($\lambda^2 t$) being small. This time range can obviously be appreciable when λ itself is small.

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DISCUSSION

Dexter, D. L.: Would you please discuss the application of this formulation to the calculation of observable quantities?

Mahanty, J.: The summation of the series that occurs as the coefficient of ($\lambda^2 t$) is rather difficult for realistic lattice models. This makes it difficult to calculate observable quantities. For the linear chain with nearest neighbour interactions the series can be summed in closed form.

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Electron-Phonon Interactions Induced by Lattice Defects in Metals

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Koshino's mechanism for inelastic scattering of electrons by impurities in metals is modified, so that the perturbation Hamiltonian depends on the strain at an impurity, not on the displacement. Expressions are derived for the resulting electrical resistance and the corresponding additional phonon scattering. The effect of the local modification of the strain field of a phonon at the impurity is discussed.

S. Koshino[1] suggested that electrons are scattered inelastically by impurity ions, because of their thermal motion. This should lead to an additional electrical resistance and to substantial deviations from Matthiessen's rule. This suggestion is further explored.

While Koshino took the displacement of the impurity ion as a measure of the perturbation Hamiltonian, the displacement of the impurity ion relative to its neighbors seems a better measure of the distortion of the impurity field. Thus the perturbation Hamiltonian is taken as

$$H_p' = \int d\boldsymbol{r}\left(\frac{\partial V}{\partial \boldsymbol{r}}\cdot\boldsymbol{\varepsilon}\right)e^{i\boldsymbol{p}\cdot\boldsymbol{r}}u(\boldsymbol{q})(e^{i\boldsymbol{q}\cdot\boldsymbol{a}}-1) \qquad (1)$$

when $V(\boldsymbol{r})$ is the impurity potential, $\boldsymbol{p}$ the difference in wave-vector of the two electron states, $\boldsymbol{\varepsilon}$ the polarization and $\boldsymbol{q}$ the wave-vector of a lattice wave, the displacement of which at the impurity is $\boldsymbol{\varepsilon}u$, and $\boldsymbol{a}$ the distance between the impurity and the nearest neighbor. The integration is broken into a number of cones, each with its appropriate direction of $\boldsymbol{a}$. This expression may be compared directly to that for elastic scattering by $V(\boldsymbol{r})$ according to the Born approximation, and yields

$$\delta\rho_i = Ae^2\rho_0 \qquad (2)$$

where $\delta\rho_i$ is the increase in the temperature-dependent part of the resistivity, ρ_0 the residual resistivity, e^2 the mean square thermal strain, and A a numerical constant which, on the present model, is about 10 to 20. Thus $\delta\rho_i \propto T^4$ at low temperatures, and varies as T at high temperatures.

Since e^2 is always small, $\delta\rho_i$ is always small compared to ρ_0, but is not small compared to ideal resistivity ρ_i, particularly at low tempera-

tures. Thus for 1% Zn in Cu, $\delta\rho_i/\rho_i$ should be about 10–15% at 30°K, and more in alloys of larger ρ_0. Actually, P. Schroeder (private communication) found $\delta\rho_i/\rho_i \simeq 0.5_5$.

Inelastic scattering of electrons by impurities can be regarded also as an additional electron-phonon interaction, without momentum conservation. It should lead to an enhancement of the scattering of phonons by electrons, and thus cause a decrease in the low-temperature lattice thermal conductivity and lattice thermoelectric power, which is distinct from the decrease at somewhat higher temperatures attributable to the elastic scattering of phonons by these defects. The additional phonon scattering is described by a relaxation time τ', such that

$$\frac{1}{\tau'} \simeq \frac{2A}{\tau_0} \frac{(\hbar\omega)^2}{Mv^2\zeta} \qquad (3)$$

where A is the same as in (2), τ_0 the electron relaxation time corresponding to ρ_0, ω the phonon frequency, Mv^2 the elastic modulus per atomic volume, ζ the Fermi energy. This may be compared to the ordinary phonon-electron relaxation time τ, where

$$\frac{1}{\tau} \simeq 2\left(\frac{C}{\zeta}\right)\frac{\hbar\omega}{Mv^2}\omega_D \qquad (4)$$

where ω_D is the Debye frequency, C the electron-phonon interaction parameter. Typically $(C/\zeta)^2 \simeq 0.1$, so that for a 1% alloy the ratio between (3) and (4) is $100A\,\hbar\omega/\zeta$, and depending on the value of A, the effect of inelastic scattering on the lattice thermal conductivity should be apparent already at 5 to 10°K for an alloy of only 5% solute content or even less.

In actual fact, the lattice thermal conductivity of alloys is usually found to decrease markedly with solute content below about 10°K, and there is a rough correlation between this decrease and the residual resistivity[2]. The present theory leads to an additional thermal resistivity proportional to T^{-1}, while the lattice thermal conductivity at low temperatures varies approximately as T^2. However, measurements of the lattice thermal conductivity are not very precise; moreover, the lattice waves involved are not infinitely long, so that departures from the frequency dependence of (3) are possible. It is at present not certain whether the mechanism proposed here can account for the anomalous variation of lattice thermal conductivity with solute content. More measurements are needed, correlating lattice thermal conductivity with deviations from Matthiessen's rule given by (2).

Now the strain due to a phonon at a defect site is modified by an impurity or defect. In the case of spin lattice relaxation, where the strain at the defect determines the interaction, this is often important, and the modifications of the theory have been discussed by Castle, Feldman, and this author[3]. Analogous considerations apply here. Every defect has at least one characteristic frequency. If it lies above the Debye continuum, a local mode is formed. This local mode contributes to the resistivity like an Einstein oscillator, leading to an increase of the slope of ρ *versus* T around half its characteristic temperature. Indications of such a change were found around room temperature in Au-2% (at) Zn by D. H. Damon (private communication).

If the characteristic frequency falls in the Debye continuum, the defect vibrates out of phase for waves above that frequency, and the interaction is correspondingly enhanced. This should lead to an increase of $\delta\rho_i$ at higher temperatures above that given by (2). A corresponding effect has been found in the spin-lattice interaction of Cr in MgO and of E'-centers in quartz.[3),4)]

There is little experimental material, except that already referred to, which has a bearing on the Koshino mechanism; in particularly $\rho(T)$ of alloys has in general not been studied in the interesting temperature range $\theta/20 < T < \theta/10$. Large changes in lattice thermal conductivity with alloy composition have frequently been observed[2)], but it would be necessary to relate them to changes in $\rho(T)$, which has not been done. The predicted decrease in lattice thermoelectric power seems consistent with measurements of Blatt, *et al.* on copper alloys.[5)]

Koshino's suggestion has pointed the way to a wide range of systematic investigations on transport properties of alloys, and a possible explanation of some existing anomalies.

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DISCUSSION

Vineyard, G. H.: From your calculations, would you expect that an observable change in resistivity could be caused by an isotopic substitution that brought into existence a local mode, where none had existed before?

Klemens, P. G.: In order that the effect of the local mode should be clearly visible, its frequency must lie well above the Debye continuum, so that its strain energy is well concentrated at the defect, and so that its effect on the temperature dependence of the resistivity is sufficiently different from the highest continuum modes. I therefore do not expect any very marked discontinuous behaviour when a local mode is raised out of the continuum. Conversely, when the impurity atom is heavy, so that the resonance frequency ω_0 of the defect is below ω_D, the maximum frequency of the continuum, it is only for the waves of frequency between ω_0 and ω_D that the interaction is enhanced, so that if ω_0 is just below ω_D, the effect will again be quite small. Thus the variation with impurity mass will be quite smooth.

Seeger, A.: I should like to call your attention to the experiments of Magnuson, Palmer and Koehler (Phys. Rev. **109** (1958)) on deuteron irradiation damage in copper. These authors found considerable positive deviations from Matthiessen's rule in the temperature range from 20°K to 60°K, which is approximately the temperature range in which the electric effect you are considering should occur. On the other hand, H. Bross (Z. Naturforschung **14a** (1959)) was able to explain the observed effects quantitatively from the anisotropy of the lattice vibration spectrum without having to adjust any parameters. A comparison of the relative magnitudes of the various effects predicting deviations from the Matthiessen's rule might be interesting.

Klemens, P. G.: Thank you for bringing to my attention. The inelastic scattering of electrons by imperfections is, of course, only one of several possible causes for deviation from Matthiessen's rule. It is for this reason that I am particularly interested in the other manifestation of this process, namely the enhanced scattering of phonons.