PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

IX-1. Cyclotron Resonance Line Width of Germanium and Silicon in the Quantum Limit

R. ITO,* M. FUKAI** and I. IMAI***

The Institute for Solid State Physics, University of Tokyo, Japan

The relaxation times of electrons in pure germanium and silicon were measured from the line width of cyclotron resonance at liquid helium temperatures at frequencies of 24 Gc/sec, 50 Gc/sec and 70 Gc/sec. Departure of the temperature dependence of the relaxation time from the $T^{-3/2}$ law characteristic of acoustic phonon scattering is explained by taking account of the quantization of the electron energy.

§ 1. Introduction

In recent years, cyclotron resonance has been used as a most powerful tool for the investigation of carrier scattering mechanisms in solids as well as their band structures. The width of the resonance line gives us direct information about the momentum relaxation time of conduction carriers. Extensive study has been done on germanium and silicon; various scattering mechanisms, such as scattering by phonons,[1] impurities,[2] or other carriers,[3] have been investigated and reasonable agreement between experiment and the usual transport theory has been obtained.

Meyer[4] was the first to point out that under "quantum" conditions, $\hbar\omega_0 \gg k_BT$, the cyclotron resonance line width cannot be described by the normal or "classical" relaxation time. This may be easily understood as follows: The classical transport theory is expected to be valid when the radius of curvature of the cyclotron orbit r_c is much larger than the force range of the scatterer, *i.e.*, in the case of phonon scattering, the wavelength λ_p of phonons which most strongly interact with the carriers. Noting that

$$r_c \cong \sqrt{\frac{\hbar}{m\omega_0}\left(n+\frac{1}{2}\right)} \cong \sqrt{\frac{\hbar}{m\omega_0}\cdot\frac{k_BT}{\hbar\omega_0}}$$

where ω_0 is the cyclotron angular frequency, m the effective mass, n the quantum number of the Landau state $|nk_yk_z>$ and the other symbols have their conventional meaning, and that,

$$\lambda_p \cong \sqrt{\frac{\hbar}{m\omega_0(n+1)}} \cong \sqrt{\frac{\hbar}{m\omega_0}\cdot\frac{\hbar\omega_0}{k_BT}},$$

we may rewrite the condition of validity of the use of the classical theory as

$$\hbar\omega_0/k_BT \ll 1$$

which also ensures the validity of the use of the classical mechanical treatment because of the very large values n of the quantum number involved. Hensel[5] measured the line width of electron cyclotron resonance in silicon in the mm-wave range at low temperatures and showed that the phonon limited relaxation time becomes shorter than that expected from the classical theory as we approach the quantum condition $\hbar\omega_0/k_BT \gg 1$ in qualitative agreement with Meyer's prediction which is based on the assumption that the low temperature line width would be determined by the transition probability from the Landau state $|n=1, k\cong 0>$ to $|n=0, k>$.

The aim of the present work is to theoretically examine Meyer's model, to find a more rigorous expression for the cyclotron resonance line width due to phonon scattering on the basis of quantum transport theory, and to compare the theoretical results with experimental data on the line width of the electron resonance in germanium and silicon. We have measured the line width as a function of temperature and resonance frequency, varying the ratio $\hbar\omega_0/k_BT$ from 0.2 to 2.1.

§ 2. Theory

In order to obtain the line shape of cyclotron resonance, we must calculate the conductivity tensor component $\sigma_{xx}(\omega)$ in the presence of the static magnetic field $H=(0, 0, H)$. The response function $\phi_{xx}(t)$, or the inverse Fourier transform of $\sigma_{xx}(\omega)$, can be evaluated by the diagram technique developed by Konstantinov and

* Present address: Central Research Laboratory, Hitachi, Ltd., Kokubunji, Tokyo.

** Present address: Central Research Laboratory, Matsushita Electric Industrial Co., Ltd., Kadoma, Osaka.

*** Present address: College of General Education, University of Tokyo, Tokyo.

Perel'.[6] $\phi_{xx}(t)$ may be expanded in powers of the electron-phonon interaction V; every term corresponds to a Feynman diagram.

We shall take V as a small perturbation and calculate $\phi_{xx}(t)$ up to the second order in V. It is easy to see that the most important contribution comes from the following terms which do not vanish in the zero's approximation:

$$\phi_{xx}(t)=\sum_{\alpha\gamma}\{(j_x)_{\alpha,\alpha+1}(j_x)_{\gamma,\gamma-1}G^{\alpha+1,\gamma-1}_{\alpha,\gamma}(-t,\beta) +(j_x)_{\alpha+1,\alpha}(j_x)_{\gamma,\gamma+1}G^{\alpha,\gamma+1}_{\alpha+1,\gamma}(-t,\beta)\} . \quad (2.1)$$

Here $|\alpha+1>$ represents the state which is different from the state $|\alpha>$ only in the quantum number n by unity, $\beta=1/k_BT$ and $(j_x)_{\alpha,\gamma}$ is the usual matrix element of the current density in the x-direction with the Landau states as the basis. G's are found to be approximately given by

$$G^{\alpha+1,\gamma-1}_{\alpha,\gamma}(-t,\beta)=\delta_{\gamma,\alpha+1}(1-f_\alpha) \times(\hbar\omega_0)^{-1}(f_\alpha-f_{\alpha+1})e^{i\omega_0 t}\left(1-\frac{1}{\tau(\alpha,\alpha+1)}\right) \quad (2.2)$$

$$G^{\alpha,\gamma+1}_{\alpha+1,\gamma}(-t,\beta)=\delta_{\gamma,\alpha}(1-f_{\alpha+1}) \times(\hbar\omega_0)^{-1}(f_\alpha-f_{\alpha+1})e^{-i\omega_0 t}\left(1-\frac{1}{\tau(\alpha,\alpha+1)}\right) . \quad (2.2)'$$

Here we have neglected the renormalization of the electron energy and a small background to the absorption signal. The last factor in eq. (2.2) may, from the damping theoretical point of view, be approximated by an exponential function $\exp[-t/\tau(\alpha,\alpha+1)]$ where

$$1/\tau(\alpha,\alpha+1)=(1/2)(\Gamma(\alpha)+\Gamma(\alpha+1)) \quad (2.3)$$

and

$$\Gamma(\alpha)=\Gamma_e(\alpha)+\Gamma_a(\alpha) =\sum_{\mu q}\frac{2\pi}{\hbar}|c_q|^2|J_{\alpha\mu}(q)|^2(1+N_q-f_\mu)\delta(\varepsilon_\alpha-\varepsilon_\mu-\hbar\omega_q) +\sum_{\mu q}\frac{2\pi}{\hbar}|c_q|^2|J_{\mu\alpha}(q)|^2(N_q+f_\mu)\delta(\varepsilon_\alpha-\varepsilon_\mu+\hbar\omega_q) . \quad (2.4)$$

In the above equations, N_q and f_μ denote the equilibrium occupation numbers of the phonon and the electron, respectively, and c_q and $J_{\alpha\mu}(q)$ are defined as follows:

$$c_q=i\Xi(\hbar/2\rho\omega)q ,$$
$$J_{\alpha\mu}(q)=<\alpha|e^{iqx}|\mu> ,$$

where ρ is the density of the crystal and Ξ is the deformation potential constant. $\Gamma(\alpha)$ may be interpreted as the transition probability from the state $|\alpha>$, $\Gamma_e(\alpha)$ and $\Gamma_a(\alpha)$ corresponding, respectively, to the phonon emission and absorption probabilities.

Using eqs. (2.1) and (2.2), we arrive at the following expression for the conductivity component $\sigma_{xx}(\omega)$

$$\sigma_{xx}(\omega)=\frac{e^2}{m}\sum_{\alpha}(n_\alpha+1)(f_\alpha-f_{\alpha+1}) \frac{i\omega+\{\tau(\alpha,\alpha+1)\}^{-1}}{\omega_0^2+[i\omega+\{\tau(\alpha,\alpha+1)\}^{-1}]^2} , \quad (2.5)$$

where n_α is the quantum number n of the state $|\alpha>$. $\mathrm{Re}\{\sigma_{xx}(\omega)\}$ is a superposition of homogeneously broadened signals with the Lorentz line shape. The width of the line arising from the photon-induced transition $|\alpha>\rightleftarrows|\alpha+1>$ is determined by the arithmetic mean of the phonon-induced transition probabilities from the two states, in contradistinction to Meyer's assumption.

We shall calculate the line width explicitly for the cases of interest.

i) Classical phonons. $N_q\cong k_BT/\hbar\omega_q\gg1$. In this case, $\Gamma(\alpha)$ reduces to

$$\Gamma(\alpha)=\frac{m^{3/2}\Xi^2\hbar\omega_0k_BT}{2^{1/2}\pi\hbar^4\rho s^2}\sum_{n'}\left[\varepsilon_\alpha-\left(n'+\frac{1}{2}\right)\hbar\omega_0\right]^{-1/2} , \quad (2.6)$$

where s is the sound velocity. It is seen that $\Gamma(\alpha)$ is proportional to the density of states corresponding to the energy ε_α. Putting eq. (2.6) into eq. (2.5) and taking its real part, we obtain an explicit expression for the line shape function

$$\sigma^*(X,\eta)=\int_0^\infty dE\sum_n(n+1)(E-n\eta)^{-1/2} \times\frac{\{(E+\eta)^{-1/2}+2\sum_{n'}(E-n'\eta)^{-1/2}\}^{-1}}{1+\dfrac{16X^2}{\{\eta[(E+\eta)^{-1/2}+2\sum_{n'}(E-n'\eta)^{-1/2}]\}^2}} \quad (2.7)$$

where

$$X=(\omega-\omega_0)(k_BT)^{-3/2}2^{-1/2}\pi\hbar^4m^{-3/2}\Xi^{-2}\rho s^2 =(\omega-\omega_0)[\tau(k_BT)]_{\mathrm{class}}$$

and

$$\eta=\hbar\omega_0/k_BT .$$

It is to be noted that the line shape thus reduced depends only on the parameter η. We have evaluated eq. (2.7) numerically for $\eta\leqq7$. $\sigma^*(X,\eta)/\sigma^*(0,\eta)=1/2$ gives us the reduced line width X_{half}, which is related to the true line width by

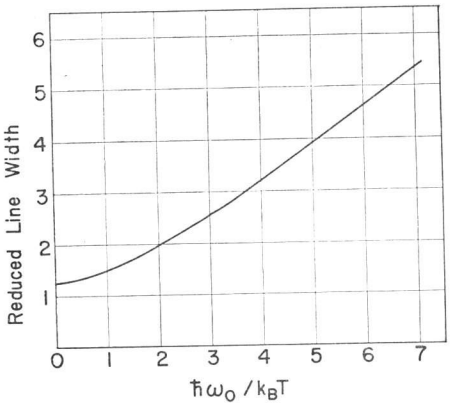

Fig. 1. The ratio of the quantum line width to the classical one as a function of $\hbar\omega_0/k_BT$.

$$(\Delta\omega)_{\text{half}}=(1/\tau(k_BT))_{\text{class}}X_{\text{half}}\ .$$

In Fig. 1 X_{half} is shown as a function of η. The limiting value 1.25 of X_{half} as $\eta\to 0$ has been obtained by Fukai *et al.*[1]

ii) *Extreme low temperatures.* $N_q\cong 0$. In this case, only phonon emission processes are possible. Moreover, for electrons with $|k_z|<ms/\hbar$, the transitions $|n,\boldsymbol{k}>\to|n,\boldsymbol{k}'>$ are forbidden by the conservation of energy and momentum. Thus, the limiting line width when $k_BT\ll ms^2$ is determined by the transition probability $|n=1,\boldsymbol{k}\cong 0>\to|n=0,\boldsymbol{k}>$, or

$$\left(\frac{1}{\tau}\right)_{\text{lim}}\cong\frac{1.7}{4\pi}\,\frac{m^2\Xi^2\omega_0}{\hbar^3\rho s}\ . \tag{2.8}$$

When electrons with $|k_z|>ms/\hbar$ contribute much to the resonance, it can be shown that scattering which conserves n adds the line width

$$\left(\frac{1}{\tau}\right)_{\text{add}}\sim\left(\frac{1}{\tau}\right)_{\text{lim}}\sqrt{\frac{\hbar\omega_0}{\varepsilon_z}} \tag{2.9}$$

for electrons of energy $ms^2\ll\varepsilon_z\ll\hbar\omega_0$.

§ 3. Experiment

Cyclotron resonance measurements were made on very pure samples (whose impurity concentration is estimated to be smaller than $10^{12}/\text{cm}^3$) of germanium and silicon at liquid helium temperatures at frequencies of 24 Gc/sec, 50 Gc/sec and 70 Gc/sec. Carriers were generated by chopped infrared light. In order to avoid line broadening due to carrier heating[7] and carrier-carrier interaction,[3] incident microwave power was lowered below 0.01 μW and light intensity was reduced until the line no longer continued to narrow. For frequencies of 50 Gc/sec and 70 Gc/sec, we employed a superheterodyne detection scheme armed with balanced mixer crystals to improve the spectrometer sensitivity. For the K band, a homodyne detection was used.

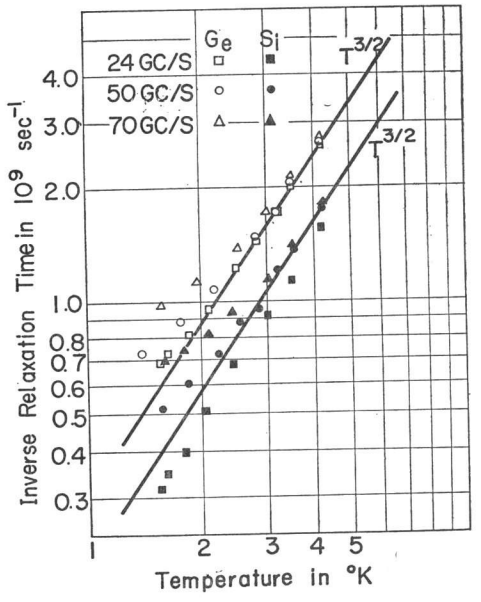

Fig. 2. Inverse relaxation time $(1/2)(\tau_\perp^{-1}+\tau_{/\!/}^{-1})$ of electrons in germanium and silicon versus temperature.

Figure 2 shows the inverse relaxation times experimentally obtained for germanium and silicon. The data were taken by applying the static magnetic field perpendicular to the longer axis of an electron ellipsoid, corresponding to the inverse relaxation time $(1/2)(\tau_\perp^{-1}+\tau_{/\!/}^{-1})$. The inverse relaxation time is defined by

$$1/\tau=(\Delta H/H_0)(\omega_0/1.25)\ ,$$

where H_0 is the resonance field and ΔH is the half-width at half power point. The factor 1.25 was noted earlier. We see that the classical $T^{3/2}$ dependence, which is characteristic of intravalley acoustic phonon scattering, is well obeyed at higher temperatures and at lower frequencies. We note that the deviation from the classical behavior is remarkable for large values of $\hbar\omega_0/k_BT$.

We also measured the line width by applying the magnetic field parallel to the longer axis of the electron ellipsoid. This yielded the inverse relaxation time $\tau_\perp^{-1}$. Combining these results, we have obtained the anisotropy of the relaxation time which turns out to be independent of both temperature and frequency. The deformation potential constants[8] determined from these

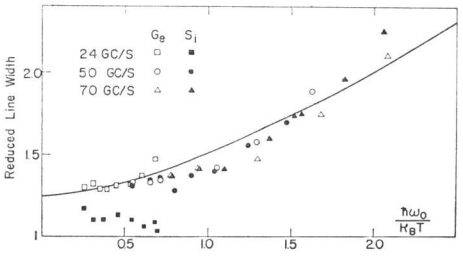

Fig. 3. Comparison between the reduced line widths experimentally obtained for germanium and silicon and the theoretical result (solid curve).

data are $\Xi_u=(18.7\pm0.2)$ eV, $\Xi_d=-(10.5\pm0.5)$ eV for germanium and $\Xi_u=(8.5\pm0.1)$ eV, $\Xi_d=-(5.2\pm0.3)$ eV for silicon. Since neutral impurity scattering has been found isotropic,[9] the constancy of the anisotropy ensures that our measured line widths are determined predominantly by acoustic phonon scattering. Thus, we believe we can ascribe the observed deviation from the $T^{3/2}$ law to the quantum effect inherent to acoustic phonon scattering.

§ 4. Comparison with Theory

In Fig. 3 the points represent the reduced line widths which are obtained by taking the ratio of the measured line width to that extrapolated from the classical region. The abscissa represents the parameter $\hbar\omega_0/k_BT$.

The data for germanium and silicon have been collected together. The solid line is the theoretical result obtained from the curve in Fig. 1. We see that the reduced line width can be well expressed as a function of the single parameter $\hbar\omega_0/k_BT$ under our experimental conditions. The good agreement between experiment and theory confirms the correctness of our model.

In conclusion, we have seen that the cyclotron resonance line width due to acoustic phonon scattering becomes broader in the quantum region $\hbar\omega_0/k_BT>1$ as a result of the orbit quantization and the accompanying increase in the state density, and that the relative line width will eventually be independent of the cyclotron frequency in the low temperature quantum limit. The width of the line arising from the photon-induced transition $|\alpha> \rightleftarrows |\alpha+1>$ is given by the arithmetic mean of the phonon-induced transition probabilities from the two states, in contradistinction to Meyer's model.

Acknowledgements

We would like to express our sincere thanks to Professor H. Kawamura for his continued interest in the work and very valuable suggestions and discussions. We are also grateful to Professor Y. Toyozawa for enlightening discussions, and to Mrs. H. Shiratori and Mr. T. Nakama for their aid in the experiment.

References

1) M. Fukai, H. Kawamura, K. Sekido and I. Imai: J. Phys. Soc. Japan **19** (1964) 30.
2) K. Sekido, M. Fukai and H. Kawamura: J. Phys. Soc. Japan **19** (1964) 1579.
3) H. Kawamura, H. Saji, M. Fukai, K. Sekido and I. Imai: J. Phys. Soc. Japan **19** (1964) 288.
4) H. J. G. Meyer: Phys. Letters **2** (1962) 259.
5) J. C. Hensel: Phys. Letters **4** (1963) 38.
6) O. V. Konstantinov and V. I. Perel': Soviet Physics-JETP **12** (1961) 142.
7) H. Kawamura, M. Fukai and Y. Hayashi: J. Phys. Soc. Japan **17** (1962) 970.
8) C. Herring and E. Vogt: Phys. Rev. **101** (1956) 944.
9) R. Ito, H. Kawamura and M. Fukai: Phys. Letters **13** (1964) 26.

DISCUSSION

Stradling, R. A.: I would point out that the assumption of elastic acoustic phonon scattering breaks down below 10°K and your results are therefore not comparable with measurements made at higher temperatures, and indeed your values of $\langle\tau_{\parallel}\rangle/\langle\tau_{\perp}\rangle$ do not agree with cyclotron resonance measurements of the same parameter at 77°K.

Ito, R.: With $m^*=0.2\,m_0$, and $s=5\times10^5$ cm/sec, the ratio of the phonon energy to the thermal energy is about 0.3 at 3°K, therefore the elastic scattering approximation partially breaks down at low temperatures as you point out. It is possible that the relaxation time anisotropy may be affected by the inelasticity of the scattering, because the effective mass is highly anisotropic and the elastic scattering approximation does depend on the effective mass. However, I would like to point out that ionized impurity scattering, which does not come into our measurements, can significantly affect the relaxation time anisotropy in those experiments which utilize thermal electrons, since the anisotropy of ionized impurity scattering is very large. We have estimated that 7% mixture of ionized impurity scattering

will be able to raise the anisotropy factor from our value 1.67 to 2.3 in Ge.

Stradling, R. A.: I agree with your remarks about ionized impurity scattering modifying the scattering anisotropy if the number of ionized sites is large. However, the high temperature experiments in both silicon and germanium were performed on ultra-high purity samples where the concentrations of ionized sites were about 10^2 to 10^4 smaller than the concentrations necessary to produce the 7% admixture you postulate. Furthermore, such an admixture would drastically modify both the temperature dependence and anisotropy of the relaxation times at lower temperatures. This was not observed.