PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

X-3. Non-Linear Galvanomagnetic Effects Due to Hot Electrons in *n*-Type InSb in the Quantum Limit

N. KOTERA, K. F. KOMATSUBARA and E. YAMADA

Hitachi Central Research Laboratory, Kokubunji, Tokyo, Japan

Non-linear galvanomagnetic effects in *n*-type InSb in the quantum limit have been studied. Non-linear current-voltage characteristics were observed at 1.5°K in transverse magnetic fields above 2 kG where ionized impurity scattering was dominant.

In degenerate samples, we observed Shubnikov-de Haas oscillations and the effects of an electric field on the oscillation maxima.

In nondegenerate samples, we studied the nonlinearity coefficient, $\beta(E_x)=\sigma^{-1}(\partial\sigma/\partial(E_x^2))_B$. Values of the maxima of $\beta(E_x)$ were oscillatory in inverse magnetic field. Electron energy losses in this region were mainly due to piezo-electric acoustic phonons. In the nondegenerate quantum limit, current oscillations were observed.

§ 1. Introduction

Since publications of the theories of Adams and Holstein[1] and Kazarinov and Skobov,[2] a number of studies of the non-linear galvanomagnetic effects in crossed electric and magnetic fields have appeared.[3] The present studies are devoted to the behavior of conduction electrons in relatively pure *n*-InSb samples located in strong electric and magnetic field at 1.5°K. Transverse magnetic fields up to 13.5 kG are employed. We will consider the heating of the electrons under quantum limit conditions. If the energy of electrons is less than $\frac{3}{2}\hbar\omega_c$, the electrons are in the lowest magnetic sub-band. This regime is called the quantum limit. Both degenerate quantum limit conditions, $\omega_c\tau>1$, $\hbar\omega_c>\varepsilon_f>kT_L$, and nondegenerate quantum limit conditions, $\omega_c\tau>1$, $\hbar\omega_c>kT_e$ were easily satisfied in the samples studied.

§ 2. Nonlinear Galvanomagnetic Effects

We used uncompensated *n*-InSb samples with impurity concentrations of about 1×10^{14}/cc. The quantum limit conditions are easily realized for the magnetic fields exceeding 2 kG. The transverse resistivity, ρ_T, caused by ionized impurity scattering in the quantum limit, for degenerate statistics, should follow the functional form[1], $\rho_T\sim B^3T_e^0$, *i.e.* temperature independent.

We have observed non-linear galvanomagnetic phenomena[4] in the current-voltage characteristics, under the quantum limit conditions. An example is shown in Fig. 1. This non-linear behavior is due to the appreciable heating of the conduction electrons which occurs under the quantum limit conditions. The electron temperature, T_e, can be expressed in the following form[2]:

$$T_e=T_L\left[1+\frac{1}{2}\left(\frac{cE_t}{sB}\right)^2\left(1+\frac{\nu_{im}}{\nu_{ph}}\right)\right], \quad (1)$$

where, T_L is the lattice temperature, c the velocity of light, s the velocity of sound, B the magnetic induction, and ν_{ph} and ν_{im} are the collision frequencies of the conduction electrons with phonons and ionized impurities, respectively. E_t is $E_x(1+\theta^2)^{1/2}$, where θ is the Hall angle and E_x applied electric field. For non-degenerate statistics, the current through the samples is[2]

$$J_x\sim E_x\left(\frac{T_e}{T_L}\right)^{3/2}. \quad (2)$$

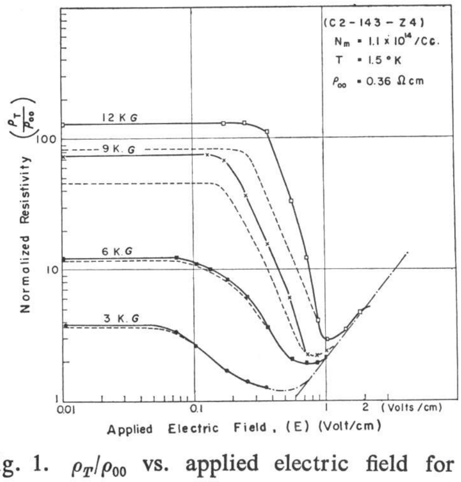

Fig. 1. ρ_T/ρ_{00} vs. applied electric field for an *n*-InSb sample with an impurity concentration of 1.1×10^{14}/cc. Freeze-out effect is corrected by dots.

Carrier heating becomes important when the applied electric field exceeds the value,

$$E_x)_1 \approx \frac{sB}{c}\left(\frac{\nu_{ph}}{\nu_{im}}\right)^{1/2}(1+\theta^2)^{-1/2}, \qquad (3)$$

and non-linear behavior becomes remarkable. The ratio of collision frequency at this electric field is about 4×10^4 in present experiments.

As the electric field is increased further, kT_e exceeds $\hbar\omega_c$, and the effect of quantization of the electron energy disappears. A classical treatment should be valid in this high electric field region. The electric field at which $\hbar\omega_c = kT_e$ is

$$E_x)_2 \approx E_x)_1 \alpha^{1/2}\left(\frac{\hbar\omega_c}{kT_L}\right)^{1/2} = 0.11B^{1/2}\alpha^{1/2}E_x)_1. \qquad (4)$$

$m^* = 0.013\, m_0$ and $T_L = 1.5$°K and α is about 4[4].

§ 3. The Shift of Electron Energy by the Electric Field

The effects of a strong electric field on the electron energy in the magnetic sub-band should be taken into consideration in heating properties of the conduction electrons in strong magnetic fields. Changes in the energy are easily observable experimentally from the changes in the transverse magnetic fields corresponding to the maxima in the Shubnikov-de Haas magneto-oscillations, for constant electric fields. We used n-InSb samples with impurity concentrations of about 10^{15}/cc. Examples of the results are shown in Figs. 2 and 3.

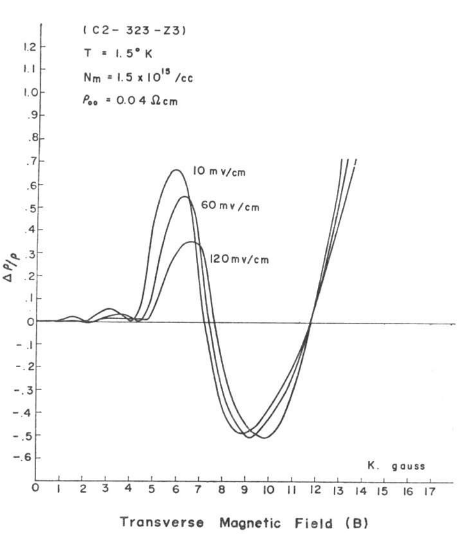

Fig. 2. $\Delta\rho/\rho$ vs. transverse magnetic field, with electric field as a parameter.

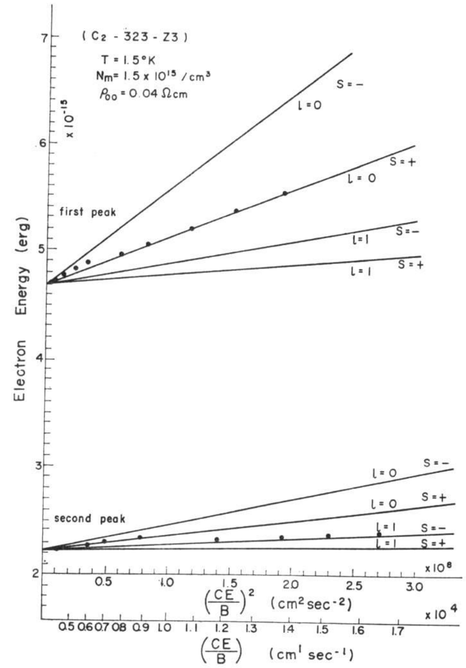

Fig. 3. The shift of electron energy in the magnetic sub-bands as a function of (cE_t/B).

The change of the electron energy in the magnetic sub-bands, $\mu_B{}^*\Delta B$, is given as follows,[5]

$$\mu_B{}^*\Delta B = \frac{\hbar k_y\left(\dfrac{cE_t}{B}\right) + m^*/2\left(\dfrac{cE_t}{B}\right)^2}{\left(2l+1 \pm \dfrac{1}{2}g\dfrac{m^*}{m}\right)}. \qquad (5)$$

Here $\hbar k_y/2m^*$ is the phase velocity of the cyclotron orbit centers drifting transversely with a group velocity, $cE_t/B \sim 10^4$ cm sec^{-1}, $\mu_B{}^*$ is the *effective* Bohr magneton, and g is the spectroscopic splitting factor ($\sim$50).

Figure 3 shows the shifts of electron energy in the magnetic sub-bands as functions of (cE_t/B).

The result clearly shows a Stark effect, where the free electron model treatment of the conduction electrons in the quantum state is applicable. Spin-dependent effects on the hot electrons may also be seen in Fig. 3.

§ 4. Energy Transfer Mechanism

We now study the quantum limit region ($\hbar\omega_c > kT_e$), but with nondegenerate statistics applicable ($\varepsilon_f < kT_e$). We can define the coefficient of non-linearity, $\beta(E_x)$, as a function of applied electric field[11,12] under the constant magnetic field,

$$\beta(E_x) = \frac{1}{\sigma}\left(\frac{\partial\sigma}{\partial(E_x{}^2)}\right)_B, \qquad (6)$$

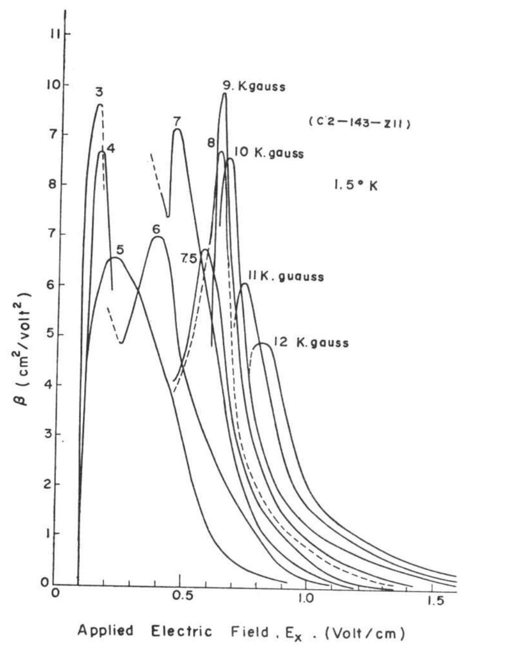

Fig. 4. The change of β as a function of E_x; curves are plotted for several magnetic fields.

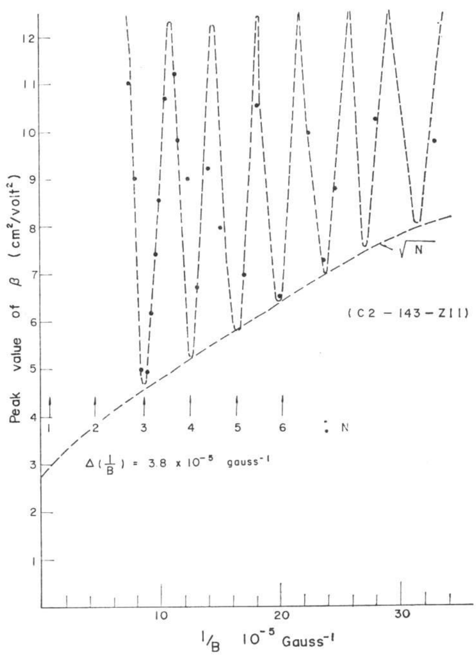

Fig. 5. The oscillatory curves of β_{peak} as a function of 1/B. This figure is replotted from Fig. 4.

where σ is the specific conductivity. Measurements of $\beta(E_x)$, plotted for several transverse magnetic fields, are shown in Fig. 4. $\beta(E_x)$ on this figure has been calculated from the current-voltage curves plotted by an *X-Y* recorder.

The expression for σ given by Adams and Holstein,[1] or Kazarinov and Skobov[2] for the case of nondegenerate statistics, is

$$\sigma=\sigma_0(B)\left(\frac{T_e}{T_L}\right)^{3/2}. \qquad (7)$$

When this is substituted into eq. (6), we have

$$\beta(E_x)=\frac{3}{2}\left(\frac{\partial(\ln T_e)}{\partial(E_x^{\,2})}\right)_B, \qquad (5)$$

where T_e is given in eq. (1). This equation is a function of ν_{im} and ν_{ph}. The presence of maxima in the β vs. E_x curves (Fig. 4) may be explained as resulting from sharp changes in the collision frequencies, ν_{im} and ν_{ph} accompanied by the changes of energy dissipation mechanisms. The values of the electric field corresponding to β_{peak} falls between the fields $E_x)_1$ and $E_x)_2$ which are defined in eqs. (3) and (4), respectively. For larger electric fields, the strong contribution of phonon scattering may lead to the observed great decrease in $\beta(E_x)$.

As for energy dissipation mechanisms, the acoustic phonon scattering is mainly of a piezo-electric nature, deformation scattering being small.[3] This is verified by the observed *B*-dependence of

$$\frac{1}{\beta(E_x)}+E_x^{\,2}=\left(\frac{\partial P}{\partial T_e}\right)_B\Bigg/\left(\frac{\partial\sigma}{\partial T_e}\right)_B \qquad (9)$$

in the region of decreasing $\beta(E_x)$[2,6]. P is the power transmitted by the electrons to the phonon system per unit volume, and is equal to $\sigma E_x^{\,2}$. Experiments showed that the quantity on the left of eq. (9) was proportional to $B^{2\sim2.5}$ and σ was proportional to B^{-1}. Theory predicts[6] that P should vary as B^1 for piezo-electric scattering and as B^2 for deformation scattering. Thus the experiments support the dominance of piezo-electric scattering in the power dissipation.

The variations of β_{peak} as a function of B are also examined. The curve of β_{peak} vs. $1/B$ shows a oscillatory nature as shown in Fig. 5.

The variation of β_{peak} as a function of B may be attributed to the variation of the power dissipation rate,

$$P=P_{\text{def.}}+P_{\text{piezo.}}+P_{\text{opt.}}\,. \qquad (10)$$

Contribution of optical phonon scattering to σ is less because the carrier density in Maxwellian tail is low. However owing to its large value of emission energy, power dissipation rate is changed remarkably when optical phonon transition rate between magnetic sub-bands is changed by the mechanism suggested by Gurevich, Firsov and Efros.[7] The value of β_{peak} becomes small when $n\omega_c=\omega_{opt.}$ because for these magnetic fields, the power dissipation is dominated by resonance emission of optical phonons, and in this case the right hand side of eq. (9) becomes large. The period $\Delta(1/B)$ is about 3.8×10^{-5} G^{-1}. The energy calculated from

$$\varepsilon=\frac{2\mu_B(m/m^*)}{\Delta(1/B)} \tag{11}$$

is about 270°K, which means the cut-off energy caused by optical phonon emission. The energy is close to the optical phonon energy of 260°K.[8] The values of cE_t/B corresponding to the β_{peak} are nearly constant and equal to 0.75×10^4 cm-sec^{-1} for different magnetic fields. This means the mean electron temperature in the electric and magnetic fields corresponding to β_{peak} is 10°K, and corresponding electric dipole energy shift (from Fig. 3) is about 20°K.

Current oscillations of frequencies in the range 10 to 100 k.cycles, were observed in the electric and magnetic fields for which the electrons could be described by nondegenerate statistics.

§ 5. Summary

Characteristic features in hot electron phenomena appeared in quantum conditions were clearly different from the classical ones. We reported on some of these features. Further quantitative and realistic theory is necessary to refine above explanations.

It is pleasure to acknowledge fruitful discussions of present problems with Dr. M. Glicksman and Prof. T. Kurosawa.

References

1) E. N. Adams and T. D. Holstein: J. Phys. Chem. Solids **10** (1959) 254.
2) R. F. Kazarinov and V. G. Skobov: Zh. eksper. teor. Fiz. **42** (1962) 1047. translation: Soviet Physics-JETP **15** (1962) 726.
3) R. J. Sladek: J. Phys. Chem. Solids **16** (1960) 1.
4) K. F. Komatsubara and E. Yamada: Phys. Rev. **144** (1966) 702.
5) A. G. Aronov: Fiz. tverdogo Tela **5** (1963) 552. translation: Soviet Physics-Solid State **5** (1963) 402.
6) Sh. M. Kogan: Fiz. tverdogo Tela **4** (1962) 2474. translation: Soviet Physics-Solid State **4** (1963) 1813.
7) V. L. Gurevich, Yu. A. Firsov, and A. L. Efros: Fiz. tverdogo Tela **4** (1962) 1813. translation: Soviet Physics-Solid State **4** (1963) 1331.
8) G. Picus, E. Burstein, B. W. Henvis and M. Hass: J. Phys. Chem. Solids **8** (1959) 282; R. F. Potter: Phys. Rev. **103** (1956) 861.