of superfluid, which can be diagonalized by the famous Bogoliubov transformation $\alpha_{\boldsymbol{k}}=u_{\boldsymbol{k}}a_{\boldsymbol{k}}+v_{\boldsymbol{k}}a^\dagger_{-\boldsymbol{k}}$. Here $\alpha_{\boldsymbol{k}}$ and $a_{\boldsymbol{k}}$ are, respectively, the annihilation operators of a quasi-particle and a free particle with momentum $\boldsymbol{k}$. $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$ are real even functions of $\boldsymbol{k}$ and satisfy the relation $u_{\boldsymbol{k}}{}^2=1+v_{\boldsymbol{k}}{}^2$.

Here we identify the fire-ball with a mixed state of low lying states with temperature of the order of the meson mass. In this case the expectation value of the meson number becomes

$$\bar{N}=N_0+\sum_{k\neq0}v_{\boldsymbol{k}}{}^2+\sum_{k\neq0}w_{\boldsymbol{k}}[1+2v_{\boldsymbol{k}}{}^2]\,, \qquad (8)$$

where $w_{\boldsymbol{k}}$ is the Planck's weight factor. Equation (8) implies that the momentum spectrum is a superposition of the ground state distributions, the first two terms, and the Planck's distribution of quasi-particles, the last term. Taking into account the finiteness of the system, we should read by $N_0$ the number of mesons with the minimum momentum of the order of $(\mu/\bar{N}^{1/3})$. For interaction of the type $\lambda\phi^4$, it is easy to show that $v_k^2$ is a function proportional to $k^{-4}$ for $k \gg \mu$. By approximating the summation by the integral, it is concluded that the momentum spectrum has a form of $(dk/k^2)$ for

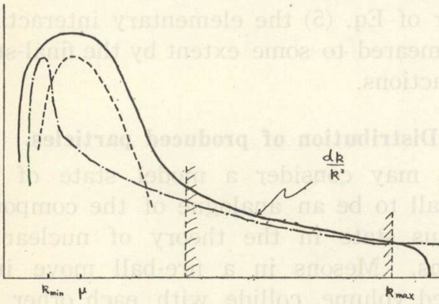


Fig. 1. Momentum spectrum: Solid line for the resultant curve, broken line for the ground state distribution and dotted line for the quasi-particle distribution.

$k > \mu$. The spectral curve is given in Fig. 1. The cut-off at $k_{max}$ of the resultant spectrum comes from the $\delta$-function of Eq. (6). One may regard $k_{max}$ as a momentum of the order of $\mu\bar{N}$ in the center-of-mass system of a fire-ball.

## § 4. Discussions on another possibility

We have so far discussed the system with a large volume but low temperature. Here we discuss briefly emission of mesons from the system with a small volume but high temperature $T$. Because of high temperature we no longer assume that $N_0$ is large. The method suggested by Bogoliubov and Zubarev[6] permits us to define the temperature dependent normal modes of quasi-particles, whose momenta distribute over a discrete spectrum because of a small volume. The number of mesons transported by one quasi-particle with momentum $k$ is $1+2v_k^2$, which is unity for $k > T$ and several for $k \leq T$. (Note that $v_k$ is a coefficient of the Bogoliubov transformation to a temperature dependent normal mode.) Now let us suppose that immediately after the collision a small and hot meson compound is left at rest in the center-of-mass system and subsequently free mesons are emitted from it through the quasi-particle modes. Such a scheme may be consistent with an interesting analysis recently given by Hasegawa[7]. Since excitation of the quasi-particles at high temperature originates in considerably high energy interactions, one may expect to derive some knowledge of high energy interactions from the experimental data of Hasegawa's type. Interpretation is, of course, open to other possibilities.

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## *Discussion*

**Zhdanov, G. B.:** Have you any consideration about the life time of the fire-ball (in the frames of your method of calculations) or should it be taken from the experiment?

**Namiki, M.:** I have not calculated the life time of the fire-ball, but it can be calculated in the framework of our theory. Certainly the life time of the fire-ball is longer than the Compton period corresponding to the fire-ball mass.

I think that, if the masses of the fire balls distribute over continuous spectrum, it is hard to determine the life-time from the experiment.