PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

XI-3. Theory of Sound Amplification in Many-Valley Semiconductors

S. V. GANTSEVICH and V. L. GUREVICH

Institute of Semiconductors, Academy of Science, Leningrad, USSR

We have developed a theory of sound amplification in many-valley semiconductors such as n-Ge in a *dc* electric field. This effect was discovered experimentally by Pomerantz.[1)]

We consider an extrinsic semiconductor with fully ionized centres so that the total number of electrons does not change under the influence of strain. The strain caused by travelling sound wave produces a small variation of the electron energy in each valley that is equal to $\delta\varepsilon_\alpha = \Lambda^\alpha_{ik} u_{ik} + e\varphi$.* Here Λ^α_{ik} is the deformation potential tensor for the α-th valley, u_{ik} is the strain tensor and φ is the electrostatic potential due to space charge redistribution, e is the electron charge. The travelling wave disturbs the intervalley equilibrium, the quantity $\delta\varepsilon_\alpha$ being different for various valleys. The processes of inter-valley relaxation (with the characteristic time τ_e) tend to reestablish the equilibrium. Such is the mechanism of sound absorption[2,3)] that can turn into amplification in the presence of a *dc* electric field. The intravalley equilibrium is also disturbed by the sound wave. It is restored by the processes of momentum relaxation (with the characteristic time τ_p) and by the processes of energy relaxation (with the characteristic time τ_i). We confine ourselves to the case of quasielastic scattering in which τ_i is much more than τ_p. On the other hand, τ_i can be of the same order of magnitude as τ_e so that one should consider both of these mechanisms of relaxation simultaneously.

We suppose that the sound wave is sufficiently long so that the electrostatic potential φ can be obtained from the neutrality condition $\sum_\alpha \delta n_\alpha = 0$. Here δn_α is the electron concentration change in the α-th valley.

The electron contribution to the sound attenuation constant Γ for the plane wave with the frequency ω and the wave vector $\boldsymbol{q}$ can be expressed through the electron part of the elastic moduli $\delta\lambda_{iklm}$ as follows:

$$\Gamma = -\frac{q_i q_m u_l u_k}{\rho \omega w u^2} \operatorname{Im} \delta\lambda_{iklm} . \tag{1}$$

* Here and henceforth we use Einstein's summation convention.

Here ρ is the crystal density, w is the (group) velocity of sound and $\boldsymbol{u}$ is the displacement vector, $\delta\lambda_{iklm}$ is given by:

$$\delta\lambda_{iklm} = \sum_\alpha \Lambda^\alpha_{ik} \frac{\partial \delta n^\alpha}{\partial u_{lm}} . \tag{2}$$

Let us consider for simplicity the case where are only two valleys (or two systems of valleys) that become non-equivalent under the strain influence. We suppose also that $\delta\varepsilon_1 = -\delta\varepsilon_2 = \delta\varepsilon$, $\sigma^{(1)}_{xx} = \sigma^{(2)}_{xx}$, where $\sigma^{(\alpha)}_{ik}$ is the conductivity tensor for the α-th valley in the infinitesimal electric field, x is the direction of sound propagation that is also the direction of the *dc* electric field $\boldsymbol{E}$.

The electron concentration change δn_α is expressible through the electron distribution function $F_\alpha(\boldsymbol{p})$ ($\boldsymbol{p}$-being the electron momentum) in the α-th valley. Omitting the terms of the second and higher order in strain one can write this function in such a form:

$$F_\alpha(\boldsymbol{p}) = F_0(\boldsymbol{p}) + F_1(\boldsymbol{p}) \frac{\delta\varepsilon_\alpha}{kT} , \tag{3}$$

(In our case $F_0(\boldsymbol{p})$ and $F_1(\boldsymbol{p})$ do not depend on α because of symmetry of the problem). Here T is the lattice temperature, k is the Boltzmann constant. In order to calculate δn_α it is sufficient to know only the symmetric part of the function $F_1(\boldsymbol{p})$ that is equal to $\frac{1}{2}\{F_1(\boldsymbol{p}) + F_1(-\boldsymbol{p})\}$. If the condition $\tau_p \ll \tau_i$ holds this function depends only on the electron energy $\varepsilon(\boldsymbol{p})$ as measured from the bottom of the corresponding valley. Let this function be denoted by $F_1(\varepsilon)$. It is the solution of the following equation:

$$-i\omega \sqrt{\frac{\varepsilon}{kT}} F_1(\varepsilon) - \frac{\sqrt{\pi}}{2} D_{xx} \Big[\Big(iq + eE\frac{d}{d\varepsilon} \Big) \frac{\varepsilon}{kT} \times \Big(iq + eE\frac{d}{d\varepsilon} \Big) F_1(\varepsilon) - iq\varepsilon \Big(iq + eE\frac{d}{d\varepsilon} \Big) \frac{dF_0(\varepsilon)}{d\varepsilon} + ieqE\frac{d}{d\varepsilon}\varepsilon\frac{dF_0(\varepsilon)}{d\varepsilon} \Big] = \bar{I} . \tag{4}$$

Equation (4) holds for the case of acoustic intravalley scattering. It is derived from the Boltzmann equation and takes into account the anisotropy of the electron and phonon spectra and the anisotropy of scattering. The method used was worked out by the authors in ref. 4). Here D_{ik} is the diffusion tensor for $E=0$, $F_0(\varepsilon)$ is the electron distribution function for an unstrained crystal in the *dc* electric field E, $\bar{I}$ is the collision operator averaged over the surface $\varepsilon=\varepsilon(\boldsymbol{p})$ in the momentum space. The collision operator I is the sum of the intravalley electron-phonon collision operator and the intervalley electron-phonon or electron-impurity collision operator. The explicit expression for the averaged operator can be found in ref. 4). The electron-electron collision operator may be added to I if necessary.

Equation (4) can be easily solved if the intravalley relaxation processes are much more rapid than the intervalley ones.

If electron-electron intravalley scattering is dominant** so that an electron temperature exists, the functions $F_0(\varepsilon)$ and $F_1(\varepsilon)$ have a Boltzmann-like form;

$$F_0(\varepsilon)=G\exp(-\varepsilon/kT_e)\ , \quad F_1(\varepsilon)=AF_0(\varepsilon)\ .$$

Here G is the normalising factor and A is given by:

$$A=-\frac{T/T_e}{1-i\{1-(V/w)\}\omega\tau_M\sqrt{T/T_e}}\ ,$$

where w is the (phase) sound velocity, V is the drift velocity of hot electrons that is equal to $V=\mu_{xx}E\sqrt{T/T_e}$ (μ_{ik} being the mobility tensor for $E=0$). The relaxation time τ_M is $\tau_M=[\tau_e^{-1}+D_{xx}q^2]^{-1}$. The electron temperature can be found from the relation:

$$T_e^2-TT_e-T^2\frac{g}{2}=0\ .$$

The "electron heating parameter" g is given by;

$$g=\frac{e\mu_{xx}E^2\tau_i}{kT}\simeq\left(\frac{\mu E}{w}\right)^2\ .$$

The expression for τ_i in the case of anisotropic scattering can be found in refs. 4) and 5).

The attenuation constant Γ for such a case is

$$\Gamma(E)=\frac{\{1-(V/w)\}(1+\omega^2\tau_M^2)}{1+\{1-(V/w)\}^2\omega^2\tau_M^2(T/T_e)}\left(\frac{T}{T_e}\right)^{3/2}\Gamma(0)\ , \tag{5}$$

where $\Gamma(0)$ is the value of an attenuation constant for $E=0$ obtained by Weinreich *et al.* in ref. 1).

If electron-electron scattering is negligible the function $F_0(\varepsilon)$ has the form[5]:

$$F_0(\varepsilon)=G\left(1+\frac{\varepsilon}{gkT}\right)^g e^{-\varepsilon/kT}\ .$$

For $F_1(\varepsilon)$ we have as before $F_1(\varepsilon)=A'F(\varepsilon)$ where A' is a complex factor analogous to A.

The attenuation constant $\Gamma(E)$ in this case is given by:

$$\Gamma(E)=\frac{\{1-(V/w)\}(1+\omega^2\tau_M^2)}{1+\{1-(V/w)\}^2\omega^2\tau_M^2(a_{1/2}^2/a_1^2)}\times\frac{a_0a_{1/2}}{a_1^2}\Gamma(0)\ , \tag{6}$$

here

$$a_m=\frac{1}{\Gamma(m+1)}\int_0^\infty x^m\left(1+\frac{x}{g}\right)^g e^{-x}dx$$
$$=g^{m+1}\Psi(m+1,\ g+m+2,\ g)\ .$$

$\Psi(a,\ b,\ x)$ is the second confluent hypergeometric function.

The expressions (5) and (6) are valid if impurity intervalley scattering plays a dominant role and the electron intervalley scattering cross-section, S, is independent of the electron energy ε. The cases of more complicated dependence $S(\varepsilon)$ can be treated with the same method. One can obtain expressions of the same structure for the case of phonon scattering. It is interesting to note, however, that the corresponding transition time τ_e becomes exponentially dependent upon the electron temperature T_e.

Let us observe that the expression for Γ changes its sign if the drift velocity of hot electrons exceeds the sound velocity. It means that the sound amplification takes place.

The general case of an arbitrary direction of sound propagation and an arbitrary direction of the *dc* electric field may be investigated in a similar manner.

It is a very difficult problem to solve eq. (4) if $\tau_i\simeq\tau_e$. Some results can be obtained for the comparatively simple case $\tau_i\gg\tau_e$. It should be noted that in this case the attenuation constant $\Gamma(E)$ changes its sign at the value of the drift velocity V that in general is not equal to the sound velocity w.

** One can easily show that intervalley electron-electron scattering is negligible.

References

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