PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

VIII-7. The Structure of the Conduction Band and the Anisotropy of the Electron Scattering in *n*-GaAs

A. F. KRAVCHENKO, W. S. SARDARJAN and W. W. EFIMOV

Institute of Physics of Semiconductors, Siberian Division, USSR Academy of Sciences, Novosibirsk, 90, USSR

The magnetoresistance and the Hall effect on the oriented specimens of *n*-type GaAs with the concentration of carriers between 5×10^{15} and $1\times10^{18}\,\mathrm{cm}^{-3}$ at temperatures ranging from 78°K to 800°K have been studied. Almost all the samples show the anisotropy of the transverse magnetoresistance and the non-vanishing longitudinal magnetoresistance. From the temperature dependence of the Hall coefficient and the calculations, the energy position of the subsidiary minima and the effective mass are estimated: $\Delta W=0.12\sim0.36$ eV, $m_2^*=1.2\,m_0$, $m_{/\!/}^*=1.98\,m_0$, $m_1^*=0.37\,m_0$. A model for the band structure in the vicinity of the conduction band edge is discussed and shown to be consistent with the experimental data when anisotropy of the electron scattering by weak oriented dipoles is taken into account. For the confirmation of this model the results of the measurements of the longitudinal Hall coefficient are analysed.

Among semiconductor compounds we have, probably, the largest available data on the structure of the energy zone in GaAs, but there is the essential divergence of the results of theoretical analysis of various authors. The principal contradictions of the results of calculations with the experimental evidence indicate that we do not know the energy spectrum of electron in gallium arsenide exactly.

The anisotropy of the transverse magnetoresistance[1,2] was explained by the existence of the conduction band minima in the direction $\langle 100\rangle$ at 0.05 eV above the absolute minimum (000). The results of the infrared absorption by free electrons predicted the presence of the energy minima in the direction $\langle 111\rangle$ at 0.05 eV above the minimum (000).[3]

In the survey paper[4] H. E. Ehrenreich has offered a model of the energy spectrum for the *n*-GaAs.

However, in offering this model there are few exactly definite parameters which characterize the energy spectrum of electrons.

In the last few years there have been published a number of papers where the influence of impurity on the energy spectrum of electrons was considered. Therefore, the further study of the structure of the conduction band is interest without doubt if one wishes to obtain a reliable information on the arrangement and the shape of the subsidiary minima.

We have studied the anisotropy of magnetoresistance and the Hall effect in the *n*-GaAs with electron concentration between 5×10^{15} and $1\times10^{18}\,\mathrm{cm}^{-3}$ at temperature range from 78°K to 800°K. The orientation of the samples has been made with x-rays or light figures. The parameters for some of the samples at $T=300$°K and the orientation are given in Table I.

Before carrying out measurements, the magneto-

Table I

NN of samples	$\frac{1}{Re}\,\mathrm{cm}^{-3}$	$\mu\,\frac{\mathrm{cm}^{-3}}{\mathrm{V\,sec}}$	Orientation	
13	8.0×10^{15}	3000	$H\,\langle 11\bar{2}\rangle$,	$H,\ I\,\langle 1\bar{1}0\rangle$
8	1.1×10^{16}	4500	$H\,\langle 010\rangle$,	$H,\ I\,\langle 100\rangle$
14	3.7×10^{16}	4500	$H\,\langle 1\bar{1}0\rangle$,	$H,\ I\,\langle 11\bar{2}\rangle$
15	3.9×10^{16}	4000	$H\,\langle 011\rangle$,	$H,\ I\,\langle 100\rangle$
10	9.0×10^{16}	3600	$H\,\langle 1\bar{1}0\rangle$,	$H,\ I\,\langle 112\rangle$
12	2.0×10^{17}	2600	$H\,\langle 111\rangle$,	$H,\ I\,\langle 11\bar{2}\rangle$
11	3.5×10^{17}	2800	$H\,\langle 112\rangle$,	$H,\ I\,\langle 1\bar{1}0\rangle$

resistance was carefully checked for homogeneity by a potential distribution along the samples. The largest inhomogeneity, $(n-\langle n\rangle)/n=0.2$ was detected in the samples which were cut out along $\langle 111\rangle$. The specimens with the orientation along $\langle 110\rangle$ had the inhomogeneity of about 0.18, and the specimens with the orientation along $\langle 100\rangle$ about 0.13. These measurements show the layer distribution of the defects. Most samples with $(n-\langle n\rangle)/n<0.05$ were chosen for measuring galvanomagnetic effects.

In magnetic fields between 2 and 16 kG, the quadratic dependence of magnetoresistance upon the magnetic field was observed. The anisotropic transverse magnetoresistance and the non-vanishing longitudinal magnetoresistance were found in all regions of temperature (Fig. 1).

The typical peculiarities for the most part of samples are essential difference between the values of the transverse and longitudinal magnetoresistances, weak temperature dependence of the carrier concentrations and comparatively weak dependence of electron mobility on concentrations. The anisotropic transverse magnetoresistance and the non-vanishing longitudinal magnetoresistance may be due to the presence of two closely situated energy minima,[1] but this model encounters great difficulties if one would analyse with the values of the effective electron mass obtained by various methods.[2] So we assume for discussing the results of magnetoresistance at low temperatures that conduction electrons are only in the absolute energy minimum because of the presence of large energy difference between the two minima.

The correlation of the symmetry between the magnetoresistance coefficients b, c and d shows

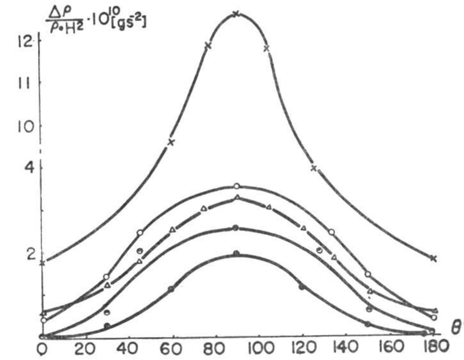

Fig. 1. Variation of magnetoresistance with orientation of magnetic field ($T=78°$K).

Table II

$T=300°$K

N of sample	K	μ cm^2/V sec	$b10^{10}$ G^{-2}	$-c10^{10}$ G^{-2}	$-d10^{10}$ G^{-2}	$\frac{1}{Re}$cm^{-3}
4E	1.4	2380	7.4	6.7	0.7	5.5×10^{15}
1	1.8	2520	4.02	3.1	0.9	3.6×10^{16}
4A	1.3	2800	5.01	4.8	0.24	3.5×10^{16}
15	1.3	4000	17	16.8	0.3	3.9×10^{16}
14	1.21	4500	16	15.1	2.0	3.74×10^{16}
5A	1.24	4000	10	9.6	0.3	2.17×10^{16}
5E	1.0	3500	16.3	0		3.8×10^{17}

Table III

$T=78°$K

N of sample	K	μ cm^2/V sec	$b10^{10}$ G^{-2}	$-c10^{10}$ G^{-2}	$-d\,10^{10}$ G^{-2}	$\frac{1}{Re}$cm^{-3}
4E	2	2500	4.25	1.25	3	4.65×10^{15}
1	2.1	2800	4.2	1.1	32	2.68×10^{16}
1C	3	3000	6.1	3.7	2.3	2.07×10^{16}
4A	3.8	3520	8	3	5	2.6×10^{16}
15	3.8	4200	11.2	7.4	3.8	2.78×10^{16}
14	3.8	6000	14	4	10	3.6×10^{16}
5A	3.8	5000	17.3	5.3	12	1.82×10^{17}

Table IV

Lattice scattering by acoustic modes.

K_m	3	4	5	6	7	10	16	18	20
$\tau_{/\!/}/\tau_{\perp}$	2.207	2.435	2.638	2.822	2.992	3.440	4.172	4.385	4.586

Impurity scattering ($\tau_{/\!/}/\tau_{\perp}$). $T=100°$K

N cm^{-3} / K_m	1×10^{14}	1×10^{15}	1×10^{16}	5×10^{16}	1×10^{17}	5×10^{17}	1×10^{18}
4	2.914	2.835	2.664	2.320	2.420	3.131	3.686
5	3.490	3.383	3.157	2.705	2.849	3.841	4.601
6	4.044	3.910	3.625	3.055	3.243	4.493	5.447
20	11.181	10.646	9.571	7.898	9.086	11.181	19.167

Table V

Dipole scattering ($\tau_{/\!/}/\tau_{\perp}$) $K_m=1$

N cm^{-3} / T°K	1×10^{14}	1×10^{15}	1×10^{16}	5×10^{16}	1×10^{17}	5×10^{17}
100	0.6261		0.3128	0.3351	0.3328	0.3326
150	0.6069	0.7579	0.2432	0.3331	0.3327	0.3327
200	0.6036	0.6746	0.6623	0.3333	0.3331	0.3332
250	0.6007	0.6445	0.7269	0.3332	0.3333	0.3326
300	0.5991	0.6294		0.3330	0.3333	0.3330

the type of $\langle 100\rangle$ minima. The values of $K=(K_m/K_\tau)=(m_{/\!/}^*/m_{\perp}^*)/(\tau_{/\!/}/\tau_{\perp})$ together with the sample characteristics are shown in Tables II and III. It may be easily seen that K increases with the decrease of temperature as observed by one of the authors.[1]

For the determination of K_m and K_τ the values of $\tau_{/\!/}$ and $\tau_{\perp}$ for the different scattering mechanisms were calculated.[5] The results of the calculations are presented in Table IV.

Assuming that at liquid nitrogen temperature the impurity scattering of electrons prevails while at room temperature and higher temperatures the phonon scattering prevails, one can see from the above calculations that the parameter K grows with increasing temperature. This is in contradiction to the experimental results. At room temperature $K\leq1.8$ and in the region of nitrogen temperature $2\leq K\leq4$. This fact shows that the energy spectrum of electrons is probably spherical, but the scattering mechanism is perhaps anisotropic.

To confirm this assumptions we calculated $\tau_{/\!/}$ and $\tau_{\perp}$ when the scattering of electrons takes place through the interaction with the acceptor-donor dipoles (Table V)[6] for the isotropical energy spectrum.

The calculations show that for specimens with the dominant scattering partly by the ions and the dipoles at nitrogen temperature we can obtain the value $K\sim4$, which agrees well with the experimental data.

Thus at room temperature the magnetoresistance anisotropy is not connected with the existence of the subsidiary minimum near the absolute minimum (000) but is stipulated by the anisotropic scattering and in some extent by the presence of the layer structure with microinhomogeneity. A small growth of K with the decrease of temperature is connected with that anisotropic dipole scattering, as gallium arsenide investigated, as a rule, is compensated. To confirm this model we investigated, the longitudinal Hall effect which is not equal to zero only when $\frac{\mu H}{c}\simeq1$ and the energy spectrum or the time of relaxation are anisotropic. In general case the electric field of the longitudinal Hall effect is given by[7]

$$E_{(j\times(H\cdot j))}=\tau jH^3\{\eta_1\eta_3\xi_2(\eta_3^2-\eta_1^2)+\eta_1\eta_2\xi_3(\eta_1^2-\eta_2^2)+\eta_2\eta_3\xi_1(\eta_2^2-\eta_3^2)\}$$

where

$$\tau=\begin{pmatrix}-1/3\\ 2/9\\ 1/12\end{pmatrix}\rho\frac{1}{K^2m_\perp^{*3}}(K-1)^2ne^4$$

$$\times\left(\ll\tau^3\gg R-\frac{e}{m_1^*}\rho\ll\tau^4\gg\right),$$

$$\ll g(w)\gg=\frac{2\sqrt{2}\,(m_\parallel^* m_\perp^{*2})^{1/2}}{3\pi^3\hbar^3 n}$$

$$\times N\int_0^\infty g(w)\left(-\frac{\partial f_0}{\partial w}\right)W^{3/2}dW,$$

N is the number of the equivalent minima, η_i, and ξ_i are the direction cosines of the magnetic field $\boldsymbol{H}$ and the current $\boldsymbol{J}$ to the axes $\{100\}$, respectively, R is the Hall coefficient, and ρ is the resistivity.

The first element of the matrix corresponds to the model when the energy minima are along $\langle 100\rangle$, the second element corresponds to the minima $\langle 111\rangle$, and the third to the minima $\langle 110\rangle$.

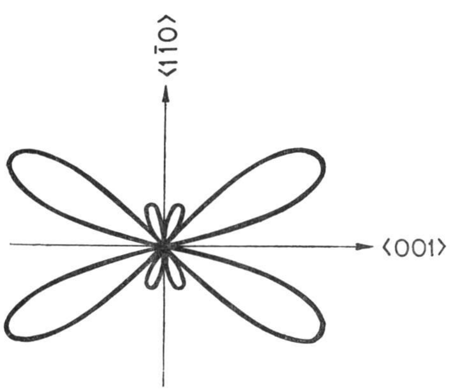

Fig. 2. The angular dependence of the longitudinal Hall effect.

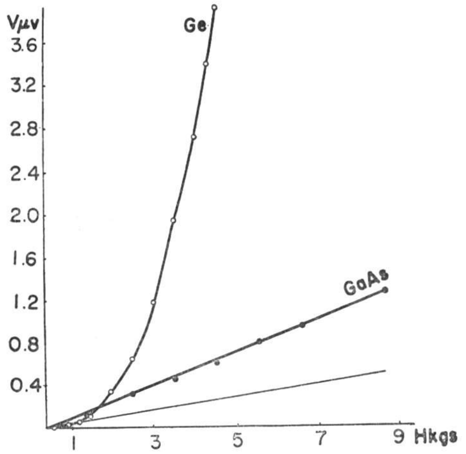

Fig. 3. The dependence of the longitudinal electric field on magnetic field value.

The angular dependence of the longitudinal Hall effect when $\boldsymbol{J}$ is parallel to $\langle 110\rangle$ and $\boldsymbol{H}$ changes in the (110) plane is shown in Fig. 2.

In our experiments the research is conducted on some samples which are cut from one ingot in the various directions but the current direction was constant.

For every sample the dependence of the value on the effect of magnetic field is measured.

This procedure permits one to separate the longitudinal Hall effect from the projection of the ordinary Hall effect which takes place through the inexact arrangement of the sample.

For weak compensated samples with electron concentration between 2×10^{15} and $2\times10^{17}\,\mathrm{cm}^{-3}$ at 300°K and 78°K the experimental points are exactly plotted on a straight line (Fig. 3). For the comparison, the dependence of longitudinal effect for Ge is plotted in Fig. 3. The ratio of the longitudinal Hall effect to the ordinary effect in GaAs is ≤ 0.0005. This limited value of the longitudinal effect shows that the parameter of the anisotropy K does not exceed 1.4 at room temperature.

The temperature dependence of the ordinary Hall effect and the magnetoresistance in the temperature region from 300°K to 800°K were measured for the study of the subsidiary minima. The temperature dependence of the transverse magnetoresistance is plotted for three specimens in Fig. 4. The transverse magnetoresistance decreases when the temperature increases. This is stipulated by the decrease in the electron mobility.

The dotted line in the figure shows the dependence T^{-3}. A good coincidence of the experimental points with theoretical curve probably shows the predominance of the scattering of the carriers by the longitudinal acoustic vibrations.

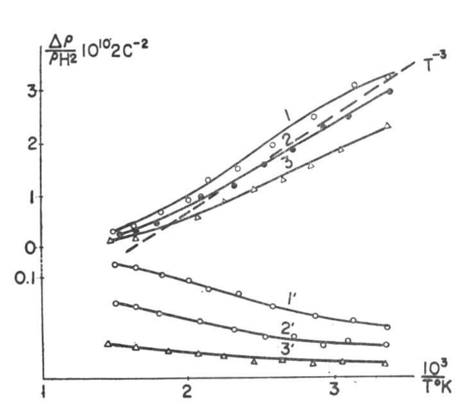

Fig. 4. Temperature dependence of transverse and longitudinal magnetoresistances.

Our results show that the ratio b/μ^2 for all the samples above room temperature is close to the theoretical value when the acoustic scattering predominates, though this comparison indicates that the optical vibration of atom takes place with the anisotropical scattering and the isotropical scattering mechanisms at the same time.

From the temperature dependence of the transverse magnetoresistance one can draw a conclusion on the situation of the subsidiary minima in the k-space. So far as the $\mu \propto T^{-3/2}$ law is fairly well satisfied even at high temperatures where the subsidiary minima may take part in the conduction, it should be expected that the intervalley scattering does not play an essential role at those temperatures.

It is possible that this is connected with the large value of the impulse for the electron transfer between the equivalent minima, which are arranged on the edge of the Brillouin zone.

The longitudinal magnetoresistance increases with temperature. This fact has been probably caused by the influence of the subsidiary anisotropic minima. The calculated values b, c and d correspond to the condition of $\langle 100 \rangle$ minima.

As electrons are scattered anisotropically in case of anisotropic dispersion law, then instead of the theoretically calculated values K_τ we can estimate the value K_m for the subsidiary minima. This value equals ~ 5.

The experimental results of the temperature dependence of the Hall coefficient at high temperature for the same samples are shown in Fig. 5.

In the samples with carriers 10^{16} and 10^{17} cm^{-3} the Hall coefficient does not change below 500°K. At higher temperatures R begins to increase. In the samples with electron concentration 10^{15} cm^{-3} the Hall coefficient decreases continuously with

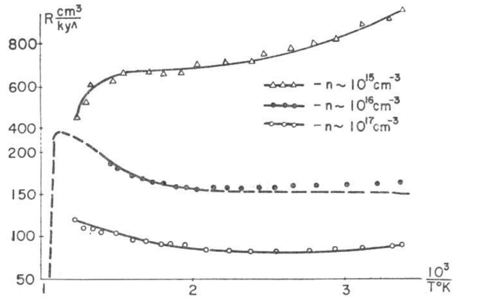

Fig. 5. Temperature dependence of the Hall coefficient.

the increase of T. Such a behaviour of the Hall coefficients in the samples with low electron concentration can be explained probably as the following: the presence of subsidiary minima with the small electron mobility is not sufficient for the growth of R with the increase of temperature. Actually, if the concentration of n_1 in the lower minimum increases with temperature faster than the concentration in the higher minima because of the release of trapped electron, the Hall coefficient will decrease. This condition is probably fulfilled in strongly compensated material with small carriers but with large concentration of impurities.

From the temperature dependence of the Hall coefficient at high temperatures we have

$$\Delta W = 0.36 \text{ eV}.$$

Assuming that $m_1^* = 0.07\, m_0$

$$m_2^* = 1.20\, m_0,$$

where m_0 is the mass of free electrons, and if $N=3$,

$$m_\perp^* = 0.37 \text{ m}_0$$
$$m_\parallel^* = 1.98 \text{ m}_0.$$

We did not take into account the degree of electron degeneration, the change of density of the states in the corresponding minima and the change of the position minima with the impurity.

Our calculations show that in general case for the carriers $10^{16} \sim 10^{18}$ cm^{-3} $\Delta W = 0.36 \sim 0.12$ eV[8] that was observed experimentally.[9]

The authors wish to express their special appreciation for the effective help of E. W. Scubnevskij in carrying our measurements.

References

1) A. F. Kravchenko and H. Y. Fan: *Proc. Int. Conf. Semiconductor Physics*, Exeter (1962) p. 737.
2) A. F. Kravchenko: Izvestija wuzov **5** (1964) 62.
3) S. A. Abagjan, A. W. Lishina and W. K. Subashiev: Fiz. tverdogo Tela **9** (1964).
4) H. Ehrenreich: Phys. Rev. **120** (1960) 6.
5) A. G. Samojlovich, I. J. Karenblit and I. W. Dahovskij *et al.*: Fiz. tverdogo Tela **3** (1961) 3285.
6) R. Stratton: *Proc. Int. Conf. Semiconductor Physics*, Exeter (1962) p. 81.
7) A. F. Kravchenko, A. V. Rhzanov and W. S. Sardarjan: Dokl. Akad. Nauk SSSR **164** (1965) 5.
8) A. F. Kravchenko and W. S. Sardarjan: Izv. Akad. Nauk SSSR **1** (1966).
9) E. Haga and H. Kimura: J. Phys. Soc. Japan **19** (1964) 9.