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Theory and Application of Spin-Wave Instability in a Microwave Magnetic Field Applied Parallel to the dc Field*

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The subharmonic generation of spin waves at high power levels is discussed with particular emphasis on the use of this technique as a research tool for probing magnon-magnon and magnon-phonon interactions. Measurements of the instability threshold as a function of the dc field make it possible to determine the wave number dependence of the spin-wave relaxation rate, and to study magnon-phonon interactions. Useful information about relaxation mechanisms can also be obtained by measuring the susceptibility beyond the instability threshold and the radiation-induced dc magnetic moment.

1. Introduction

It has recently been recognized that a strong microwave magnetic field applied parallel to the dc field can give rise to parametric excitation of spin waves[1),2)]. The mechanism is quite similar to the one proposed by Suhl[3)] as an explanation of the nonlinear phenomena observed in conventional ferromagnetic resonance experiments with the rf magnetic field perpendicular to the dc field. Because of its relative simplicity the "parallel pumping" technique has proved very useful in determining rather fundamental properties of ferromagnetic materials.

According to the theoretical predictions of Kaganov and Tsukernik[4)] a ferromagnetic material will absorb power from a microwave magnetic field applied parallel to the dc field even at arbitrarily low power levels. This absorption is very small, however, and within the accuracy of the experiments described below it has not been detected. The nonlinear absorption discussed in this paper appears only at high power levels, namely, as soon as the rf magnetic field exceeds a well-defined threshold given by[1),2),5)]

$$H_{1crit}=\frac{\omega}{\omega_M}\mathrm{Min}\frac{\Delta H_k}{\sin^2\theta_k}. \qquad (1)$$

Here ω is the pump frequency, ω_M equals $\gamma 4\pi M$, γ is the gyromagnetic ratio, M is the saturation magnetization, ΔH_k is the equivalent linewidth of a spin wave characterized by the propagation vector $\boldsymbol{k}$, and θ_k is the angle between $\boldsymbol{k}$ and the dc magnetic field. In Eq. (1) the minimum with respect to the propagation vector $\boldsymbol{k}$ must be taken subject to the side condition that the spin wave

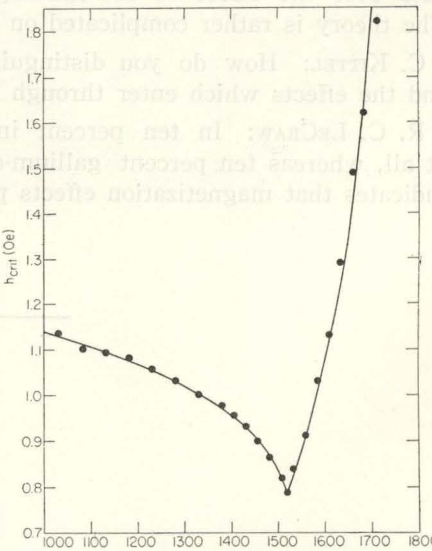

Fig. 1. Critical field for spin-wave instability under parallel pumping as a function of the external dc field. Measurements were made by U. Milano on a spherical single crystal of yttrium iron garnet at 9.42 kMc with the magnetic field applied parallel to a [111] direction. The measurements were taken with a pulsed microwave source with pulses of 4μs length.

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frequency ω_k be equal to half the pump frequency.

2. *k*-dependence of relaxation rate

By varying the dc magnetic field, keeping the frequency constant, the wave number of the spin wave having the lowest instability threshold can be varied. In this way the *k*-dependence of the relaxation rate can be inferred from measurements of the critical rf field as a function of the dc field. Figure 1 shows typical experimental results obtained on a spherical single crystal of yttrium iron garnet. Following a suggestion of LeCraw and Spencer we shall refer to such curves as "butterfly curves". The sharp minimum of the butterfly curve occurs at a dc field for which the upper edge of the spin-wave band ($\theta_k=\pi/2$) extrapolated to $k=0$ corresponds to half the pump frequency. The sharp increase on the high field side of the minimum is due to the fact that the frequency condition cannot be satisfied any more for $\theta_k=\pi/2$. The slower increase on the low field side reflects the wave-number dependence of the relaxation rate. In Fig. 2 a similar set of experimental

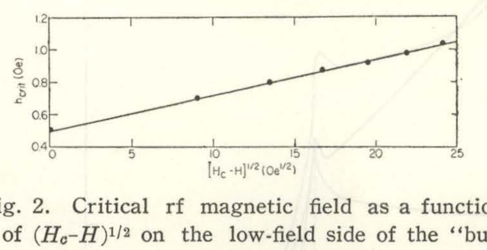

Fig. 2. Critical rf magnetic field as a function of $(H_c-H)^{1/2}$ on the low-field side of the "butterfly curve". Measurements were taken by U. Milano at 9.19 kMc on a spherical sample of yttrium ipon garnet, using a square-wave modulated source.

data is plotted as a function $(H_c-H)^{1/2}$. The fact that the points fall on a straight line indicates that to a good approximation the relaxation rate of spin waves propagating at right angles to the dc field increases linearly with the wave number[6),7),8)]. This behavior is theoretically expected since the three-magnon confluence process leads to a relaxation rate which is proportional to the wave number[9)]. Green and Schlömann[10)] have made a detailed quantitative comparison between theory and experiment and have found reasonably good agreement. From the experimental data one can also infer the dependence of the relaxation rate on the angle θ_k between the direction of propagation and the dc field for very small wave numbers. Experiments on yttrium iron garnet containing appreciable amounts of rare-earth substitution (0.5% Yb or 0.3% Tb) have indicated[11)] that the contribution of the rare earth ions to the relaxation rate is not dependent on the magnitude or the directions of the propagation vector.

3. Phonon interaction

By adjusting the dc field it is possible to realize a situation wherein the unstable spin waves have the same frequency and wave number as lattice phonons. It is expected theoretically that under these conditions the instability threshold will be raised by the interaction (one magnon—one phonon process)[12)]. This effect was first observed by Turner[7)] in experiments at 35 kMc. At lower frequencies the effect is considerably less conspicuous in agreement with the theoretical predictions[12)]. Figure 3 shows experimental

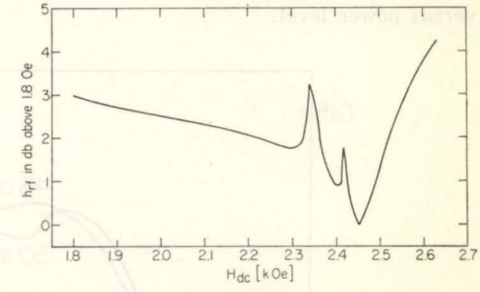

Fig. 3. Phonon peaks in the butterfly curve measured at 18 kMc in yttrium iron garnet by J. R. Zeender.

results obtained at 18 kMc. The large peak in the butterfly curve arises from interaction with transverse phonons, the small peak from interaction with longitudinal phonons. The presence of the longitudinal peak is not completely understood at this moment. According to the theory[12)] only the transverse branch of the phonon spectrum should interact with the magnons excited in this experiment ($\theta_k=\pi/2$). From the location of the phonon peaks and the velocity of sound the exchange constant can be determined[7),13)].

4. High power susceptibility

The experiments described in the preceding sections are basically a determination of the instability threshold. According to the theory described in refs. 1, 2, and 5, the potentially unstable spin waves grow exponentially as soon as the instability threshold is exceeded. If this were rigorously correct, the absorption coefficient (imaginary part of the susceptibility) would be infinite in the region above the threshold. Actually, the absorption coefficient remains fairly small and, in the region well above the threshold (approximately 10 db), decreases with increasing power level[14]. Figure 4 shows a typical saturation curve. The bumps in the experimental curve are caused by the appearance of "relaxation oscillations" and should be disregarded for the present purpose. The theoretical curve is obtained by taking account of the fact that the power absorbed by the sample increases the temperature of the modes (magnon and phonon) that determine the relaxation rate of the parametrically excited spin waves. Thus a steady-state situation is established, in which the damping is just sufficient to balance the power absorbed from the electromagnetic field. Calculations based on this picture[15] give results which are at least qualitatively in agreement with the experimental data.

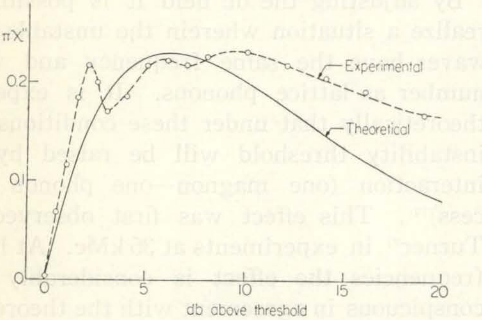

Fig. 4. Imaginary part of the longitudinal susceptibility of a sphere of yttrium iron garnet versus power level.

Of particular interest in this connection is a suggestion by Suhl and Gottlieb[16] that under certain conditions the three-magnon confluence process may contribute directly to the saturation of the absorption. The physical conditions for this to occur are that two parametrically excited magnons (both propagating perpendicular to the dc field and having a frequency equal to half the pump frequency)

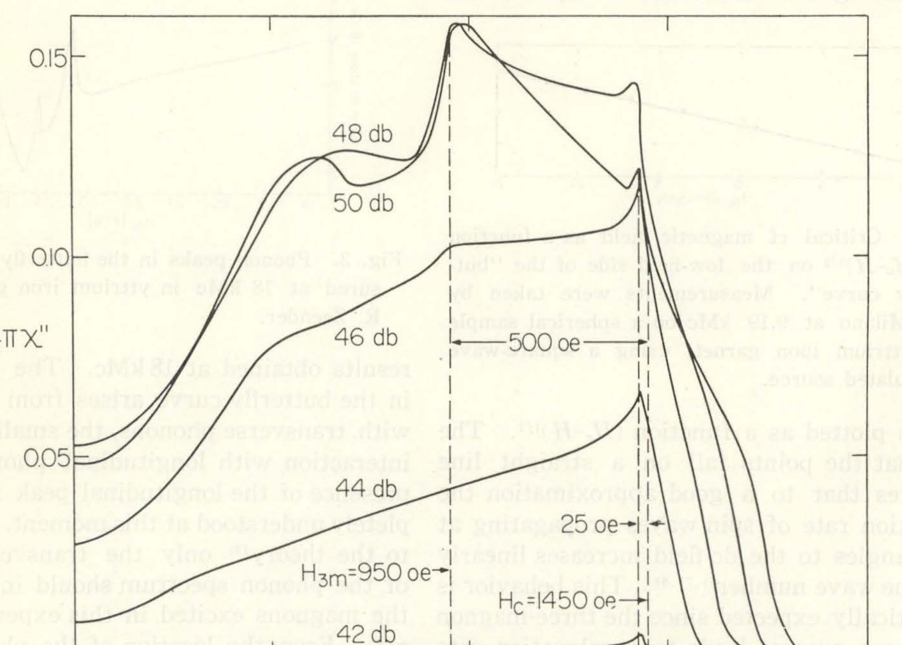

Fig. 5. Imaginary part of the susceptibility vs dc magnetic field. Measurements by J. J. Green at various incident power levels on yttrium iron garnet with ½ percent holmium substitution at a frequency of 9.082 kMc. The lowest instability threshold corresponds to an attenuator setting of 41 db.

must be able to coalesce into a third magnon under conservation of energy and momentum. The detailed calculation first carried out by Suhl and Gottlieb[15),16)] shows that this should occur at all dc fields smaller than a certain characteristic value (which depends on the frequency and the saturation magnetization). This effect has been experimentally detected by Green[17)] who has measured the high power absorption coefficient as a function of dc field in various materials at three frequencies (5.6, 9.1 and 18 kMc). Figure 5 shows typical experimental results demonstrating that the curves obtained at very high power levels have a characteristic structure with three well defined peaks. The lowest one of these apparently reflects the fact that the spherical samples does not become fully magnetized until the applied field reaches approximately 600 oe. No conclusive explanation of this peak can be offered at the present time. The peak at 950 oe. is attributed to the three-magnon saturation process described above. It has been found[17)] that the characteristic field H_{3m} varies with frequency and saturation magnetization in the manner theoretically expected. The high field peak in Fig. 5 is attributed to magnon-phonon interaction.

5. Radiation-induced dc magnetic moment

Spin-wave excitation should generally be accompanied by a decrease of the z-component of the magnetic moment. The magnitude of this effect to be expected under parallel pumping conditions may be calculated with the help of Fig. 6. On the low field side of the butterfly curve the two axes of the precession ellipse are proportional to $(H+Dk^2)^{1/2}$ and $(H+Dk^2+4\pi M)^{1/2}$ respectively. A simple calculation shows that this implies that for reasonably small precession angles the ratio of the radiation-induced dc moment M_1 to the amplitude M_2 of the rf moment should be

$$M_1/M_2=[1+(\omega/\omega_M)^2]^{1/2} \qquad (2)$$

provided that the change in the dc magnetic moment is entirely due to the parametrically excited spin waves. This condition breaks down if the relaxation is largely caused by the "splitting process."[9),10)] Recent experiments by Kohane[18)] have shown that in the region just above the instability threshold the magnitude of the radiation-induced dc

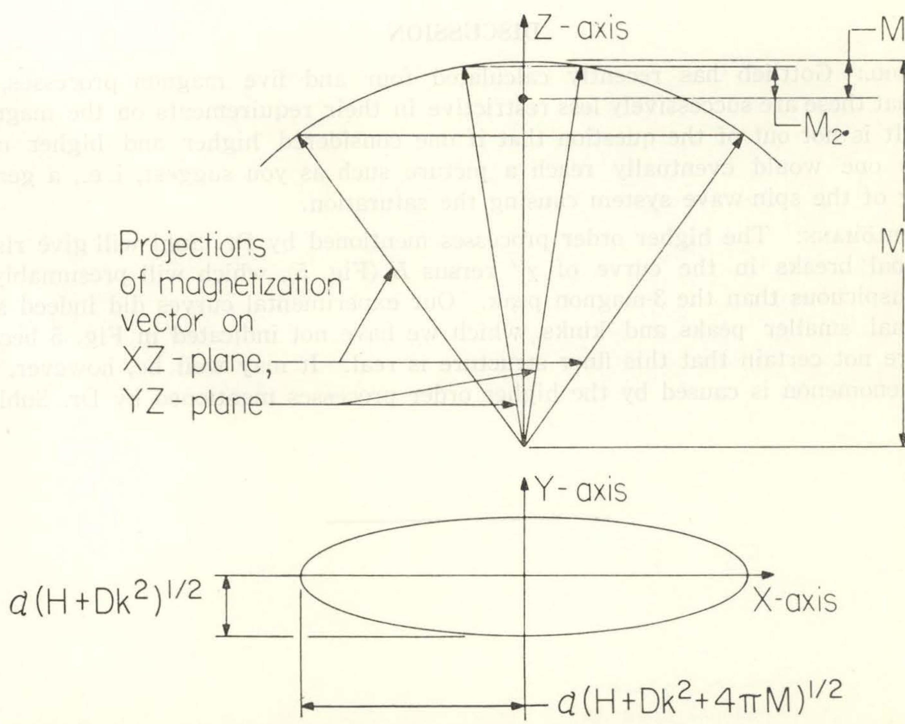

Fig. 6. Definition of dc and rf magnetic moments induced by a strong rf magnetic field applied parallel to the dc field.

moment is substantially in agreement with Eq. (2).

Acknowledgment

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DISCUSSION

H. SUHL: Gottlieb has recently calculated four and five magnon processes, and finds that these are successively less restrictive in their requirements on the magnetic field. It is not out of the question that if one considered higher and higher order process one would eventually reach a picture such as you suggest, i. e., a general heating of the spin-wave system causing the saturation.

E. SCHLÖMANN: The higher order processes mentioned by Dr. Suhl will give rise to additional breaks in the curve of χ'' versus H (Fig. 5), which will presumably be less conspicuous than the 3-magnon peak. Our experimental curves did indeed show additional smaller peaks and kinks, which we have not indicated in Fig. 5 because we were not certain that this finer structure is real. It may well be, however, that this phenomenon is caused by the higher order processes mentioned by Dr. Suhl.