# IX-2. Magnetoresistance Oscillations Due to Inelastic Scattering of Electrons on Optical Phonons

I. M. TSIDILKOVSKI and M. M. AKSELROD

Institute of Metal Physics, Sverdlovsk, USSR

The transverse  $(\rho_{xx})$  and longitudinal  $(\rho_{zz})$  magneto-resistance in *n*-InSb and *n*-InAs are investigated in the magnetic fields up to ~500 kG. It is shown that the minimum of  $\rho_{zz}$  at 75 kG in InSb and at 480 kG in InAs, and the maximum of  $\rho_{zx}$  at 82 kG in InSb are due to spinmagnetophonon resonance (SMR), while the maximum of  $\rho_{zz}$  in InAs at 76 kG is caused by magnetophonon resonance (MPR).

### §1. Introduction

When the distance between two Landau levels becomes equal to the energy of the longitudinal optical phonon  $\hbar \omega_0^{\ l}$ , the probability of inelastic scattering of electrons on optical phonons increases sharply and the dependence of the resistance on magnetic field should have some extrema<sup>1,2)</sup>. These extrema may be observed if  $\Omega \tau \gg 1$ , where  $\Omega = eH/m^*c$  is the cyclotron frequency,  $\tau$  the relaxation time and  $m^*$  the effective mass of the electrons.

The condition of appearance of magnetoresistance oscillations may be written as

$$\varepsilon_N - \varepsilon_K = \hbar \omega_0^{\ l}, \qquad (1)$$

where  $\varepsilon_N$  and  $\varepsilon_K$  are the energies of the *N*-th and *K*-th Landau levels. For the simple parabolic band eq. (1) takes the form

$$M\Omega = \omega_0^{l}, \quad M = 1, 2, 3...$$
 (2)

This effect, named magnetophonon resonance (MPR), was observed first in n-InSb by Puri and Geballe<sup>3)</sup> and then investigated in details by Shalyt et  $al.^{(4)}$  and Tsidilkovski et  $al.^{(5)}$ . Shalyt<sup>4)</sup> and Tsidilkovski<sup>5)</sup> have found that the transverse magnetoresistance of n-InSb has five maxima located at  $\sim 6$ ,  $\sim 8$ ,  $\sim 11$ , 17 and 35 kG, and the longitudinal, approximately at the same fields, five minima. The MPR oscillations of the longitudinal magnetoresistance of n-InAs were observed by Tsidilkovski et al.6) and Shalyt et al.<sup>7)</sup>. Since the values of  $m^*$  and  $\omega_0^l$  for InSb are known, it is easy to see that the mentioned extrema correspond to resonance transitions of electrons from the Landau level N=0 to the levels N=5 (6 kG), 4, 3, 2 and 1 (35 kG).

In the case of MPR transitions the electron spin orientation does not change. But one can imagine a situation when  $\hbar \omega_0^{\ l}$  is equal to the

spacing between the spin sublevels of the same Landau level. Then one can expect an effect similar to MPR, but accompanied by a spin inversion—SMR. The resonance condition (1) may be written then as

$$\varepsilon_{NS} - \varepsilon_{NS'} = \hbar \omega_0^{l}, \qquad (3)$$

where s, s' = +1/2 or -1/2 depending on spin orientation.

It is obvious that the SMR could be expected in semiconductors with a strong spin-orbit interaction, for direct spin-phonon interaction is impossible. In addition the spectroscopic splitting factor g should be large enough in order to satisfy the resonance condition at really attainable magnetic fields. From this point of view InSb and InAs are the most suitable materials because of their small electron effective masses and relatively large spin-orbit splittings.

Here will be reported the results of investigation of the MPR and SMR oscillations in InAs. The data so far existing on the subject have not been satisfactorily explained. Besides the recent data on the SMR oscillations in InSb will be presented.

## §2. Analysis

#### Magnetophonon resonance

In high magnetic fields  $(\Omega \tau \gg 1)$  transverse magnetoresistance  $\rho_{xx} = \sigma_{xx}/\sigma_{xy}^2$ , where  $\sigma_{xx}$  and  $\sigma_{xy}$ are components of the conductivity tensor. Since  $\sigma_{xy} = \text{enc/H}$  is independent of scattering,  $\rho_{xx}$  is proportional to the scattering probability, just as  $\sigma_{xx}$ . The scattering probability sharply increases, when the resonance condition (1) is fulfilled, and therefore  $\rho_{xx}$  should always have a maximum. If several scattering mechanisms are present, their contributions to the scattering probability, and hence to  $\rho_{xx}$ , are additive. The longitudinal magnetoresistance  $\rho_{zz} = \sigma_{zz}^{-1}$ , and  $\sigma_{zz}$  is inversely proportional to the scattering probability. Therefore, the contributions of different scattering mechanisms to  $\rho_{zz}$  are not additive. As a result  $\rho_{zz}$  may have both a minimum and a maximum under the resonance conditions. In the case of mixed scattering by optical and acoustic phonons  $\rho_{zz}$  should have minima at the resonance values of H when the parameter

$$\Gamma^{-1} = \frac{3\sqrt{\pi}kT}{4\hbar\Omega} \left(\frac{\hbar\omega_0^l}{kT}\right)^{1/2} \frac{\mu}{\mu_0} \alpha < 1 , \qquad (4)$$

and maxima, when  $\Gamma^{-1} > 1$ ;  $\mu_0 = e/m^* \omega_0^{\ l}$ ,  $\mu$  is the electron mobility due to scattering by acoustic phonons and  $\alpha$  is the electron-phonon coupling constant.

#### Spin-magnetophonon resonance

The calculation of Tsidilkovski et al.8) and Pavlov et al.<sup>9)</sup> have shown that the resonance scattering on optical phonons with spin inversion should cause extrema in the dependencies of both  $\rho_{xx}$  and  $\rho_{zz}$  on the magnetic field. The  $\rho_{xx}$ should always have a maximum whether the scattering on optical or acoustic phonons prevails. The essential feature of the SMR oscillation of  $\rho_{xx}$  is that the main contribution to the interaction with the electrons in the resonance region make the transverse phonons (frequency  $\omega_0^t$ ). Under the SMR conditions  $\rho_{zz}$ should always have a minimum, except the ideal case when scattering on optical phonons is the sole interaction mechanism and the unreal situation when the resonance scattering on optical phonons with spin inversion is the dominant mechanism.

The position of the SMR minimum can be displaced from the resonance position  $H = \hbar \omega_0^{\ l}/g\mu_B$  within the limits  $\Delta H \approx kT/g\mu_B$ , depending on the relations between the coupling constants of different scattering mechanisms.  $\mu_B$  is the Bohr magneton and g is the spectroscopic splitting factor.

### § 3. Experimental Data

The measurements of the magnetoresistance were made with pulsed magnetic fields up to ~500 kG in the temperature range  $80-370^{\circ}$ K. Experimental details were described previously<sup>5,8)</sup>. The big monotonic part (background) of  $\rho_{xx}$ was diminished by both differentiation of the signal with respect to time and subtraction from the measured signal a voltage proportional to H. Due to all precautions, we succeeded in



Fig. 1. Longitudinal ( $\parallel$ ) and transverse ( $\perp$ ) magnetoresistance for *n*-InSb ( $n=2.4 \times 10^{14} \text{cm}^{-3}$ ) at 120°K in arbitrary units. The extrema at 17, 35, 75 and 82 kG are shown by arrows.

finding the negative region of  $\rho_{zz}(H)$  in InSb and InAs, which takes place at magnetic fields corresponding to the intermediate region between the classical and quantum limit<sup>5,6,10</sup>.

#### Indium antimonide

Figure 1 shows that  $\rho_{zz}$  has a minimum at 75 kG and  $\rho_{xx}$  has a weak maximum at 82 kG. The minimum of  $\rho_{zz}$  at 75 kG and the maximum of  $\rho_{xx}$  at 82 kG, first observed by us<sup>5,8)</sup>, are due to SMR. For the nonparabolic conduction band of InSb

$$\varepsilon_{N} = -\frac{A}{2} + \sqrt{\frac{A^{2}}{4} + B[(N+1/2)\hbar\Omega \pm (1/2)g_{0}\mu_{B}H]},$$
(5)

where  $A = \varepsilon_g \frac{\varepsilon_g + \Delta}{2\varepsilon_g + \Delta}$ ,  $B = \left(1 - \frac{m_0^*}{m}\right)A$ ,  $\varepsilon_g$  is

the forbidden gap,  $g_0$  the g-factor at the bottom of the band. With  $\varepsilon_g = 0.21 \text{ eV}$ ,  $\Delta = 0.9 \text{ eV}$ , and  $\omega_0{}^t = 3.48 \times 10^{13} \text{scc}^{-1}$  one can obtain from (3) and (5) at  $H_{\min} = 75 \text{ kG}$  that  $|g_0 \exp| = 70$  for  $m_0{}^* =$ 0.014 m and 73 for  $m_0{}^* = 0.012 \text{ m}$ . At  $H_{\max} =$  $82 \text{ kG} |g_0 \exp| = 61$  and 63 correspondingly  $(m_0{}^*$ is the effective mass at the bottom of the band). According to Roth<sup>11</sup>  $|g_0 \text{theor}| = 50$  for  $m_0{}^* =$ 0.014 m and 58 for  $m_0{}^* = 0.012 \text{ m}$  at  $100{}^\circ\text{K}$ . The agreement between the values  $g_0 \text{theor}$  and  $g_0 \exp$ indicates that both extrema are really caused by SMR.

#### Indium arsenide

The transverse magnetoresistance of *n*-InAs has a maximum at 76 kG (Fig. 2a). The dependence of  $d^2 \rho_{xx}/dt^2$  on H confirms the existence of a maximum near 70 kG (Fig. 2b). The estimation of H<sub>max</sub> should be made on the base of the expression (5). Assuming  $\omega_0^{\ l}=4.6\times10^{13}$ 

sec<sup>-1</sup>,  $\varepsilon_g(300^{\circ}\text{K}) = 0.34 \text{ eV}$ ,  $m_0^* = 0.023 \text{ m}$ ,  $\Delta = 0.43 \text{ eV}$ ,  $|g_0^{\text{theor}}| = 18$  we obtain from (1) and (5) that  $H_{\min} = 74 \text{ kG}$ , which corresponds to the transition  $\varepsilon_0 \rightarrow \varepsilon_1$ .



Fig. 2. Transverse magnetoresistance of *n*-InAs  $(n=2.2\times10^{16} \text{ cm}^{-3})$  at 300°K in arbitrary units. The arrows show the maxima at 76, ~66 and ~30 kG.

Taking into account the transitions  $\varepsilon_1 \rightarrow \varepsilon_2$  and  $\varepsilon_2 \rightarrow \varepsilon_3$  we find that  $H_{max} = 78 \text{ kG}$ .

The minimum at 78 kG on the curve  $\rho_{zz}(H)$ (Fig. 3) is naturally to associate with the  $(\varepsilon_0 \rightarrow \varepsilon_1)$ -type transition. But for the last MPR peak  $\Gamma^{-1} \approx 30$ , if the deformation potential constant is taken equal to 10 eV<sup>12</sup>). At such a value of  $\Gamma^{-1} \rho_{zz}$  should have a maximum near  $\Omega = \omega_0^{l}$ . Although the criterion of the existence of a



Fig. 3. Longitudinal magnetoresistance for *n*-InAs  $(n=2.2\times10^{16} \text{ cm}^{-3})$  at 300°K (the lower curve) and 83°K (the upper curve). The arrows show the maximum at 110 kG and the minimum at 480 kG.

maximum,  $\Gamma^{-1} > 1$  is apparently rather approximate, it is fulfilled here with a big redundancy. The maximum of  $\rho_{zz}$  is located at 110 kG (Fig. 3).

The analysis of the expression for  $\rho_{zz}$  in the case of pure optical scattering shows that the maximum of  $\rho_{zz}$  should be displaced from its position at  $kT \ll \hbar \omega_0^{\ l}$  toward higher fields by ~ 50% when  $kT \approx \hbar \omega_0^{\ l}$ . At 300°K for InAs  $kT/\hbar \omega_0^{\ l} = 0.85$ . On the other hand, the existence of nonresonant scattering mechanisms can also cause a displacement of the extremum of  $\rho_{zz}$  within the limits  $\Delta H = \pm kTm^*/2\mu_Bm$ . At low temperatures,  $T \le 100$ °K, when the contribution of optical scattering is small, the maximum of  $\rho_{zz}$  vanishes (Fig. 3).

In the region of intermediate magnetic fields  $(\hbar \Omega \sim kT) \rho_{zz}(H)$  has a negative part (Fig. 3), which is of the form of a broad minimum. Therefore, the presence of the MPR maximum of  $\rho_{zz}$  leads to the appearance of two subsidiary nonresonant minima at  $\sim$ 78 and  $\sim$ 160 kG.

Finally, the minimum at 480 kG (Fig. 3), as it is readily seen, corresponds to SMR transitions  $\varepsilon_{0-} \rightarrow \varepsilon_{0+}$ . Indeed, from (2) and (5) follows that at H<sub>min</sub>=480 kG  $|g_0^{exp}|=19$ . This is in good agreement with  $|g_0^{\text{theor}}|=18$ .

#### References

- V. L. Gurevich and J. A. Firsov: Zh. eksper. teor. Fiz. 40 (1961) 199.
- M. I. Klinger: Fiz. tverdogo. Tela 3 (1961) 1342.
- S. M. Puri and T. H. Geballe: Bull. Amer. Phys. Soc. 8 (1963) 309.
- S. S. Shalyt, R. V. Parfenev and V. M. Muzdaba: Fiz. tverdogo. Tela 6 (1964) 647.
- 5) I. M. Tsidilkovski, V. I. Sokolov and M. M. Akselrod: Fiz. tverdogo. Tela 7 (1965) 316.
- M. M. Akselrod, V. I. Sokolov and I. M. Tsidilkovski: Phys. Status solidi 8 (1965) K 15.
- D. V. Mashovets, R. V. Parfenev and S. S. Shalyt: Zh. eksper. teor. Fiz. 1 (1965) N 3, 2.
- I. M. Tsidilkovski, M. M. Akselrod and S. I. Uritsky: Phys. Status solidi 12 (1965) 667.
- S. T. Pavlov and J. A. Firsov: Zh. eksper. teor. Fiz. 49 (1965) 1664.
- I. M. Tsidilkovski, M. M. Akselrod, V. I. Sokolov and G. I. Harus: Phys. Status solidi 9 (1965) K 91.
- L. Roth, B. Lax and S. Zwerdling: Phys. Rev. 114 (1959) 90.
- 12) E. Haga, H. Kimura: J. Phys. Soc. Japan 19 (1964) 471.

## DISCUSSION

Moss, T. S.: How did you produce the 500 KG magnetic field?

Tsidilkovski, I. M.: The high magnetic fields are produced in the conventional way with a beryllium-copper coil and pulsed current from a condensor bank.

Fritzsche, H.: Disregarding for a moment the effects of the MPR on the magnetoresistance, can you explain the gross feature of the field dependence of the magnetoresistance? Both the perpendicular and the longitudinal magnetoresistance of InSb seem to consist of a component which is nearly proportional to H and a component which appears to saturate at larger H. The second component has opposite signs for the two orientations in contrast to the anomalous part of the magnetoresistance (see paper XIII-1 by W. Sasaki), which is found in many semiconductors at intermediate impurity concentrations.

Tsidilkovski, I. M.: Our experiments are performed at relatively high temperatures and the effects that you refer to occur at helium temperatures and are due to scattering on localized electrons with magnetic moments. In our case the main contribution to scattering is from lattice vibrations and ion impurities. The minimum of the longitudinal magnetoresistance at the intermediate magnetic fields (between the classical and quantum limits) is explained in our paper in Phys. Status solidi 8 (1965) 15.