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Inhomogeneities in Semiconductors Exposed to Fast Neutrons*

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The permittivity and conductivity are determined for a semiconducting sol of spheroidal inhomogeneities, with the symmetry axes oriented at random, the sols being those which might be induced by fast neutrons, *e. g.* p-type particles in n-type material, a dispersion of metallic precipitates, or ring dislocations. The presence of these localized inhomogeneities in a semiconductor would significantly alter its electrical properties. Both the displacement of majority carriers and minority carrier storage about the inhomogeneities would contribute to the permittivity. The potential barriers, which might accompany the inhomogeneities, would not only reduce the conductivity of majority carriers, but simultaneously increase the conductivity of minority carriers.

1. Introduction

Defect clusters, induced by fast neutrons in n-type germanium, have been discussed[1)-6)], using a model for spherical clusters. As Silk and Barnes[7)] have observed tracks from fission fragments and recoil atoms in mica, and Barnes[8)] has indicated that clusters of interstitials form dislocation loops in neutron irradiated copper, it is reasonable to suppose that defect clusters in neutron irradiated semiconductors might approximate the form of needles or rings in certain cases. Furthermore, it seems not unlikely that the ionization produced by recoiling lattice atoms could print out metallic precipitates in certain compound semiconductors in the manner that the photolytic process activates a photographic film, and that these precipitates would assume the form of spheres, needles or platelets. A defect cluster, or precipitate, in a semiconductor should be surrounded by a double layer, the outer boundary being a prolate spheroid about a needle shaped inhomogeneity, or an oblate spheroid about a ring dislocation or metallic platelet. In order to treat these cases, the model for inhomogeneities in semiconductors is extended in this paper to spheroids.

Inhomogeneities in semiconductors, *e.g.* defect clusters, or precipitates, may be surrounded by inversion layers of opposite conductivity type, as suggested by studies of Weisberg[9)]. Therefore, the model which is described here explicitly for p-type particles in n-type germanium should be applicable to certain other inhomogeneous semiconductors.

The extent to which the double layer enclosing an inhomogeneity follows its contour depends on the size of the inhomogeneity relative to the Debye length based on the donor concentration in the surrounding germanium. With material in the resistivity range of 0.03 to 10 ohm cm, the Debye length varies from 1500 to 3500 angstroms, making the exterior boundary of the double layer approximately spherical for inhomogeneities with linear dimensions of a few hundred angstroms. However, an inhomogeneity of the size of a Debye length can influence the exterior boundary of its double layer. Needles or platelets with linear dimensions up to 3000 angstroms could be surrounded by double layers with spheroidal outer boundaries of eccentricity up to about 0.85. A typical potential barrier supporting the double layer around an inhomogeneity rises about a half volt. As the height and width of the barrier are large compared with the thermal energy and wave length of holes and conduction electrons, the theory of p-n junctions applies to the complex polarizability of the barriers.

It is assumed in the following discussion that

(A) the neutral core of an inhomogeneity is either metallic or has a conductivity which is of opposite type, and much larger in magnitude than that of the surrounding semiconductor,

(B) the double layer is supported by a

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potential barrier much higher than kT/q*,

(C) the holes in the inversion layer, and charge carriers in the neutral core, provide a real polarizability which may be represented by a conducting spheroid,

(D) the inhomogeneous germanium under discussion represents a dispersed phase of p-type spheroids suspended in a homogeneous n-type crystal.

The terms "aligned system" and "random system" will be used hereafter to designate a dispersed phase of identical spheroids with the axes of symmetry in parallel, and oriented at random respectively.

2. Complex Polarizability of an Inhomogeneity

Consider the current density $\vec{J}$ in the bulk germanium without inhomogeneities as

$$\vec{J}(t)=i\omega\varepsilon_0^*\vec{E}e^{i\omega t} \tag{1}$$

where the electric field is $\vec{E_0}e^{i\omega t}$ and the complex permittivity is

$$\varepsilon_0{}^*=\varepsilon_0-\frac{i}{\omega}(\sigma_{n0}+\sigma_{p0}) . \tag{2}$$

The effect of the inhomogeneities on current flow enters as a contribution to the complex permittivity, and it is therefore pertinent to develop the polarizability of a single inhomogeneity.

The polarizability of a uniform spheroid, with volume V_c and permittivity ε_1, surrounded by a uniform medium with permittivity ε_2, may be written[10]

$$M_j=\frac{(\varepsilon_1-\varepsilon_2)V_c\varepsilon_2}{\varepsilon_2+(\varepsilon_1-\varepsilon_2)L_j} \tag{3}$$

with subscripts j designating Cartesian axes, with $j=1$ the symmetry axis of the spheroid. The depolarization factors L_j have two distinct elements, L_1 and $L_2=L_3$. The term "spheroid" and symbols V_c, M_j and L_j refer to an inhomogeneity together with the surrounding double layer, for the remainder of the discussion.

An applied electric field polarizes the spheroid, increasing the barrier where the field applies reverse bias, and decreasing it where there is forward bias. It follows from assumption (C) that this contribution to the polarizability of a spheroid M_j' is described by the limit of (3) as $\varepsilon_1\rightarrow\infty$, and with ε_2 replaced dy ε_0, *e.g.*

$$M_j'=\frac{V_c\varepsilon_0}{L_j} . \tag{4}$$

According to assumption (B), the potential barrier effectively blocks the flow of electrons, and makes an imaginary contribution to the polarizability of a spheroid M_j'' as given by the limit of (3) as $\varepsilon_1\rightarrow 0$, and with ε_2 replaced by $\sigma_{n0}(i\omega)^{-1}$, *e.g.*

$$M_j''=\frac{-V_c\sigma_{n0}}{i\omega(1-L_j)} . \tag{5}$$

The high potential barrier presented to electrons by the spheroid is simultaneously such a deep well to holes that it short circuits the flow of holes, and makes an imaginary contribution to the polarizability through dipolar diffusion[11]. It has been shown elsewhere[12], that the polarizability of a spherical barrier with radius r_0, from dipolar diffusion, is given by the product of the admittance $Y(\omega)$ and $2r_0^2(i\omega)^{-1}$, *e.g.*

$$M_j'''=\frac{2\pi r_0^3\sigma_{p0}}{i\omega}\left(1+\frac{r_0}{\Lambda}+\frac{1}{1+r_0/\Lambda}\right) \tag{6}$$

with

$$\Lambda=\left(\frac{1+i\omega\tau}{D\tau}\right)^{1/2} \tag{7}$$

the diffusion length, and τ the recombination time of holes. The following contribution to the polarizability is a generalization dased on (6)

$$M_j'''=\frac{V_c\sigma_{p0}}{2i\omega L_j}\left(1+\frac{r_j}{\Lambda}+\frac{1}{1+r_j/\Lambda}\right) \tag{8}$$

in which $r_1=a$, the semi-major axis and $r_2=r_3=b$, the semi-minor axis of a prolate spheroid, and $r_1=b$, and $r_2=r_3=a$, for an oblate spheroid. The high frequency ($\omega\gg D/a^2$) applications of (8) have been treated in a discussion of optical extinction[12]. In the low frequency range ($\omega\ll D/a^2$), which ordinarily includes microwave bands, (8) reduces to

$$M_j'''=\frac{V_c\sigma_{p0}}{i\omega L_j}\left(1+\frac{i\omega r_j^2}{2D}\right) . \tag{9}$$

The total polarizability of a spheroid is the following sum of (4), (5) and (9)

$$M_j=\frac{V_c}{L_j}\left[\varepsilon_0+\frac{\sigma_{p0}r_j^2}{2D}-\frac{i}{\omega}\left(\sigma_{p0}-\frac{\sigma_{n0}L_j}{1-L_j}\right)\right] . \tag{10}$$

* The assignment of symbols is listed in an appendix.

3. The Complex Permittivity

The permittivity of an aligned system of N spheroids per unit volume is written

$$\varepsilon_{ij}^* = \delta_{ij}(\varepsilon_0^* + NM_j) \tag{11}$$

with the concentration subject to the restriction $NV_c \ll 1$. As the concentration N increases, the effective field which polarizes a spheroid exceeds the applied field by the "Lorentz field" contributed from the polarized neighboring spheroids. The following expression for the permittivity of an aligned system accounts for the "Lorentz field".

$$\varepsilon_{ij}^* = \delta_{ij}\left(\varepsilon_0^* + \frac{NM_j}{1 - \dfrac{NM_jL_j}{\varepsilon_0^*}}\right). \tag{12}$$

The density of power dissipation w, and density of stored energy u, in an aligned system, under an applied alternating electric field with amplitude (E_{01}, E_{02}, E_{03}), are given respectively by the relations

$$w = \omega \operatorname{Im} \varepsilon_{ij}^* E_{0j}^2 , \tag{13}$$

and

$$u = 1/2 \operatorname{Re} \varepsilon_{ij}^* E_{0i}^2 . \tag{14}$$

With a random system, the mean square components of applied field comply with the relation

$$\langle E_{01}^2 \rangle = \langle E_{02}^2 \rangle = \langle E_{03}^2 \rangle = |\vec{E}_0|^2/3 . \tag{15}$$

Therefore, with a random system, the power and energy densities reduce respectively to

$$w = \omega |\vec{E}_0|^2 \operatorname{Im}\left(\frac{\varepsilon_{11}^*}{3} + \frac{2\varepsilon_{22}^*}{3}\right), \tag{16}$$

and

$$u = \frac{1}{2} |\vec{E}_0|^2 \operatorname{Re}\left(\frac{\varepsilon_{11}^*}{3} + \frac{2\varepsilon_{22}^*}{3}\right). \tag{17}$$

Hence, it follows from (12), (16) and (17) that

$$\langle \varepsilon^* \rangle = \varepsilon_0^* + \frac{1}{3}\frac{NM_1}{1 - \dfrac{NM_1L_1}{\varepsilon_0^*}} + \frac{2}{3}\frac{NM_2}{1 - \dfrac{NM_2L_2}{\varepsilon_0^*}} \tag{18}$$

gives the mean permittivity of a random system. The dc conductivity given by (18) is written, to the second order, as follows

$$\langle \sigma \rangle = \sigma_{n0} + \sigma_{p0} + \frac{f}{3}\left[\sigma_{p0}\left(\frac{1}{L_1} + \frac{2}{L_2}\right) - \sigma_{n0}\left(\frac{1}{1-L_1} + \frac{2}{1-L_2}\right)\right] + \frac{f^2}{3(\sigma_{n0}+\sigma_{p0})} \times \left[L_1\left(\frac{\sigma_{p0}}{L_1} - \frac{\sigma_{n0}}{1-L_1}\right)^2 + 2L_2\left(\frac{\sigma_{p0}}{L_2} - \frac{\sigma_{n0}}{1-L_2}\right)^2\right] \tag{19}$$

with $f = NV_c$, the fraction of volume occupied by spheroids. It may be shown that a random system of spheroids has a more pronounced effect on the conductivity for a given f than spheres. The permittivity, by (20), is written to the first order

$$\langle \varepsilon^* \rangle = \varepsilon_2^* + \frac{f}{3}\left\{\left(\varepsilon_0 + \frac{r_1^2\sigma_{p0}}{2D}\right)\frac{1}{L_1} + \left(\varepsilon_0 + \frac{r_2^2\sigma_{p0}}{2D}\right)\frac{2}{L_2} - \frac{i}{\omega} \times \left[\sigma_{p0}\left(\frac{1}{L_1} + \frac{2}{L_2}\right) - \sigma_{n0}\left(\frac{1}{1-L_1} + \frac{2}{1-L_2}\right)\right]\right\}. \tag{20}$$

The increments in permittivity of the type $fr_j^2\sigma_{p0}(DL_j)^{-1}$ come from "hole storage", an effect which has been treated extensively in the literature on the admittance of rectifying barriers.

List of Symbols

δ_{ij}	Kronecker delta
ε_0^*	complex permittivity of germanium
ε_0	real permittivity of germanium
ε_1	permittivity of a spheroid
ε_2	permittivity of a medium surrounding a spheroid
ε_{ij}^*	permittivity of an aligned system
$\langle \varepsilon^* \rangle$	permittivity of a random system
Λ	diffusion length of holes
σ_{n0}, σ_{p0}	conductivity, of electrons and holes respectively
$\langle \sigma \rangle$	conductivity of a random system
τ	recombination time of holes
ω	frequency with factor 2π
a, b	semi-major and semi-minor axis of a spheroidal inhomogeneity
D	diffusion coefficient of holes
$\vec{E}_0$	applied electric field
E_{01}, E_{02}, E_{03}	components of $\vec{E}_0$
e	base of Naperian logarithms
f	fraction of volume occupied by inhomogeneities
Im	imaginary part of
i	$\sqrt{-1}$
$\vec{J}$	current density
k	Boltzmann's constant
L_j	depolarization factor for principal axis j
M_j	polarizability for field component along principal axis j
N	number of inhomogeneities per

	unit volume
q	electronic charge
Re	real part of
r_0	radius of spherical inhomogeneity
r_j	$r_1=a$, $r_2=r_3=b$, prolate $r_1=b$, $r_2=r_3=a$, oblate
T	absolute temperature
t	time
u	energy density
V_c	volume of a colloidal inhomogeneity
w	power density

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DISCUSSION

Bragg, R. M.: I assume your calculations were done for a randomly oriented collection of ellipsoids. A possible experimental way of checking your calculations is to measure single crystals given *directional* irradiation. Directional effects attributed to elongated regions in silicon were reported by Truell and Levi some years ago.

Gossick, B. R.: Yes. My calculations treat both an "aligned system" and "random system". Thank you for pointing out the work of Truell and Levi.

Baruch, P.: Is there some possibility that the electric field in the dipole layer be large enough to sweep electrically charged defects from the clusters and thus add an extra contribution to annealing?

Gossick, B. R.: I have considered models only for a stable cluster, in which the field within the barrier is insufficient to produce migration of defects, independent of cluster shape. Your question is very broad, but, at least in typical experimental cases, it appears that the contribution of the external field is smaller than the equilibrium field within the barrier.

Fan, H. Y.: Have you applied these calculations on spheroidal p-region to any of the experimental observation?

Gossick, B. R.: I have not, but my paper cites several papers in which the spherical model has been used to interpret experimental data. In the light of the more general model for spherical clusters, it seems that previous interpretations based on the spherical model may have overestimated the volume occupied by clusters.