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VIII-6. Galvanomagnetic Effects and Band Structure of CdSb

T. KAWASAKI

*Research Laboratory, Matsushita Electronics Corp., Osaka, Japan**

Galvanomagnetic effects of *p*-CdSb were measured above 77°K up to a magnetic field of 18.7 kG. No anisotropy of the Hall coefficient was observed within the experimental error (less than 5%). The Hall and magnetoresistance coefficients change little up to 18.7 kG at 77°K. The longitudinal magnetoresistance coefficients M_{11}^{11} and M_{22}^{22} do not vanish, while M_{33}^{33} is zero within the experimental error. A four valley structure with ellipsoidal energy surfaces with axes of the ellipsoids inclined 11°~15° from the [100]-axis of the crystal can explain the observed galvanomagnetic effects of *p*-CdSb.

§ 1. Introduction

Compound semiconductor CdSb crystallizes in an orthorhombic structure, $P_{bca}-D_{2h}^{15}$, which has 16 atoms in a unit cell. The lattice parameters are $a_0=6.471$, $b_0=8.253$ and $c_0=8.526$Å.[1] The energy gap has been determined by optical measurements to be 0.47 eV at room temperature.[2] The resistivity and the Hall mobility have been measured by several authors and proved to be highly anisotropic.[3-9]

Stevenson proposed for the band structure a spheroidal energy surface with the rotation axis parallel with the *b*-crystal axis based upon his cyclotron resonance experiments.[10] Frei *et al.* suggested an ellipsoidal energy surface with axes parallel to the principal axes of the crystal based upon magnetoresistance and optical measurements.[11]

The present investigation is aimed at a clearer picture of the band structure of CdSb. For this reason in the following measurement of galvanomagnetic coefficients will be described, as well as investigations on the dependence of the Hall and magnetoresistance coefficients on the magnetic field.

§ 2. Definitions and Experimental Procedure

It is convenient to express the transport relations with the aid of the resistivity tensors, ρ_{ij}, ρ_{ijk} and ρ_{ijkl}. For the electric field we get:

$$E_i=\rho_{ij}J_j-\rho_{ijk}J_jH_k+\rho_{ijkl}J_jH_kH_l+\cdots, \quad (1)$$

where the usual summation is conventionally applied. In the following the values 1, 2 and 3 for i, j and k refer to the directions of the [100], [010] and [001] crystal axes, respectively. For the orthorhombic, D_{2h}, system the number of independent components of the resistivity tensor is 3 for the zero magnetic field case (ρ_{ii}), 3 for the first power Hall term (ρ_{ijk}) and 12 for the magnetoresistance term (ρ_{ijkl}). The Hall coefficient R_{ijk} is equal to ρ_{ijk} for weak magnetic fields. The nine magnetoresistance components ρ_{iikk} can be related to the magnetoresistance coefficients M_{ii}^{kk}. The latter are connected with the relative changes of the resistivity by,

$$M_{ii}^{kk}=\frac{\Delta\rho_{ii}}{\rho_{ii}H_k^2}=\frac{\rho_{iikk}}{\rho_{ii}}. \quad (2)$$

The other three magnetoresistance components $\rho_{ijji}(i\neq j)$ can be related to the magnetoresistance coefficient M_{ij}^{ij} by,

$$M_{ij}^{ij}=\frac{E_j}{\sqrt{\rho_{ii}\rho_{jj}}J_iH_iH_j}=\frac{\rho_{ijji}}{\sqrt{\rho_{ii}\rho_{jj}}}. \quad (3)$$

The independent Hall coefficients R_{ijk} and $R_{ikj}(=-R_{kij})$ were determined for different samples by using the electrode arrangement shown in Fig. 1 (a), whereas the magnetoresistance coefficients M_{ii}^{kk} and M_{ij}^{ij} were measured by using the electrode arrangement shown in Fig. 1 (b).

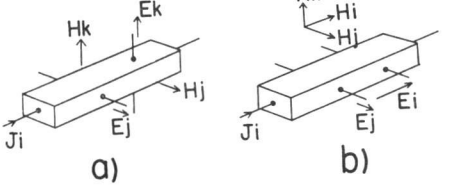

Fig. 1. Two types of electrode arrangements (a) for measurement of the Hall coefficients R_{ijk} and R_{ikj}, (b) for measurement of the magnetoresistance effects.

§ 3. Results

Non-doped CdSb crystals were always found to be *p*-type with carrier concentrations 2~5×

* A part of the experiments was performed in the Tanaka Laboratory, Kyoto Univ., Kyoto.

10^{15} cm^{-3}. For three samples Fig. 2 shows the temperature dependences of resistivity and of Hall coefficients for current directions along the [100], [010] and [001]-axes, respectively. At around 250°K the Hall coefficients begin to decrease. This must be due to the onset of intrinsic conduction. As can be seen from Fig. 2, in the extrinsic region, the ratios $R_{123}/-R_{132}$, $R_{231}/-R_{213}$ and $R_{312}/-R_{321}$ are for each sample very close to one. These ratios were determined for at least three samples for each current direction and they proved to be equal to one within an estimated experimental error of less than 5%.[9]

Since no difference was observed between the Hall coefficients R_{ijk} and $-R_{ikj}$, the Hall mobilities $\mu_{Hijk}=R_{ijk}/\rho_{ii}$ and $\mu_{Hikj}=-R_{ikj}/\rho_{ii}$ must be nearly equal and are denoted henceforth by μ_{Hi}. The Hall mobility is highest along the [001]-direction (μ_{H3}) and lowest along the [010]-direction (μ_{H2}). Between 77°K and 250°K the temperature dependence of the Hall mobility observed can be expressed by $\mu_{Hi}=A_iT^{-n_i}$. The observed values of μ_{Hi}, the ratios of these values and the determined values of n_i are summarized in Table I. The values of n_i are smaller than 1.5. The latter value is expected, if scattering is by longitudinal acoustic lattice vibration.

Between 77°K and 250°K the change of Hall mobility μ_{H3} with varying carrier concentration was very small up to concentrations of about 3×10^{16} cm^{-3}.[9] According to Hruby *et al.* the Hall mobility μ_{H2} is nearly constant up to a hole concentration of about 1×10^{17} cm^{-3}.[7] These results together show that the effect of ionized impurity scattering can be neglected at 77°K for samples with carrier concentrations less than 1×10^{16} cm^{-3} (this concentration corresponds to a Hall coefficient 6.2×10^{2} cm^3/coulomb).

The observed dependence of the Hall coefficient on the magnetic field at a temperature of 77°K is plotted in Fig. 3 for 3 different oriented samples. The Hall coefficients change little with the magnetic field up to a field of

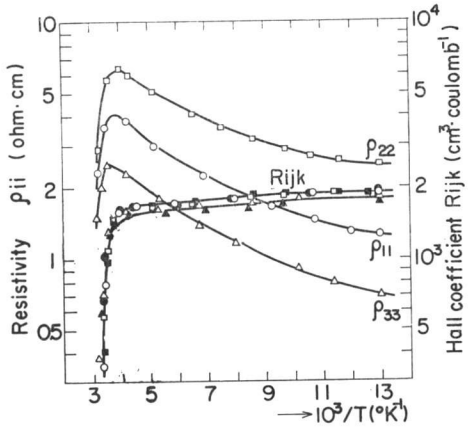

Fig. 2. Temperature dependences of resistivity and Hall coefficients along the principal axes for sample 1 (○: R_{123} ●: $-R_{132}$) sample 2 (□: R_{231} ■: $-R_{213}$) sample 3 (△: R_{312} ▲: $-R_{321}$).

Table I. The Hall mobility μ_{Hi} (cm²/V·sec), the mobility ratio and values of n_i at 77°K.

	μ_{H1}	μ_{H2}	μ_{H3}
μ_{Hi}	1720	843	2710
ratio	2.0	1.0	3.2
n_i	1.21	1.03	1.22

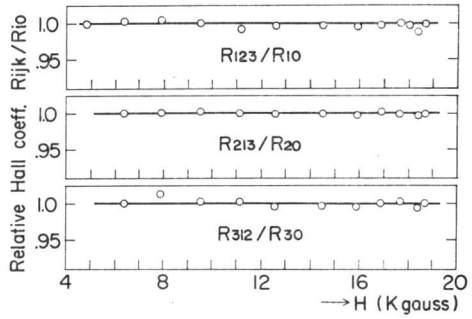

Fig. 3. Dependence on magnetic field of the Hall coefficient of *p*-CdSb at 77°K for sample 4 (R_{10} =1600), sample 5 (R_{20}=1710) and sample 6 (R_{30} =2820 cm³/coulomb).

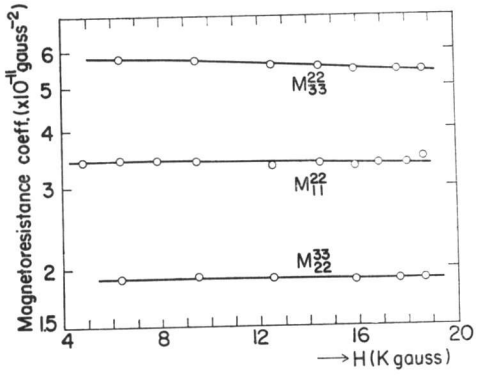

Fig. 4. Dependence on magnetic field of the magnetoresistance coefficients M^{22}_{11}, M^{33}_{22} and M^{22}_{33} of *p*-CdSb at 77°K. The Hall coefficients measured on the same samples are 1990 (sample 7), 1710 (sample 5) and 2820 cm³/coulomb (sample 6), respectively.

Table II. Magnetoresistance coefficients of *p*-CdSb at about 80°K ($\times 10^{-12}$ gauss^{-2}). The different sets of values were obtained with different samples. The Hall coefficients of samples measured were $1\sim3\times10^{3}$ cm^3/coulomb.

M_{11}^{11}	M_{11}^{22}	M_{11}^{33}	M_{12}^{12}	M_{31}^{31}	M_{22}^{11}	M_{22}^{22}	M_{22}^{33}	M_{23}^{23}	M_{12}^{12}	M_{33}^{11}	M_{33}^{22}	M_{33}^{33}	M_{31}^{31}	M_{23}^{23}
4.3	31	8.7		13	14	12	12		28	28	53	<1	27	
4.5	45	13	41		17	12	14	18						23

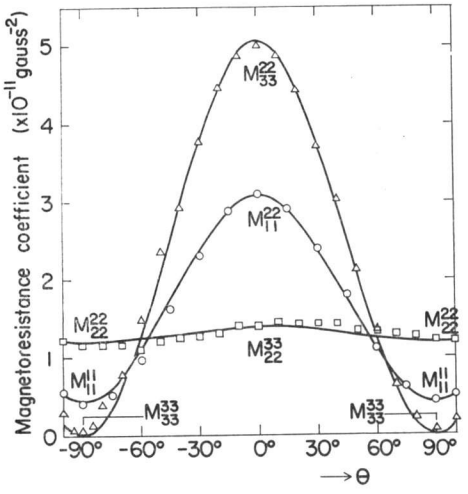

Fig. 5. Variation of the magnetoresistance effect of *p*-CdSb with the direction of a magnetic field of strength 6 kG at about 80°K. (θ is the angle between H and principal crystal axis).

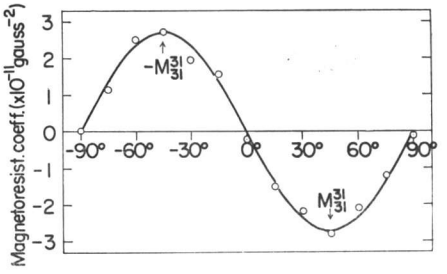

Fig. 6. Variation of the magnetoresistance effect with the direction of a magnetic field of strength 6.5 kG at 77°K. (θ is the angle between H and principal crystal axis).

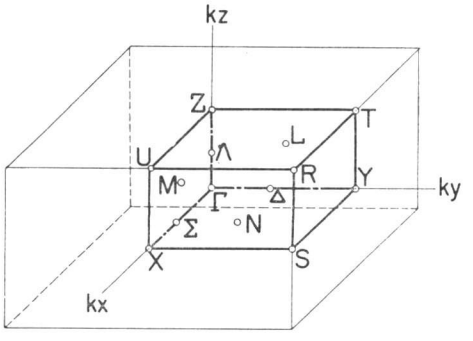

Fig. 7. Diagram indicating the first Brillouin zone of CdSb after e.g. Tovstyk and Gemus.[12]

18.7 kG. Dependence on the magnetic field of the magnetoresistance coefficients is shown in Fig. 4 for similar samples. Changes of the coefficients are very small. The largest change occurs in M_{33}^{22}, and the decrease at 18.7 kG was about 5% of the value at 6 kG.

The magnetoresistance is highly anisotropic as shown in Fig. 5 and Table II. As can be seen from the figure the longitudinal magnetoresistance coefficients M_{11}^{11} and M_{22}^{22} have finite values, while the value of M_{33}^{33} is vanishingly small. An example of the measurement of the coefficient M_{ij}^{ij} is shown in Fig. 6.

§ 4. Discussion

The first Brillouin zone of an orthorhombic, D_{2h}, crystal is shown in Fig. 7. The symmetry points are named Γ, Σ etc..

Stevenson[10] proposed a spheroidal energy surface with rotation axis parallel to the *b*-crystal-axis, based upon cyclotron resonance experiments. But he did not identify the sign of the carriers responsible for the resonance.

Frei *et al.*[11] found finite values of one of the longitudinal magnetoresistance coefficients. They attributed the behavior of the temperature dependence of the magnetoresistance effect to a change of scattering mechanism. Their optical data have been explained not by degenerate energy surfaces with warping but by an ellipsoidal energy surface with axes parallel to the principal axes of the crystal. As has been shown by Herring and Vogt,[13] however, the non-zero values of the coefficients M_{ii}^{ii} can not be adequately explained by those ellipsoidal (including spheroidal) energy surfaces even when taking into account

an anisotropic relaxation time. The obtained non-zero values of M_{11}^{11} and M_{22}^{22} suggest that the energy surfaces of the top of the valence band are different from ellipsoids with axes parallel to the principal axes of the crystal. One possible valence band structure now is the many valley structure. The energy surfaces in this case are ellipsoids whose axes are inclined with respect to the crystal principal axes. Another possibility is given taking account of the structure of a degenerate band with warped energy surfaces.

In the case of a degenerate band structure a remarkable magnetic field dependence of the galvanomagnetic coefficients can be expected, even in the weak magnetic field range ($\mu H<1$). Such large effects have been observed for p-Ge[14] and p-Si.[15] In the present investigation the maximum magnetic field used was 18.7 kG which corresponds to $\mu H\simeq 0.5$ along the [001]-direction at 77°K. If so called light holes exist, μH should be larger than 1 for the light holes along the [001]-direction. As shown in Figs. 3 and 4 the dependence of the galvanomagnetic coefficient on the magnetic field is small for fields up to 18.7 kG. Furthermore for a warped energy surface a zero value of one of the coefficients $M_{ii}^{ii}(i=1, 2, 3)$ is improbable. The observed value of M_{33}^{33}, however, is within experimental error equal to zero. These facts together strongly suggest that degenerate bands with warped energy surfaces do not exist at the top of the valence band of CdSb.

The negligible value of M_{33}^{33} together with the non-zero values of M_{11}^{11} and M_{22}^{22} suggests a four valley model as shown in Fig. 8 for the valence band structure. Equations have been given by the present authors[9] for the galvanomagnetic coefficients for this model in terms of the conductivity tensors of a single valley. The Hall coefficient in general is anisotropic for the case $\sigma_{11}\neq\sigma_{22}$ and $\theta\neq 0$ even when the coefficient for a single valley is isotropic. However, this anisotropy is so small that it is difficult to observe if θ is less than about 15° (for $\theta=15°$ and $\sigma_{11}/\sigma_{22}=2$, we find $R_{123}/R_{231}=0.96$). As has been shown elsewhere[9] for these small values of θ M_{ii}^{ii} can be expressed by:

$$-M_{11}^{11}/\rho_{11}=-M_{22}^{22}/\rho_{22}$$
$$=4\sin^2\theta\cos^2\theta(\sigma_{1122}^{\alpha}+\sigma_{2211}^{\alpha}+2\sigma_{1221}^{\alpha})$$
$$\simeq\sin^2\theta\cos^2\theta\,(\sigma_{1122}+\sigma_{2211}+2\sigma_{1221}) \qquad (4)$$

$$-M_{33}^{33}=0\,, \qquad (5)$$

(σ_{1122}^{α} refers to the value of σ_{1122} of the α-th valley).

The conductivity tensors were calculated using the measured values of $M_{ii}^{kk}(i\neq k)$, $M_{ij}^{ij}(i\neq j)$ and μ_{Hi}. The values of θ were chosen such that the right hand term is equal to the observed values of M_{11}^{11} and M_{22}^{22}. The value derived from M_{11}^{11} is $\theta=11°15'\sim 12°32'$ and from M_{22}^{22} is $\theta=13°10'\sim 14°35'$. The values of θ derived from the two coefficients agree fairly well.

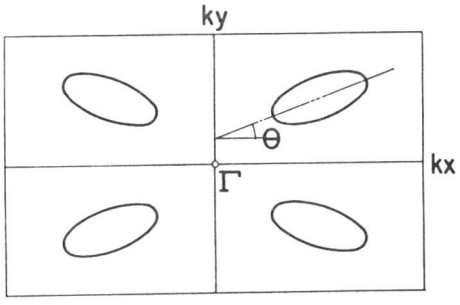

Fig. 8. Plain view of a four valley model for the valence band structure of CdSb. (See Kawasaki and Tanaka[9]).

§ 5. Conclusion

The non-zero values of the longitudinal magnetoresistance coefficients cannot be explained by an ellipsoidal energy surface with axes parallel to the crystal principal axes. The degenerate band model asks for a number of extra preconditions before being able to explain the experimental results and seems to us to be highly improbable. A four valley structure with ellipsoidal energy surfaces with the axes of the ellipsoids inclined 11°∼15° from the [100]-axis of the crystal can explain without extra assumption observed galvanomagnetic effects of p-CdSb.

Acknowledgement

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DISCUSSION

Stourac, L.: I should like to mention that in our laboratory we have measured some time ago the magnetoresistance and Hall constant of CdSb and have obtained the same results as Dr. Kawasaki. (M. Matyas and A Mueller: Czech. J. Phys. **B 16** (1966); L. Stourac: *ibid.* **B 15** (1965); A. Hruby, I. Kubelik and L. Stourac: *ibid.* **B 15** (1965).

However, we thought that the non-zero value of the longitudinal magnetoresistance might be connected with some scattering mechanism and we were situated on the minima at the axes. This seemed to be also in better agreement with optical data. (J. Tauc and A. Abraham: Czech. J. Phys. **B 15** (1965). I think that further investigations are required before final assignment can be made.

Kawasaki, T.: I would like to say that it is questionable to ascribe the non-zero values of the longitudinal magneto-resistance coefficient to the scattering mechanism, even if the anisotropic relaxation time is taken into account. I agree with your suggestion about the necessity of further investigation.

Vodopyanov, L. K.: What was the value of the parameter $\omega_H \cdot \tau$ in your experiments?

Kawasaki, T.: The value of $\omega\tau$ was about 0.5 at 77°K for the [001] direction at magnetic field of about 19 kG.