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IX-6. Shubnikov-de Haas Effect in Tellurium

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The transverse and longitudinal magnetoresistance of heavily doped tellurium single crystals ($p=1\times10^{17}$–$6\times10^{18}/\mathrm{cm}^3$) was investigated in pulsed magnetic fields up to 220 kOe at 4.2°K. Pronounced Shubnikov-de Haas oscillations were detected and used to explore the topology of the constant energy surfaces of the valence band. Analysis of the anisotropy of the measured periods in H^{-1} shows that the energy surfaces have rotational symmetry about the trigonal axis and cannot be described by ellipsoids. These findings are incompatible with conclusions which have been drawn from previous measurements of the low field magnetoresistance in tellurium.

§ 1. Introduction

Various attempts have been made to elucidate the valence band structure of tellurium by low field magnetoresistance experiments.[1-5] The success has been dubious, however, and no accord was obtained between different workers; 1-, 3-, 6-, and 12-ellipsoid models have been proposed to interpret the data. This is not surprising from the experimental point of view, because the electrical properties of pure single crystals are extremely sensitive to slight mechanical strain. But even if this difficulty is circumvented by using doped or annealed samples, another obstacle is, that the relaxation time τ enters the results. This is important, if τ is anisotropic, a situation, which probably holds for tellurium.

It seemed therefore worth-while, to investigate the energy surfaces of tellurium with a more fundamental method,[6] f. i. the oscillatory magnetoresistance (Shubnikov-de Haas effect), which may be observed in statistically degenerate semiconductors in strong magnetic fields at low temperatures. For ellipsoidal energy surfaces it is advantageous to relate the Shubnikov-de Haas periods to the components of the effective mass tensor. For more general surfaces of constant energy it is more appropriate, however, to discuss the results in terms of extremal cross-sections of the Fermi-surface, according to Onsager's treatment[7] of the de Haas-van Alphen effect. If A is an extremal cross-section in $\boldsymbol{k}$-space ($\boldsymbol{k}=$ wave vector) of the Fermi-surface (which, of course, depends on the doping level in semiconductors) perpendicular to the magnetic field, the following relation holds for the period ΔH^{-1} of the oscillations:

$$\Delta H^{-1}=\frac{2\pi e}{\hbar c}\cdot\frac{1}{A}\,. \qquad (1)$$

H is the magnetic field strength, the other symbols have their usual meaning. Hence, much information about band structure may be obtained by measuring the anisotropy of the Shubnikov-de Haas effect.

§ 2. Experimental

The samples used were cut from single crystals which were heavily doped with bismuth or antimony. The hole concentration varied between $1\times10^{17}/\mathrm{cm}^3$ and $6\times10^{18}/\mathrm{cm}^3$ and was determined from low- and high-field Hall effect measurements. The specimens had a cross-section of about 1 mm^2 and were between 5 and 10 mm long. They were suspended in a strain-free manner from current and potential leads consisting of platinum wires with 0.15 mm diameter, which were attached by spot welding. Because the hole mobility at helium temperatures was only a few thousand cm^2/Vs, magnetic fields of the order 100 kOe had to be used in order to observe the Shubnikov-de Haas effect. Pulsed fields up to 220 kOe were employed, the details of the experimental setup have been published elsewhere.[8] In addition to previously used "longitudinal" magnet coils "transverse", split coils, which could be rotated by almost 360° around specimens in the center, were found convenient for measuring angular dependences. The resistance of the tellurium crystals was plotted as a function of the magnetic field on an oscilloscope. A typical oscillogram is shown in Fig. 1.

The current through the samples is switched on, when the magnetic field starts to rise, and

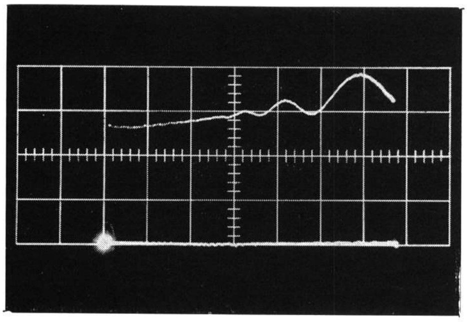

Fig. 1. Electrical resistance of a tellurium single crystal with $p=1.5\times10^{18}$ cm³ as a function of a transverse magnetic field, at a temperature of 4.2°K. H is a perpendicular to the trigonal axis. Horizontal scale: 15 kOe/large division

is switched off, when the field has its maximum value. This allows to increase the sensitivity and to control spurious induction voltages.

§ 3. Results

Most of the measurements have been made in two configurations: H being parallel or perpendicular to the trigonal c-axis of the samples. In the perpendicular orientation the positions of

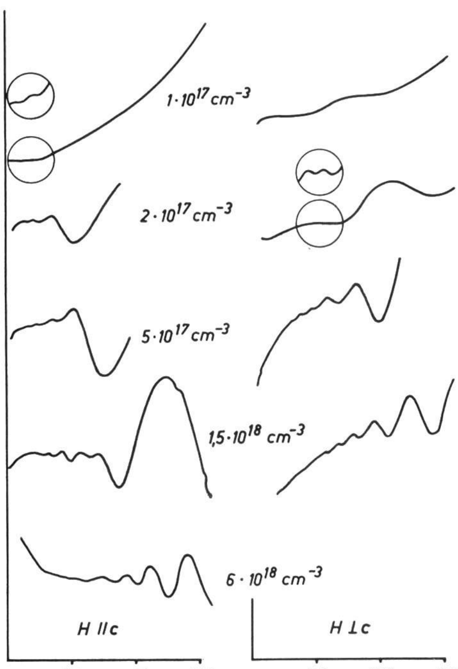

Fig. 2. Electrical resistance of various tellurium single crystals with different doping as a function of a magnetic field at helium temperatures. The vertical scale differs for each curve. The encircled parts appear with larger magnification.

the observed extrema in the resistance were equal for H parallel to a binary and a bisectrix direction. Surprisingly maxima and minima occurred at the same fields, regardless if the sample current was parallel or perpendicular to H. Typical results for $H||c$ and $H\perp c$ are shown in Fig. 2 for a series of specimens with different doping.

From top to bottom the hole density increases from $1\times10^{17}/\text{cm}^3$ to $6\times10^{18}/\text{cm}^3$. The data were generally taken at 4.2°K. The curves were copied from oscillograms, the baselines have been omitted for the sake of clarity. The degree of magnification is not the same for different measurements, so that no conclusions should be drawn from the magnitude of the amplitudes. The data for $H\perp c$ are the most simple. Only a single period $\varDelta H^{-1}$ is present, which decreases with increasing doping. The most heavily doped sample with $p=6\times10^{18}/\text{cm}^3$ showed no oscillations in this configuration. Within the limit of error—which is not small especially for the lighter doped samples—the period is proportional to $p^{-2/3}$. This result suggests, that all carriers which contribute to the Hall effect, are responsible for the oscillations. Because of the measurement uncertainty it cannot be argued, that the valence band is parabolic. The sample with $p=2\times10^{17}/\text{cm}^3$ shows a finestructure, which is revealed at 1.85°K and larger amplification. The structure is possibly caused by spin-splitting of the Landau-level $n=1$, although we are not sure. For $H||c$, the purest specimen with $p=1\times10^{17}/\text{cm}^3$ shows two weak oscillations only on a magnified scale (see circle). Beyond 30 kOe, only the lowest Landau-level with the quantum number $n=0$ is occupied, the quantum limit is reached. The smooth variation of the resistance in this region will not be discussed here. It is clearly visible, that for the crystals with $p=2\times10^{17}$ and $5\times10^{17}/\text{cm}^3$ the last, deep minima shift to higher field strengths. The oscillations, which are observed for the sample with $p=1.5\times10^{18}/\text{cm}^3$ are periodic in H^{-1} up to about 70 kOe. Then the periodicity is interrupted, which can immediately be recognized from the figure. The position of the "interference" is shifted outside the available field range for the most heavily doped crystal. As for $H\perp c$, for all samples a $p^{-2/3}$ relation holds for the periodic part of the Shubnikov-de Haas oscillations within the limit of error, which is not only caused by the uncertainty of $\varDelta H^{-1}$, but also by possible errors in the hole density p. If the structure, which appears in the speci-

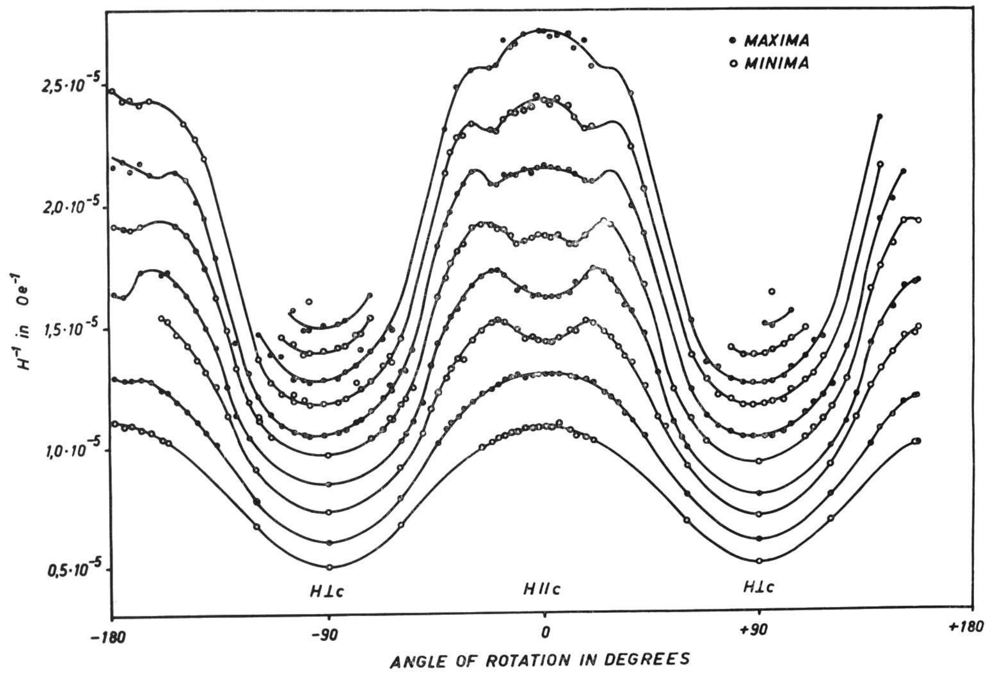

Fig. 3. Maxima and minima of Shubnikov-de Haas oscillations for a tellurium sample with $p=1.5\times10^{18}/\text{cm}^3$ at 4.2°K as a function of the angle of rotation of the magnetic field in the trigonal-bisectrix plane.

mens with $p=1.5\times10^{18}/\text{cm}^3$ and which is probably unresolved in the purer crystals, is ignored, it follows from eq. (1) that the constant energy surfaces are prolate with respect to the c-axis and that the ratio of the extremal cross-sections amounts to

$$A(H\perp c)/A(H||c)=2.2\pm10\% \;.$$

For a more complete investigation of the Fermi-surface the crystal with the hole density $1.5\times10^{18}/\text{cm}^3$ seemed most suitable. First it was established, that the $E(\boldsymbol{k})$-surfaces have rotational symmetry about the c-axis, because no change was found in the period, when H was rotated in small steps in the binary-bisectrix plane. The angular variation of the oscillations in the trigonal—bisectrix plane is shown in Fig. 3.

Observed maxima and minima in H^{-1} have been plotted against the angle of rotation. Structure appears within a range of about $\pm30°$ off the c-axis, except for the lowest quantum numbers, suggesting, that the constant energy contours are relatively complicated. The small maxima shift with decreasing quantum numbers to smaller angles.

§ 4. Discussion

Analysis of the angular dependence data shows, that they are incompatible with ellipsoidal energy surfaces. The theoretical possibility, that the structure is due to spin splitting, can be ruled out. Even if the "high" field data are excluded from the analysis, with the reasoning, that quantum limit effects may show up, f. i. through a field dependent Fermi energy, no fully satisfying picture emerges. The difficulties arise from the region $\pm30°$ off the trigonal axis. Having in mind eq. (1), the small maxima in Fig. 3 suggest that the Fermi surface has two symmetrical belts. However, the position of the maxima depends on the quantum number. Moreover at least one additional period should be present. Certain indications in this direction were found. Plotting the positions of the extrema in H^{-1} against whole numbers gives in the critical region two different straight lines with different slopes, which, however, do not overlap sufficiently.

We conclude, therefore, that our data are at present not complete enough, to allow a quantitative determination of the surfaces of constant energy. The rotational symmetry with respect to the trigonal axis seems to be well established, however. Our speculations center around a barrel shaped Fermi surface at $\boldsymbol{k}=0$, with additional finestructure. Certain reservations should be made, because our samples have a high carrier density. Within the investigated doping range, our data seem to be consistent, however. It is interesting to note, that recent magneto-optical data[9] also have lead to consideration of warped energy surfaces. In addition new low field magnetoresistance data[5] have cast doubt on the validity of previous ellipsoidal models. Cyclotron resonance experiments on pure samples[10] yielded prolate, ellipsoidal energy surfaces with rotational symmetry with respect to the *c*-axis. The anisotropy in the cross-section of the constant energy surfaces $A(H \perp c)/A(H \| c)$ was found to be smaller than in our case and amounted to about 1.4. The resonance lines were rather broad, however,[11] so that warping of the energy contours cannot principally be ruled out.

At present we are trying to measure the absolute value of the cyclotron mass for $H \perp c$ from the temperature dependence of the damping of the oscillations.

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References

1) H. Roth: J. Phys. Chem. Solids **8** (1959) 525.
2) A. Nussbaum and R. J. Hager: Phys. Rev. **123** (1961) 1958.
3) C. Rigaux: J. Phys. Chem. Solids **23** (1962) 805.
4) R.V. Parfen'ev, A.M. Pogarskii, I. I. Farbshtein and S. S. Shalyt: Soviet Physics—Solid State **4** (1963) 2630.
5) I. I. Farbshtein, A. M. Pogarskii and S. S. Shalyt: Soviet Physics—Solid State **7** (1966) 1952.
6) E. Braun and G. Landwehr: Z. Naturforsch. **21a** (1966) 495.
7) L. Onsager: Phil. Mag. **43** (1052) 1006.
8) G. Landwehr and P. Drath: Z. angew. Phys. **20** (1966) 392.
9) C. Rigaux: personal communication.
10) J. H. Mendum and R. N. Dexter: Bull. Amer. Phys. Soc. **9** (1964) 632.
11) R. N. Dexter: personal communication.