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XII-6. Mixed Scattering in III-V Compounds

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The electron mobility in AlSb, GaP and GaSb is calculated by combining the effects of optical polar scattering and deformation potential scattering (acoustic, optical, and inter-valley). Good agreement is obtained with the observed values for AlSb and GaSb. For GaP the observed mobility is about half of that calculated.

The field dependence of mobility is complex when mixed scattering operates. It is shown that under some circumstances a negative resistance effect can occur. The conditions necessary for this are examined, and results of calculations on GaSb are given.

First experimental results at high fields on GaSb and GaP show only small departures from Ohm's law.

§ 1. Introduction

Since early work established that polar optical scattering was the dominant process limiting mobility in pure InSb at room temperature, it has been generally assumed that other lattice scattering processes are of little significance in III–V compounds. Calculations of electron mobility in InAs, GaAs, and InP showed that polar scattering was sufficiently large to account for the values obtained by experiment. However, for GaSb, GaP and AlSb the observed mobilities are much smaller than the values calculated for polar scattering alone. In this paper we show that better agreement with experiment is obtained if one includes the effects of deformation potential scattering.

At high electric fields mixed scattering can have complex results. With particular mixtures the interplay of the different effects can lead to a differential negative resistance. This occurs with the electrons remaining in the same type of conduction band minimum, and is in contrast with Gunn effect, which is due to intervalley transfer. The conditions required to obtain a mixed scattering negative resistance are analyzed.

First experimental results on GaP and GaSb show little departure from Ohm's law at fields where a noticeable departure should occur.

§ 2. Method of Calculation

It is in compounds with a large effective mass that deformation potential scattering, which is proportional to $m^{*5/2}$, becomes as important as polar scattering, with a $m^{*3/2}$ dependence. These compounds have the lowest conduction band minima away from the centre of the zone, and the resultant multiplicity of minima leads to the possible complication of intervalley scattering. In a pure III–V compound at room temperature there then are four processes which need to be considered, polar optical scattering and the three deformation potential scattering processes, acoustic scattering, optical scattering and intervalley scattering. The appropriate expressions for the mobility have been summarized by Paranjape and Alba.[1] The contribution of polar scattering can be calculated from the optical constants, as has been done previously.[2] The other processes can be included only if the appropriate deformation potentials are known. The assumption we make is that for compounds with (100) minima the lowest, the deformation potentials will be the same as in silicon. For GaSb, where most of the electrons are in (111) minima, the better analogy is with germanium, and germanium deformation potentials are used.

In silicon optical scattering is small and the two processes to be considered are acoustic scattering and intervalley scattering which limit the mobility to 4000 cm^2/V sec and 2000 cm^2/V sec respectively.[3] Acoustic scattering can therefore be estimated for GaP and AlSb directly, extrapolating for the change in effective mass and the elastic constants. Intervalley scattering is a little more difficult to assess since there is some ambiguity about the type of phonon taking part in the process. The two longitudinal branches of the phonon spectrum are degenerate in silicon at the X point, but the degeneracy is lifted in the compounds, and there is an appreciable difference between the LO and LA temperatures. The calculated mobility is quite sensitive to this temperature. Recently Berman, Lax and Loudon[4] have deduced selection

rules which enable us to choose the appropriate phonon. The LO phonon should be used for AlSb and the LA phonon for GaP. For comparison, the mobility in the 100 minima of GaAs has also been calculated. This has been calculated more rigorously by other workers.

In germanium at room temperature acoustic scattering is dominant. The averaged deformation potential is 17 eV, and the acoustic mobility 5500 cm^2/V sec.[5] Intervalley scattering is small with a mobility near 100,000 cm^2/V sec. Optical scattering, with a mobility of about 15000 cm^2/V sec cannot here be neglected. The calculation of mobility in the (111) minima of GaSb is therefore made by extrapolating from germanium, using the appropriate effective mass, elastic constants and phonon frequencies to deduce the acoustic and optical mobilities, and determining the polar mobility from the optical constants. The phonon spectrum of GaSb has not been studied as thoroughly as those of the other compounds and the value used throughout, 350°K, is only approximate.

The calculation of field dependence of mobility is more complex. Stratton has shown that the mobility can be deduced for hot electrons by solving equations for the conservation of momentum and energy, and in two papers[6,7] has given expressions appropriate to polar optical scattering and deformation potential scattering respectively. We combine the results of the two papers and obtain

$$p_0F=\frac{3k\theta_{p0}}{4\mu_{p0}}A(\gamma_p)+\frac{3m^*S^2}{\mu_a}\left(\frac{T}{T_0}\right)^{3/2}\left(1-\frac{T_0}{T}\right)+\frac{3k\theta_0}{4\mu_0}C(\gamma)\ , \qquad (1)$$

$$\frac{m^*F}{p_0}=\frac{1}{2\mu_{p0}}B(\gamma_p)+\frac{1}{\mu_a}\left(\frac{T}{T_0}\right)^{1/2}+\frac{1}{2\mu_0}D(\gamma)\ . \qquad (2)$$

In these equations p_0 is the average displacement in momentum space due to the electric field F, T is the electron temperature and T_0 the lattice temperature, and S is the speed of sound. The first term on the right hand side is due to polar scattering, the second to acoustic scattering and the third to optical (or intervalley) scattering. The appropriate optical mode temperature θ must be used in the first and third terms, and $\gamma=\theta_0/T$, $\gamma_p=\theta_p/T$. The mobilities μ_{p0}, μ_a, μ_0 are the zero field polar, acoustic and optical (or inter-valley) mobilities respectively. $A(\gamma)$, $B(\gamma)$, $C(\gamma)$ and $D(\gamma)$ can be expressed in terms of Bessel functions. Equations (1) and (2) can be solved numerically by substituting in each a value for T, and deducing F by multiplication and p_0 by division. The current density J is equal to nep_0/m^*, so that for each T linked values of F and J are obtained.

In the derivation of these expressions it is assumed that the electron density is sufficiently high for energy and momentum exchanges to be controlled by interelectronic collisions.

§ 3. Mobility at Low Electric Fields

The results of the calculation are given in Table I.

Table I. Mobility in off-centre minima of III–V compounds.

Compound	Minimum	m^*	μ_0	μ_A	μ_{p0}	Combined μ	Observed μ
AlSb	(100)	0.39	280	1550	850	190	200
GaP	(100)	0.36	570	3000	750	290	150
GaSb	(111)	0.2	3700	1070	3450	670	645
GaAs	(100)	0.4	320	1900	600	190	155

The mobilities are in reasonable agreement with the highest observed values, with the exception of GaP. We should note that the effective mass values used are not very accurate, since they were determined on a small number of samples with high impurity content. In particular the relative values[8] for AlSb and GaP seem open to question, for AlSb has smaller energy gap values than GaP, and we might therefore expect it to have a smaller effective mass.

In GaSb the lowest conduction band minimum is at the centre of the zone, with the (111) minima about 0.074 eV higher. The purest n-type samples of GaSb contain about 10^{17} electrons per cm^3, and over 80% of these are in the (111) minima. The mobility value given in Table I is that deduced by Sagar[9] from an analysis of conduction in two bands.

§ 4. Mobility at High Electric Fields

Polar scattering is not strongly dependent on electric field if the optical mode temperature is about the same as the lattice temperature. This is true for all the compounds considered here. Deformation potential scattering is strongly field dependent, the mobility falling rapidly with field. The effects due to mixed scattering can be ex-

tremely complex, and depend on the relative contributions of the two mechanisms. Particularly interesting are the effects due to a mixture of acoustic and polar scattering. We see that p_0^2 has the form $[A(\gamma)+x(T/T_0)^{3/2}]/[B(\gamma)+y(T/T_0)^{1/2}]$ at high electron temperatures. At such temperatures $A(\gamma)$ is only slowly varying with electron temperature, and is a little greater than unity while $B(\gamma)$ approaches $3\sqrt{\gamma}$. It is simple to show that the p_0-T curve can have a negative slope if y is large and x small, and a more detailed study shows y must be greater than unity and x less than $y^{3/2}/100$ approximately. Since y is equal to $2\mu_{p0}/\mu_a$ and x to $2m^*S^2y/k\theta_p$, the negative resistance is best seen in a weakly polar material. It can be shown that optical or inter-valley scattering tends to reduce the negative resistance effect.

The effect is therefore best demonstrated in a material with predominant acoustic scattering, appreciable polar scattering and weak optical or inter-valley scattering. This combination cannot be obtained in (100) minima, where inter-valley scattering is strong, but is approached in GaSb, the least polar of the III–V compounds. In Fig. 1 we show the predicted current density-field characteristic. (The carrier concentration has been taken as 10^{16} cm^{-3}). The dotted curve applies to GaSb, with mobilities as in Table I. The full curve shows the effect of reducing the optical scattering. These calculations ignore electron transfer from the central minimum. This should not have a drastic effect, but would give an extra heating process. As a result the departures from Ohm's law should occur at lower field.

When the two most powerful mechanisms are polar scattering and inter-valley scattering, as in GaAs, AlSb, and GaP the negative resistance is hardly perceptible. Instead the current density saturates at high fields. It has been shown previously[10~12] that at high fields the mobility in the (100) minimum of GaAs falls to less than 50 cm^2/V sec. There is good experimental evidence for this from the Gunn effect, and indeed it is difficult to explain the observed results without invoking mixed scattering.

§ 5. High Field Experiments on GaP and GaSb

Initial experiments have been made on n-type

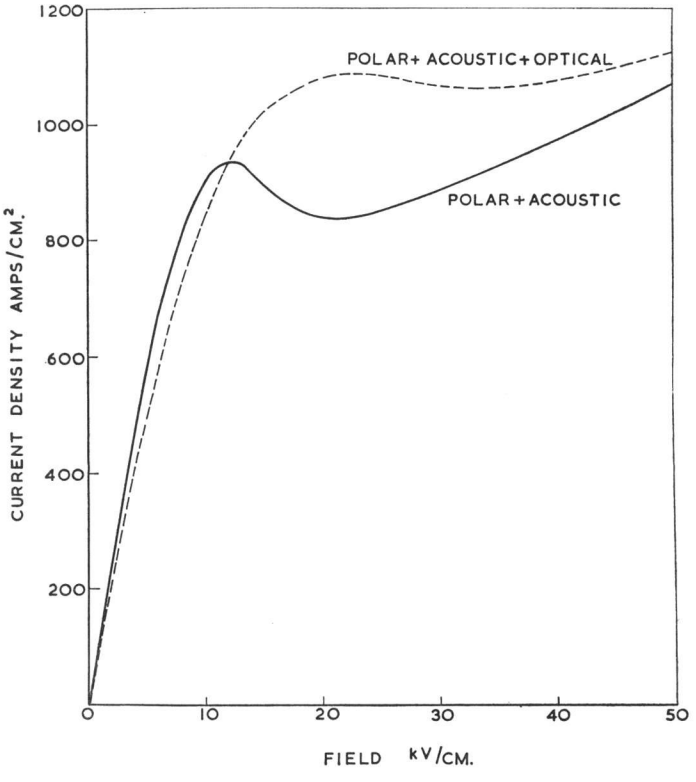

Fig. 1. Current density-field characteristic for GaSb (300°K) including and excluding optical scattering.

samples of GaSb and GaP. The samples were of rather lower resistivity than was convenient, and it was difficult to apply fields greater than 10 kV/cm to the GaSb, and 25 kV/cm to the GaP. The pulses, 3 ns in length, were applied from a high voltage source through a mercury wetted relay and a delay line. The samples, thin crystals approximately 0.5 mm sq. had one side alloyed to a 1 mm tin plated molybdenum stub. The other contact was a small alloyed tin dot. Each sample was mounted in a specially designed holder to allow current and voltage pulses to be measured on a sampling oscilloscope.

The GaSb ($n=10^{17}$, $p=0.05$ ohm cm) showed no appreciable departures from Ohm's law at fields up to 10 kV/cm. At such fields our calculations indicate departures of over 25%, which should have been observable. Samples of GaP showed a slight change in mobility at a field of 25 kV/cm. These samples were compensated, with a resistivity of 4000 ohm cm. Again, the departures from Ohm's law which the theory leads us to expect were not observed. Further more detailed measurements are in progress.

Acknowledgements

The authors are grateful to P. Butcher, W. Fawcett and S. McQuillan for valuable discussions, to D. Effer for the supply of the GaP and to J. Morgan for the manufacture of the samples and the sample holder. Crown Copyright Reserved. Reproduced with the permission of H. M. Stationery Office.

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DISCUSSION

Conwell, E. M.: The calculations of Stratton on which the present calculation is based were done for a displaced Maxwellian distribution, which is of course not correct. It is not certain that a negative differential resistance would appear if the correct distribution were used, particularly since the predicted negative resistance is small.

Bonch-Bruevich, V. L.: I should like to put a question both to Dr. Conwell and Dr. Hilsum. Both calculations are approximate. Therefore some parameter must exist the smallness of which makes the result valid. What is the parameter in question?

Hilsum, C.: The high-field calculation followed the treatment of Stratton, which is essentially a first order theory. However, this negative resistance effect is a consequence of the breakdown due to the nature of polar scattering. At some critical field the carriers can no longer be restrained by polar scattering. The sudden increase loss of energy to acoustic modes causes a sharp decrease in mobility, and as a result a negative differential resistance occurs.

This conclusion is unlikely to be affected by the detailed nature of the calculations, as long as these calculations predict polar breakdown.

Conwell, E. M.: (Reply to Bonch-Bruevich question): In answer to the question of Prof. Bonch-Bruevich, our calculations are approximate only in the field region for which the average electron energy is comparable to optical phonon energy. This is the region around the threshold field in the case of the Gunn effect in GaAs. The parameter which must be small is the ratio of optical phonon energy to electron energy, or equivalently the ratio of drift to random velocity. I believe the same is true of the calculation of Stratton on which Hilsum's work was based.

Stratton, R.: In my original papers (referred to Dr. Hilsum) the displaced Maxwellian

was only one of the cases considered. Expressions for the distribution function considering acoustic, nonpolar and optical polar scattering were also derived as were expressions involving an arbitrary dependence of the scattering matrix element on phonon wave number. I showed that the concept of the runaway field did not depend on the assumed form of the distribution.

Moss, T. S.: Concerning my effective mass values which you quoted, you may be able to improve them by putting accurate values for the scattering parameter into the Hall coefficient formula which I used.

Hilsum, C.: I agree.