PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

VIII-4. Oscillatory Magneto-Conductance in Si Surfaces

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We have measured magneto-oscillatory effects in inverted (100) surfaces of p-type silicon. These measurements substantiate the quantization of the electronic states perpendicular to the surface resulting in a two-dimensional electron gas. The expected mass parallel to the surface was consistent with the temperature variation. Splitting due to lifting of the spin and valley degeneracies was also observed at high fields.

For semiconductor surfaces with sufficiently low densities of surface states, a strong electric field applied perpendicular to the surface will produce an inversion layer. Thus a large negative electric field produces an n-type surface or n-channel, on a p-type substrate crystal. This is the basis of operation of insulated-gate field-effect transistors.[1]

Figure 1 a shows the energy band diagram for such an inverted surface on p-type Si. The left edge of the diagram corresponds to the surface or, more generally, to the Si-insulator interface. We show the low temperature case, in which the bending of the bands due to the electric field is sufficient to bring the Fermi level into the conduction band producing a degenerate electron distribution in the surface. The region of interest in the dotted square has been expanded on the z-scale into Fig. 1b.

It has been accepted in principle that electrons in such a potential well should not be treated as classical point particles.[2,3] In general, treatments of quantization in surfaces have emphasized the onset of quantization[4-6] in which one expects to find deviations from predominantly classical behavior. Recent calculations of the levels in n-channels on p-type Si[7] suggested that distinctly non-classical behavior should be observable in such channels, particularly at low temperature. The results of Howard and Fang require, for (100) n-channel surfaces that the six-fold degeneracy of the conduction valleys be lifted, so that one has two-fold degenerate series of sub-bands (ignoring spin) corresponding to the motion of the electron perpendicular to the surface. Furthermore, the valley splitting between the two lowest sub-bands with the heavy mass perpendicular to the surface is sufficiently large that, except for the highest electric fields, all the electrons should be in the two-fold degenerate sub-band at low temperatures. Figure 1 b shows the relative position of the two sub-bands for a typical field and substrate

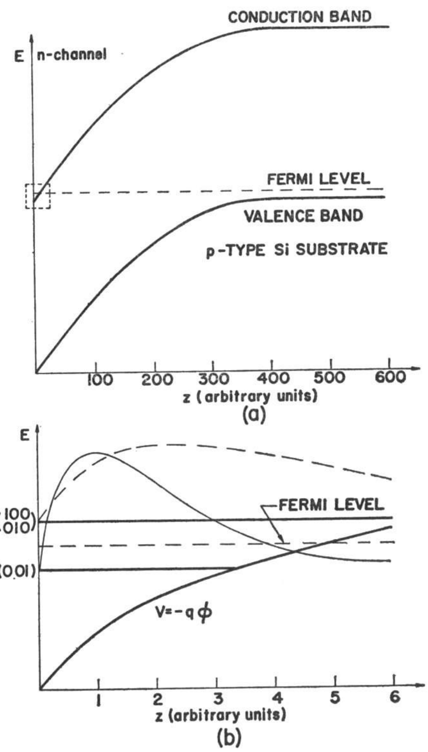

Fig. 1. (a) Schematic band diagram of the inverted Si surface. Dotted square indicates area shown in (b). (b) Detail of potential well at surface, showing bottoms of subbands and approximate wave functions for a typical applied field.

* Part of this work was performed while the authors were guest scientists at the National Magnet Laboratory which is supported at M. I. T. by the Air Force Office of Scientific Research.

doping. The bottoms of the two-fold and four-fold bands are identified by $E_0(001)$ and $E_0\begin{pmatrix}100\\010\end{pmatrix}$, respectively. Approximate z-wave functions are shown. As an example of the actual magnitudes involved, for a charge of 3×10^{11} electrons/cm^2 on a 10 ohm-cm p-type substrate, $E_0(001)$ is about 25 meV and the z-scale is then in units of about 10 Å.

Thus one expects of the conduction band on a (100) Si surface at low temperature a constant density of states, resulting from a single 2-dimensional sub-band, a two-fold valley degeneracy, and an isotropic effective mass of $0.194\,m_0$, the transverse mass for bulk Si.

The Shubnikov-de Haas effect or oscillatory magneto-conductance has been observed in several metals, semi-metals, and semiconductors. Pippard[8] has shown that it should generally result from a periodic density of states and Adams and Holstein[9] have discussed magnetoconductance for detailed scattering mechanisms. In 3-dimensions, each successively higher Landau level contains more electronic states because the average density of states is preserved when the distribution is perturbed by the magnetic field. In a 2-dimensional gas, the density of states at zero magnetic field is a constant as a function of energy above the conduction band minimum. Its density, n_0, is given by $2\pi g_s g_v m/h^2$, where g_s and g_v are the spin and valley degeneracy respectively, and m is the effective mass. If there were no broadening, the Landau levels (split by the spin) would result in a density of states given by a series of δ-functions at $(eH/mc)\,\hbar\,(l+1/2)\pm g\beta H/2$, where H is the magnetic field, l is the order, g is the Lande g-factor, and β is the Bohr magneton. If collision broadening is important (if the magnetic splittings are of the order of $\hbar/\tau$, where τ is the scattering time) the density of states is given by a series of Lorentzians.

Each Landau level has to conserve the states available in an energy interval equal to the Landau energy at zero field. A given Landau level contains $n_0 h\omega_c$ states, where the spin splitting is ignored (as it may be at fields where $h/\tau \gg g\beta H$). In the simplest case, the period of oscillations may be determined by

$$E_F=(l+\gamma)\hbar\omega_c=\frac{Nh^2}{2\pi g_v g_s m}\,, \qquad (1)$$

where N is the surface electron density, corresponding to either a maxima or minima, and γ is a phase factor which depends on the scattering mechanism.

Experiments were carried out on circular field effect transistors, as shown in the insert in Fig. 2, for which the source-drain width, w, was 10 μ and the length or circumference, L, was 500 μ. Most samples were n-surface devices on 100 ohm-cm p-type substrates made on (100) surfaces. The samples were made with both 5330 Å and 1150 Å oxide thicknesses.

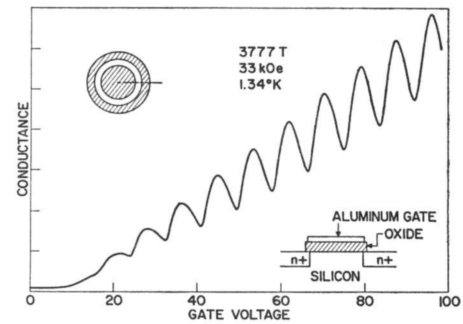

Fig. 2. The conductance as a function of gate voltage or surface field at 33 kOe. In the upper left hand corner, a plan of the source-drain electrode configuration is shown. In the lower right hand corner, a section through the structure is shown.

The total carrier density and the carrier density change between maxima could be calculated using $N=k(V_g-V_0)/4\pi\delta e$ in cgs units, where k is the silicon dioxide dielectric constant, V_g and V_0 are the gate voltage and threshold gate voltage for free carrier induction, respectively, and δ is the oxide thickness.

In these samples at low temperatures the apparent conductance threshold occurred at higher fields than the free carrier induction threshold because the electrons had extremely low mobility near the bottom of the band. This is demonstrated below and also by Hall measurements[10] on similar samples.

The maximum mobilities observed at low temperature were 3500–4000 cm^2/V·sec and 3000 cm^2/V·sec for the 5330 Å and 1150 Å oxide devices, respectively. At low gate voltages the mobility was less, probably because of impurity scattering, and at higher voltages it decreased because of surface scattering. The maximum scattering time was thus about 4×10^{-13} sec. Thus Shubnikov-de Haas oscillations could be observed. ρ with the surface field or carrier density held constant.

The period of the oscillations at fixed magnetic field and varying gate voltage was studied. This measurement corresponds to passing the Fermi surface through successive Landau levels by inducing more electrons. Figures 2, 3 and 4, show the conductance as a function of gate voltage at 33, 90, 143.7 and 165 kOe for a 5330 Å oxide sample at 1.34°K or less. At the lowest magnetic fields, the constant period in gate voltage and thus the electron density

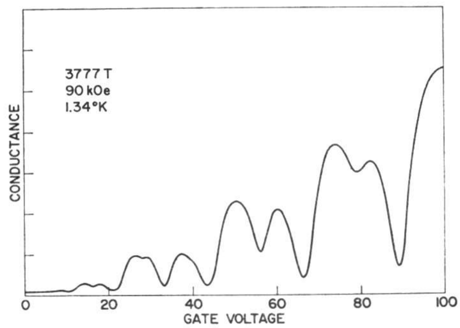

Fig. 3. The conductance as a function of gate voltage at 90 kOe.

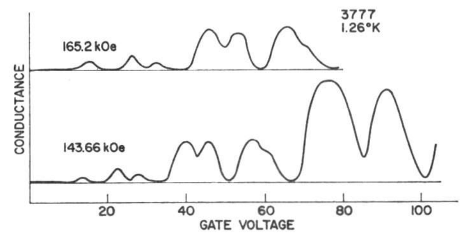

Fig. 4. The conductance as a function of gate voltage at 143.6 and 165.2 kOe.

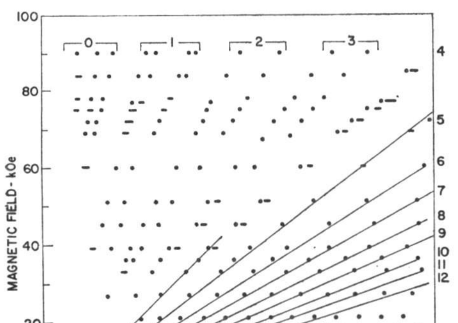

Fig. 5. The position of the conductance maxima plotted as a function of magnetic field and gate voltage. The orders of the Landau levels are shown.

confirms the existence of a 2-dimensional gas because each period contains an equal number of electrons. The degeneracy can be determined and is 4.0 ± 0.2. This means that $g_s g_v = 4$ so that the valley degeneracy, g_v, is 2.

As the field is increased, the peaks split at about 5 times the fields necessary to observe the Landau splitting. This is roughly consistent with a spin splitting 0.2 times as great as the Landau splitting, which would correspond, for $g=2$, to $m=0.2\,m_0$. Thus, in Fig. 3, the maxima at 60 and 50 V and at 83 and 74 V are the spin split levels of the single peaks at 20 and 28 V, respectively, in Fig. 2.

At the highest fields, a further splitting was observed. This is probably associated with a lifting of the valley degeneracy, and it must be a magnetic field dependent effect. The magnetic field dependence was no more than linear, as was demonstrated by following the peaks to 165 kOe and observing that this splitting always remained less than half the spin splitting.

Figure 5 is a plot of the peak positions as a function of magnetic field and gate voltage. The higher order levels satisfy eq. (1) over a large range and may be used to determine the phase factor, γ, in the absence of resolved spin splitting. The phase factor was 0.5 to 0.8 for samples of different mobility.

Lines drawn through the different order levels all extrapolate to the same gate voltage at zero magnetic field. They should because all of the Landau levels coalesce at the carrier threshold rather than the conductance threshold.

The oscillation amplitude of the effective mobility in the magnetic field increased as the magnetic field increased. The increase was initially exponential and then linear until the spin splitting was significant. The dependence on scattering time was more complicated but did not correspond to a Dingle[11] type variation of the form $\exp(-kT_D/\hbar\omega_c)$ where the Dingle temperature, T_D, was inversely proportional to the scattering time, τ.

Adams and Holstein[9] have shown that the temperature dependence is approximately

$$\frac{T}{\sinh 2\pi^2 kT/\hbar\omega_c},$$

provided $2\pi E_F/\hbar\omega_c > 1$. This is the well known result for de Haas-van Alphen effect in the range of sinusoidal oscillations.[12]

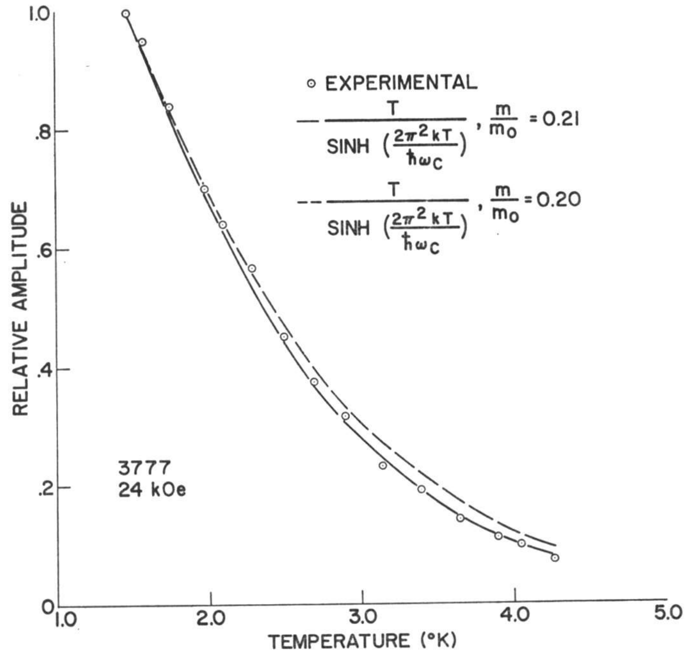

Fig. 6. Normalized temperature dependence of oscillations in transconductance. Solid dotted lines are theoretical curves for effective masses as shown.

In Fig. 6, we have plotted $T/\sinh(2\pi^2kT/\hbar\omega_c)$ for masses of $0.21\,m_0$ and $0.20\,m_0$, for 24 kOe. We have also plotted experimental data taken at 24 kOe. The data points represent average normalized amplitudes for several values of gate voltage. The agreement is surprisingly good and leads us to assign a mass of $(0.21\pm0.01)\,m_0$.

No evidence was seen for the four valleys split upwards from the two valleys with heavy masses perpendicular to the surface. At high enough Fermi energy, an overlap was expected which should have resulted in multiple periodic oscillations and in larger periods.[7] Even at voltages twice those expected for overlap, no such effect was seen. We have no explanation for this discrepancy.

In summary, we have demonstrated that a 2-dimensional band model describes the electrons in these surfaces (thus, there is extreme surface quantization), that the valley degeneracy is two, and that the mass parallel to the surface is $0.21\,m_0$.

We would like to acknowledge helpful discussions with our colleagues, G. J. Lasher, T. D. Schultz, and W. P. Dumke. Finally, we would like to thank G. Cheroff, E. Wurst, F. Hochberg, and their co-workers for providing samples. We would also like to thank J. Cummings for technical assistance.

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DISCUSSION

Stradling, R. A.: In cyclotron resonance experiments we have observed an increase of transverse mass with temperature which we attribute to a lack of parabolicity of the silicon conduction band. Do you think that the change in mass from the bottom of the band value which you report is due to non-parabolicity or to a change in band structure close to the sample surface?

Fowler, A. B.: I do not believe our measurements of mass are sufficiently accurate to state that there must be a deviation from the usual light electron mass, $0.194\,m_0$. Furthermore slight misorientation of either the field or surface would cause the apparent mass to be larger than $0.194\,m_0$. The temperature variation was fitted at several values of surface field corresponding to Fermi energies ranging from about 10 to 40 meV. The same masses were measured at all fields within the error. Thus there is no evidence for non-parabolicity.

Einspruch, N. G.: Can this tool be used to ascertain the relative contributions of specular and diffuse scattering at the surface?

Fowler, A. B.: At the present time we have no theory at all for transport in this two-dimensional system and therefore no theory of transport when in magnetic field. If the Adams-Holstein theory is adapted to this system it may shed some light on the scattering.

Weiss, H.: What was the thickness of the surface layers which would be the limiting thickness for a two-dimensional electron gas?

Fowler, A. B.: The thickness of the electron layer in the surface varies with field, substrate doping, and temperature. It may range from 10 Å to several hundred Angstroms.