$$\rho_H=(3\lambda/\Delta E)(V/N)^2(2m^5)^{1/2}\langle(M-\langle M\rangle)^3\rangle \times F_2F_0^2/\hbar^4e^2E_F^{1/2}, \quad (10)$$

where M denotes the z component of a localized spin. The factor, $\langle(M-\langle M\rangle)^3\rangle$, represents the temperature variation of the Hall resistivity. It is remarkable that the lattice and impurity scatterings do not contribute to the Hall resistivity, although they do to the Hall conductivity. If we take

$$(\lambda/\Delta E)F_2F_0^2=-0.03\ (\text{eV})^3 \quad \text{for nickel}$$
$$=\ 0.12\ (\text{eV})^3 \quad \text{for iron,}$$

the agreement between (10) and experimental results[5] is such as shown in Fig. 1.

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DISCUSSION

J. ELLIOTT: Is the temperature dependence of your result the same as the ρ^2 dependence predicted by Luttinger?

J. KONDO: Incidentally it has the similar temperature dependence as that by Luttinger as long as the content of impurity atoms is small.

J. ELLIOTT: Some years ago, a student of mine, Mr. Roycroft, began a similar calculation, which was not completed. He noted that the spin orbit coupling in the conduction electrons produced an exactly similar but probably smaller matrix element.

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Diagrammatic Expansion for the Heisenberg Ferromagnet with Arbitrary Spin and Range of Interaction

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A diagrammatic expansion method is presented, in which each diagram has a particularly simple structure and in which "excluded volume" corrections are eliminated. Summation of renormalized loop diagrams gives an approximation applicable in all temperature ranges.

The statistical mechanics of a ferromagnet has been treated satisfactorily only in the limits of low temperatures (the spin wave theory) or of high temperatures (expansion in T_c/T). We here describe an approximate theory based on a new form of linked cluster expansion for spin variables, and on a partial summation of selected sub-series. Separate criteria for the selection of the sub-series are applicable in different temperature regions, so that the series to be summed are those containing all diagrams which are dominant at any temperature. The resultant theory is then approximately valid through the entire temperature range, including the particularly interesting neighborhood of the transition temperature.

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The Hamiltonian of a Heisenberg ferromagnet is the sum of the Ising Hamiltonian $H_I = -\sum_{ij} J_{ij} S_{iz} S_{jz} - g\mu_0 H \sum_i S_{iz}$ and of the "transverse Hamiltonian" $H_T = -\sum_{ij} J_{ij} S_i^+ S_j^-$. The exchange integral J_{ij} is assumed to be a function of the relative distance $|\boldsymbol{R}_i - \boldsymbol{R}_j|$ only, but is not restricted to nearest neighbors. We introduce the spin deviation operator $\sigma_i = \bar{S} - S_{iz}$, where $\bar{S}$ is to be chosen subsequently. The Ising Hamiltonian becomes the sum of the zero-order Hamiltonian $H_0 = -N\bar{S}(J_0\bar{S} + g\mu_0 H) + (2J_0\bar{S} + g\mu_0 H)\sum_i \sigma_i$ and of the Ising perturbation $V = -\sum_{ij} J_{ij}\sigma_i\sigma_j$. The free energy can be written as $-\beta F = -\beta F_0 + \ln \langle e^{-\beta V} e^{-\beta H_T} \rangle$, where the average is taken with the distribution function $e^{-\beta H_0}/\operatorname{tr} e^{-\beta H_0}$, and where $-\beta F_0 = \ln \operatorname{tr} e^{-\beta H_0}$. Series expansion produces three types of diagrams; pure Ising diagrams, transverse diagrams, and mixed diagrams involving a product of an Ising diagram and a transverse diagram.

By semi-invariant expansion and a particular type of dissociation and regrouping of the terms we establish that the Ising contribution to the free energy is a sum of all possible linked diagrams. Each diagram denotes a product of 1) the reciprocal of the order of the symmetry group of the diagram, 2) a product of semi-invariants for each spin in the diagram, the order of the semi-invariant equaling the number of bonds joined to that spin, 3) a sum of products of exchange integrals $\sum_{ijk\cdots} (2\beta J_{ij})(2\beta J_{kl})\cdots$, without restrictions on the summation indices (no "excluded volume" corrections). In the vicinity of the transition temperature Brout's criterion of expansion in the reciprocal of the effective number (z) of neighbors indicates that the dominant diagrams are the Caylee trees (having no closed loops). Summation of these diagrams gives the familiar molecular field approximation. However considerations of self-consistency suggest renormalization procedures whereby each vertex contains implicitly the contribution of all simple diagrams affixed to that vertex in a reducible manner (i.e., so that the vertex is an articulation point). Such renormalizations can be carried out for the Ising diagrams, and appropriate choice of $\bar{S}$ then eliminates all reducible diagrams. Generalizing Brout's $1/z$ criterion to the renormalized diagrams, and summing all simple loops (first order in $1/z$), gives a resultant free energy contribution which is quite similar to that of the spherical model.

The transverse diagrams are intrinsically restricted to closed loops. The transverse loops can be renormalized partially, and sufficiently to maintain the desired self-consistency and certain variational properties of F. Renormalization of the transverse loops with Ising loops includes the combination diagrams. Summation of those diagrams containing only a single transverse loop gives the simple spin wave result at low temperatures. The Dyson corrections to spin wave theory arise from multiple transverse loops, but we have not yet calculated these effects explicitly.

DISCUSSION

W.C. MARSHALL: I do not see how the Ising and transverse parts of the Hamiltonian can commute as you have assumed.

H. B. CALLEN: My statement that they commute is not correct, but the non-commutativity is easily accounted for in the approximation which we have used, that is, of single transverse loops. The non-commutativity of the transverse operators is the chief difficulty in the extension of the approximation to higher orders.