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IX-8. Quantum Effects on Capacitance of *p-n* Junctions

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The theoretical study of the capacitance of n-p^+ junction predicts a decrease of the capacitance when a magnetic field is applied, caused by Landau quantization and spin degeneracy removal.

For a degenerate n type semiconductor at $T=0°\text{K}$, an oscillatory behaviour is expected at weak magnetic field followed by a monotonic decrease in the quantum limit.

At higher temperatures, where Maxwell Boltzmann statistics apply, we predict a monotonic decrease.

The experiments were carried out at $T=4.2°\text{K}$ and $T=77°\text{K}$. The results are in good agreement with the theory.

§ 1. Introduction

In a material, in which a magnetic field is applied the Landau quantization and removal of the spin degeneracy affect the electronic energy levels. This phenomenon was observed in very many ways. We propose in this paper to observe it by a change in the work-function of the material. This may be detected by a variation of the capacitance either of a sandwich structure semi-metal or semi-conductor/insulator/metal or of *p-n* junction.

We have carried out a detailed study of the capacitance of n-p^+ InSb junctions.

The theory was achieved for two limiting conditions:

—At 0°K we apply Fermi-Dirac statistics. We may predict an oscillatory variation of the capacitance for magnetic fields such as $\hbar\omega \sim E_F$ followed by a monotonic decrease in the quantum limit.

—At higher temperatures when the Maxwell-Boltzmann statistics is applicable we must observe a monotonic decrease of the capacitance. It is interesting to note that this purely quantum effect may be observed at liquid nitrogen temperature.

In our theory, in both cases, collision—broadening of levels is neglected.

Experiments were carried out at 77°K and 4.2°K. We shall see below that these experimental conditions fit rather well with the two limiting conditions of our theory.

§ 2. Theory

In the following we shall assume that the p^+ type side is highly degenerate and that the voltage drop occurs only in the n type side which is assumed homogeneous.

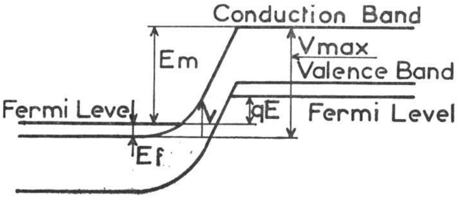

Fig. 1. Assumed band scheme.

We will use the band scheme shown in Fig. 1 where the junction is assumed to be reverse biased by a voltage E, for a degenerate n type material. The validity of such a band scheme will be discussed below. We will only consider quantum effects in the n type side of the junction.

With these conditions it is easy to show that the capacitance for unit area is given by $C=\varepsilon q N/4\pi Q_B$ where N is the donors density and Q_B is the stored charge in the space charge region. Other symbols have their usual meanings. When the magnetic field varies all parameters except Q_B are constant and we may set

$$\Delta C/C = -\Delta Q_B/Q_B \,. \tag{1}$$

Q_B is determined by

$$Q_B{}^2 = \frac{\varepsilon}{2\pi}\int_0^{V_{\max}} \rho(V)dV \,. \tag{2}$$

We notice that $V_{\max}=E_F+E_m$, Fig. 1, is the total energy drop for an electron across the junction.

A. Fermi-Dirac statistics at $T=0°K$

With magnetic field the energy levels of free electrons are given by

$$\varepsilon = \left(n+\frac{1}{2}\right)\hbar\omega \pm \frac{1}{2}g\mu_B H + \frac{\hbar^2 k_{/\!/}{}^2}{2m^*} \,, \tag{3}$$

($k_{\parallel}$ is the component of the $\boldsymbol{k}$ vector parallel to the field) where $\omega = eH/m^*c$ is the cyclotron frequency, g is the gyromagnetic factor and μ_B the Bohr magneton.

In the following we set

$$\frac{1}{2} g\mu_B H/\hbar\omega = \frac{1}{4} gm^*/m_0 = \lambda \, . \qquad (4)$$

The density of states in a sub-level is proportional to

$$\left[E - \left(n + \frac{1}{2}\right)\hbar\omega \pm \frac{1}{2} g\mu_B H\right]^{-1/2} . \qquad (3)$$

We determine the Fermi-energy by the condition that the total density of free carriers is constant.

It is easy to show that the total number of carriers is given by

$$N = N_c \pi^{-1/2} (\hbar\omega/kT)^{3/2} (S^+_{1/2} + S^-_{1/2}) \, , \qquad (6)$$

where we set $N_c = 2(2\pi m^* kT/h^2)^{3/2}$ and

$$S_x{}^{\pm} = \sum_{n=0}^{n_{\max}} \left[(E_F/\hbar\omega) - \left(n + \frac{1}{2} \pm \lambda\right)\right]^x .$$

Going to the limit $H \to 0$ we get the relation

$$\frac{4}{3} (E_{F_0}/\hbar\omega)^{3/2} = S^+_{1/2} + S^-_{1/2} \, ,$$

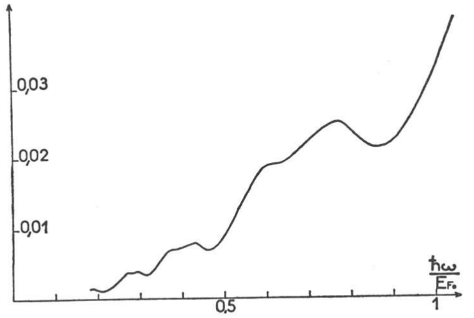

Fig. 2. Variation of the bracketed term versus $\hbar\omega/E_{F_0}$, eq. (8).

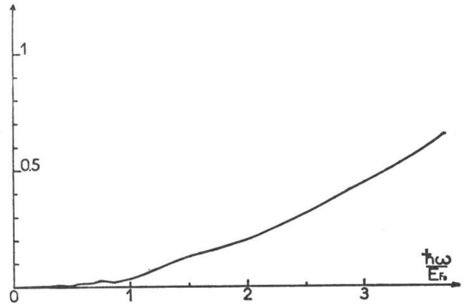

Fig. 3. Variation of the bracketed term versus $\hbar\omega/E_{F_0}$, eq. (8).

where E_{F_0} is the Fermi energy for $H=0$ and where in $S^{\pm}_{1/2}$ we use the Fermi energy E_F for $H \neq 0$.

For the computation of Q_B from eq. (2) we have to take into account the space charge $+qN$ of donor impurities and the space charge of free carriers for $V < E_F$.

The latter one is easily computed from eq. (6) where we have just to modify E_F in $E_F - V$.

The final result is

$$Q_B{}^2 = \frac{\varepsilon N}{2\pi} \left(V_{\max} - \frac{2}{3} \hbar\omega \frac{S^+_{3/2} + S^-_{3/2}}{S^+_{1/2} + S^-_{1/2}}\right) . \qquad (7)$$

When the applied dc bias is constant a straightforward calculation gives the change in Q_B vs. H, whence the relative change in capacitance

$$\frac{\Delta C}{C} = -\frac{1}{2} \frac{E_{F_0}}{E_m + (3/5) E_{F_0}} \times \left[\frac{\hbar\omega}{E_{F_0}} \left(\frac{2}{3} \frac{S^+_{3/2} + S^-_{3/2}}{S^+_{1/2} + S^-_{1/2}} - \frac{E_F}{\hbar\omega} + \frac{3}{5} \frac{E_{F_0}}{\hbar\omega}\right)\right] . \qquad (8)$$

In Figs. 2 and 3 we have plotted the bracketed term of (8) as a function of $\hbar\omega/E_{F_0}$ for respectively low and high values of this parameter. We assume the following values for InSb: $m^*/m_0 = 1.45 \times 10^{-2}$, $g = -48$.

When $\hbar\omega/E_{F_0}$ is about unity we may expect an oscillatory variation of the capacitance. At higher fields in the quantum limit we must observe a monotonic decrease. The observed variations must be small because the change in capacitance arises from the region where the space charge of free carriers is efficient. This occurs on a depth of about the Debye length which is much smaller than the junction width.

B. Maxwell-Boltzmann statistics

The calculation is carried in the same way as above but is much easier.

It is easy to show that the number of carriers in the conduction band is given by $N = N_c[\exp(-E_F/kT)]\phi$, where $\phi = [(\hbar\omega/2\,kT)/(sh\,\hbar\omega/2\,kT)] \times [(ch\, g\mu_B H/2\,kT)]$ whence we deduce the change of Fermi energy $\Delta E_F = E_F(H) - E_{F_0} = kT \log \phi$.

The stored charge Q_B is given by eq. (2) where now $\rho(V) = N - n_{el} = N[1 - \exp(-V/kT)]$ where n_{el} is the contribution of free electrons to the space charge: $Q_B{}^2 = (\varepsilon N/2\pi)(V_{\max} - kT)$.

In the following we neglect kT with respect to $V_{\max}$. Since $E_F + V_{\max}$ is constant for a constant bias

$$\Delta C/C = -\Delta Q_B/Q_B = -\frac{1}{2}\Delta E_F/V_{\max}$$

or

$$\Delta C/C = kT/2\,V_{\max}\times\log\phi\,. \qquad (9)$$

§ 3. Experimental Procedures

A. Samples

The *p-n* junctions were grown by the liquid phase epitaxy method described by Calawa and Melngailis[1] by deposition of p^+ InSb on an n type substrate from an In-InSb-Cd bath.

After masking and subsequent etching with CP_4 we get a mesa structure whose diode characteristics are generally very poor. With the passivation technique described by Mueller, Jacobson and Maffitt[2,3] we may improve these characteristics and we get reverse resistance of 2 MΩ for junctions of an area of about 3 mm^2.

B. Magnetic field

Experiments reported in this paper were carried out with a conventional electromagnet with fields up to 14 kOe. Experiments at higher fields with a superconducting coil are now under way.

C. Measurements

Our measurements were carried out with a Boonton Bridge (Type 75 AS 8) at a frequency of 480 kHz. The output signal is measured with a phase sensitive detector. This allows separate measurements of changes of capacitance or of losses. The variation of C versus H is observed on an X-Y recorder.

§ 4. Experimental Results and Discussion

We will give results for n-p^+ junctions in which the carrier densities ranges from 2×10^{14} to 4×10^{15} in the n type material.

A) At 4.2°K we failed to observe the predicted oscillations as shown in Figs. 2 and 3. At best we may observe an accident at the magnetic field value for which the theory predicts a large oscillation, Fig. 4 for a sample. This probably arises from collision broadening of levels.

At higher fields, in the quantum limit, the broadening of levels is less important and if we compare the total change in capacitance when the field varies from 0 to 14 kOe with the predicted one for various biases (Table I), the agreement is very good. The E_{F_0} value was computed from the carrier density measured by Hall effect. E_m was determined from the known value of the energy gap (the Fermi energy on the p-type side is neglected). The use of the theory achieved for $T=0$°K seems valid at 4.2°K because we

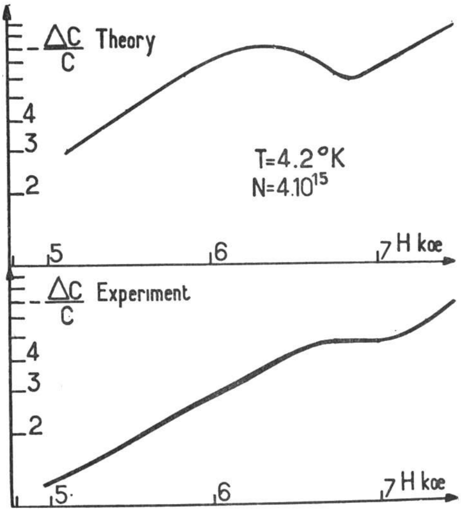

Fig. 4. Theoretical and experimental variations of the capacitance at 4.2°K.

Table I. Total variation of capacitance at 4.2°K.

Sample He 07	$N=4\times10^{15}$		$T=4.2$°K
V_{DC} mv	C_{pf}	$-10^{-2}\frac{\Delta C_{\max}}{C}$ Exper.	$-10^{-2}\frac{\Delta C_{\max}}{C}$ Theory
−200	1452	1.40	1.43
−100	1670	1.87	1.89
0	2003	2.62	2.40
+100	2605	4.35	4.15

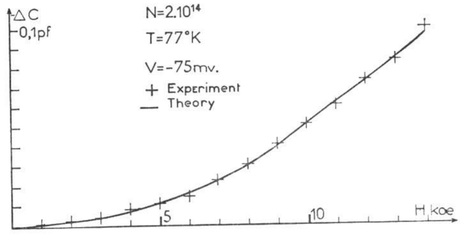

Fig. 5. Capacitance variation vs. magnetic field at 77°K.

Table II. Total variation of capacitance at 77°K.

Sample N 2-12	$N=2\times10^{14}$		$T=77$°K
V_{DC} mv	C_{pf}	$-10^{3}\frac{\Delta C_{\max}}{C}$ Exper.	$-10^{3}\frac{\Delta C_{\max}}{C}$ Theory
−100	107.6	0.92	0.86
− 75	112.1	0.94	0.93
− 50	118.0	1.05	1.03
− 25	124.8	1.20	1.14
0	134.2	1.19	1.29

did not observe significant changes when pumping on helium.

B) At 77°K the agreement is very good. On Fig. 5 we plot the capacitance variation versus magnetic field as compared with the computed curve. On Table II we compare the change in capacitance for the maximum magnetic field for various biases. $V_{\max}$ is deduced from the known value of the energy gap.

C) We checked our junctions by a plot of C^{-2} versus bias. We generally observe a perfectly straight line which enables us to determine $E_g=0.2$ eV at 77°K and $E_g=0.24$ eV at 4.2°K.

However in some cases we observed anomalously small values for E_g (e.g. 0.165 eV at 77°K). Furthermore we sometimes observed a kink in the C^{-2}/E characteristic. This probably arises from a spreading of the hole space charge region across the stoechiometric junction. The theory of this effect is now in progress.

Acknowledgement

We are very greatly indebted to Professor P. Aigrain for many stimulating discussions.

References

1) I. Melngailis and A. R. Calawa: J. Electrochem. Soc. **113** (1966) 58.
2) R. K. Mueller and R. L. Jacobson: J. appl. Phys. **35** (1964) 1524.
3) K. N. Maffitt and R. K. Mueller: J. appl. Phys. **35** (1964) 1563.

DISCUSSION

Reuber, C.: Did you measure also the losses as function of B and what were the results? What is the frequency-response of your effect? Is there any dependence of $\Delta c/c$ on the 480 kc voltage?

Thuillier, J. C.: We measured losses in some cases at 4°K. Oscillations are observed. The theory of this phenomenon is underway. The frequency variations of capacitance is used to check any possible channel effect. Within 100 to 1,000 kc, the capacitance variation versus B is independent of frequency. The high frequency voltage applied to the sample is kept as small as possible (a few mV).