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XIV-4. Optical Absorption, Tunnel Current and Diagonal Tunneling in Crossed Electric and Magnetic Fields

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A problem of the electron motion in crossed fields with an arbitrary ratio of electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{H}$ can be solved only by taking into account both the valence and the conduction bands simultaneously as it was pointed at first by B. Lax[13]. In the simplest case of non-degenerated bands, in which extrema are situated in one and the same point of k-space, the two-band model is to be described by the equation, which differs from the Dirac equation only in substitution of $s=(\varepsilon_g/2m^*)^{1/2}$ instead of c and anisotropy of the effective mass, which can be easily cancelled by the appropriate transformation of the coordinate system. Here m^* is the density-of-states mass of the electrons and the holes, which are equal in this approximation, and ε_g is the energy gap. Such a model is probably applicable for semiconductors of IV-VI groups (PbTe, PbSe, PbS), if the relativistic terms are taken into account, and for diamond type lattice or ZnS structure, if the sign of the spin-orbit splitting is opposite to that in Ge and InSb.

This equation is invariant in transformations, which differ from the Lorentz-transformation only in substitution of s instead of c, the quantity H' and iE' being transformed as the components of the appropriate field tensor in theory of relativity.

Consequently choosing the appropriate moving coordinate system, we can reduce the problem in crossed fields to that in the effective electric field $E^*=E'(1-\beta^{-2})^{1/2}$ only, if $\beta=E'/H'>1$, and to that in the effective magnetic field $H^*=H'(1-\beta^2)^{1/2}$ at $\beta<1$. Here

$$E'=\left(\sum_i E_i^2\frac{m^*}{m_i}\right)^{1/2} \qquad H'=\frac{s}{c}\left(\sum_i H_i^2\frac{m_i}{m^*}\right)^{1/2},$$

where m_i—tensor components of reverse effective mass in principal axes of ellipsoid; E_i, H_i—the component of external fields in the same axes, $m^*=(m_x m_y m_z)^{1/3}$. Besides $(\boldsymbol{E}'\cdot\boldsymbol{H}')=(\boldsymbol{E}\cdot\boldsymbol{H})=0$. In the first case the motion in the direction of $\boldsymbol{E}$ is infinite, in the second case it is finite and the Landau quantization takes place. It is possible to carry out the calculation of the number of transition at once in a moving coordinate system, as four-dimensional volume element $dVdt$ is an invariant with respect to Lorentz transformations. That is why at $\beta<1$ the tunnel current in homogeneous field is equal to zero and it can be caused only by scattering on impurities and phonons. It is possible to show that this conclusion does not depend on the accepted model, and at any dispersion law $\varepsilon_0(\boldsymbol{p})$ if only the electron orbits in a magnetic field are closed, at given $\boldsymbol{H}$ there exists a critical value of $\boldsymbol{E}$, below this value the direct tunnel transitions are forbidden by the laws of conservation of energy and momentum. It is to be noted that the change of character of the electron motion with increasing electric field can be also derived from the quasiclassical approximation. As Lifshits and Kaganov[12] showed, it is possible in this approximation to consider the motion in the crossed fields as that in a magnetic field only with the dispersion law

$$\varepsilon=\varepsilon_0(\boldsymbol{p})-V_0\boldsymbol{p}\,, \qquad V_0=\frac{c[\boldsymbol{E}\times\boldsymbol{H}]}{H^2}\,.$$

In such case the period of the motion along the orbit is $T=(c/eH)(\partial S/\partial\varepsilon)$, where $S(\varepsilon)$ is the area of the cross-section of ε-surface by the plane $P_z=\text{const}$. Consequently, the condition $(\partial S/\partial\varepsilon)^{-1}=0$ determines the critical value of the electric and magnetic fields, at which the period of the motion becomes infinite for the given group of electrons, *i.e.* $T\to\infty$. It follows naturally from this condition that in the present model the critical field for all electrons is determined by the condition $\beta=1$. At $\beta>1$ the tunnel current decreases with increasing H due to decrease of effective field E^*.

As the calculations show, the number of transitions per 1 cm^3 per 1 sec in such case is equal to:

$$J=\frac{e^2(E'^2-H'^2)}{36\pi\hbar^2 S}\exp\left\{-\frac{\pi m^{*2}s^3}{e\hbar(E'^2-H'^2)^{1/2}}\right\}. \tag{1}$$

At $H'\to 0$ this expression reduces to the formula given in ref. 1). Current dependency of tunnel diode $J(H)$ at given E is determined by an expression analogous to (1) precisely to pre-exponential factor.

In Fig. 1. $J(H,E)/J(0,E)$ dependence versus H is plotted for PbTe diode to two H orientations, and in Fig. 2—the dependence of this ratio versus H orientation. Energy band parameters of PbTe are chosen according to ref. 2), and E value is determinated from comparison of calculated ratio $J(H_{\bar{1}01})/J(H_{\bar{2}11})$ at $H=53.2$ kG with the experimental data in ref. 3), which are shown in Figs. 1 and 2 by points.

It is necessary to note that at $\beta>1$ the absence of quantization explains the unsuccessful attempt to observe oscillations of tunnel current in a transverse magnetic field,[4] and also the independence of maximum position of tunnel current on H value.[5] In a longitudinal magnetic field, where the quantization always takes place, both effects are being observed well.[4,5]

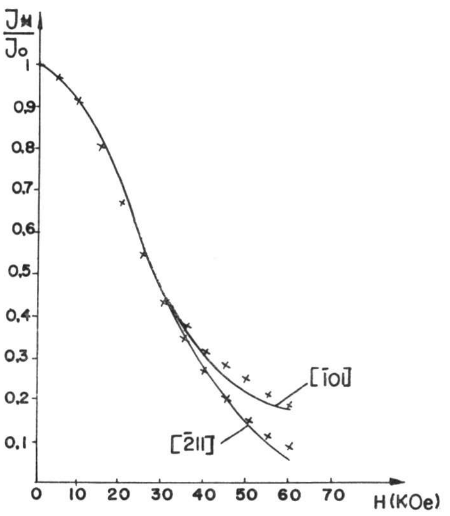

Fig. 1.

As a result of principle difference in the dynamics of electrons at $\beta>1$ and at $\beta<1$ at examining of interband light absorption in crossed fields it is necessary to distinguish two effects: at $\beta>1$ magnetic field Franz-Keldysh effect takes place, *i.e.*, exponential decrease of absorption coefficient $\alpha(\omega)$ at $\hbar\omega=\varepsilon_g$. When passing to movable coordinate system, where an effective magnetic field is equal to zero, as a result of wave phase invariance with respect to Lorentz transformation, the light frequency ω' and wave vector q' in a movable coordinate system connected with ω and q in unmovable system by ratio (if $q\simeq 0$).

$$\omega'=\frac{\omega}{\sqrt{1-\beta^{-2}}} \quad q_y'=\beta^{-1}\frac{\omega/s}{\sqrt{1-\beta^{-2}}}\,,\ q_x'=q_z'=0\,. \tag{2}$$

In this case, as the calculations show, absorption edge does not shift, and at $(\varepsilon_g-\hbar\omega)/\varepsilon_g\ll 1$

$$\frac{\alpha(E,H)}{\alpha(E,0)}=\exp\left\{-\frac{4}{15}\frac{(\varepsilon_g-\hbar\omega)^{5/2}}{em^{*1/2}s^2\hbar}\frac{H'^2}{E'^3}\right\}\,, \tag{3}$$

where $\alpha(E,0)$ dependence is determined by Franz-Keldysh formula.[6]

At $\beta<1$, when the transitions take place between Landau levels, it is necessary to speak about the electric field influence on magneto-optical transitions first considered by Aronov[8]. In such case in movable coordinate system, where $E^*=0$

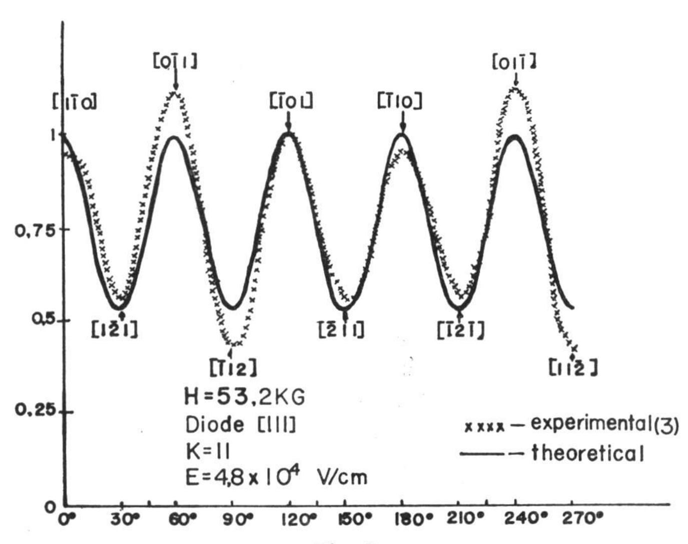

Fig. 2.

$$\omega'=\frac{\omega+s(\boldsymbol{q}\cdot\boldsymbol{\beta})}{(1-\beta^2)^{1/2}}, \qquad q_y'=\frac{q_y+\beta\omega/s}{(1-\beta^2)^{1/2}},$$
$$q_x'=q_x,\ q_z'=q_z$$

where $\boldsymbol{\beta}=[\boldsymbol{E}'\times\boldsymbol{H}']/H'^2$ *i.e.*, ω' and q' grow with increasing E' owing to Doppler effect. As accordingly to ref. 7) the line positions ($k_{z1}=k_{z2}=0$) are determined by the law of energy conservation

$$\frac{\hbar\omega+\hbar s(\boldsymbol{q}\cdot\boldsymbol{\beta})}{(1-\beta^2)^{1/2}} =\left[(m^*s^2)^2+2\hbar esH^*\left(n_1+\frac{1}{2}\pm\frac{1}{2}\right)\right]^{1/2} +\left[(m^*s^2)^2+2\hbar esH^*\left(n_2+\frac{1}{2}\pm\frac{1}{2}\right)\right]^{1/2} \quad (4)$$

so as with growing of β at the same ω the transitions become possible with greater n. The position of each line shifts in the direction of lower frequencies, and therefore the absorption edge shifts as $(1-\beta^2)^{1/2}$, and the distance between the lines decreases simultaneously as $(1-\beta^2)$ to small n.

As is seen from (4) the line position and consequently α at $(\boldsymbol{q}\cdot\boldsymbol{\beta})\neq 0$ varies slightly with the reversal of direction of $\boldsymbol{H}, \boldsymbol{E}$ or $\boldsymbol{q}$.

Owing to Doppler effect in a moving coordinate system the light wave vector $\boldsymbol{q}'$ differs from zero (even if $\boldsymbol{q}\simeq 0$). That is why the transitions take place with changing of the center of gravity of oscillator and the forbidden transitions become possible with changing of quantum number n. The probability of forbidden transitions increases with the growing of E, and that of allowed transitions decreases, so as

$$\sum_{n_1=0}^{\infty}|M_{n_1n_2}|^2=1 ,$$

where $M_{n_1n_2}$—matrix element of the transition $n_1\to n_2$.

In such case $|M_{n_1n_2}|^2$ depends non-monotonic on E and has a number of zeroes equal to the lesser one of n_1 and n_2. While in movable coordinate system the shift of absorption edge and appearance of forbidden transitions are caused by Doppler effect, in a stationary one, where the transitions take place with momentum conservation, in an electric field the center of oscillator shifts along the field during the transition, that leads to decreasing of energy required for the transition, and the appearance of the forbidden lines.[8,10] We must note that for indirect transitions, when the position of center of oscillator does not change, the absorption edge shifts to higher frequences with increasing electric field.[9]

In conclusion, it is to be noted that a magnetic field affects not only the absorption, but also the spontaneous emission of light at frequency $\hbar\omega<\varepsilon_g$ in *p-n* junction as the result of diagonal tunneling.[11] It is possible to show that the number of photons, emitted in *p-n* junction at bias V in $d\omega d\Omega$ range is given by,*

$$dN=d\omega d\Omega\frac{n^2\omega^2}{4\pi^3 eEc^2}\alpha(\omega)A , \quad (5)$$

where a value A is determined by the bias and the position of Fermi levels μ_N and μ_P in *n*- and *p*-regions. At $T\to 0$,

$$2A=(\mu_N+\mu_P)-|\hbar\omega-eV+\mu_N|-|\hbar\omega-eV+\mu_P| ,$$

and emission maximum shifts relatively to the highest frequency $\hbar\omega_v=eV$ by

$$\omega_v-\omega_{\max}=\frac{eE}{\sqrt{2}\, m^*[\varepsilon_g^2-(\hbar\omega_{\max})^2]^{1/2}} .$$

This shift was observed in a number of papers.[11] In a magnetic field $dN(H, E)$ decreases as well as $\alpha(H, E)$ according to (3).

In such experiments it is easier, probably, to find out the magnetic field effect, as it is possible to observe the light emission in *p-n* junction at the considerably lower frequency than the light absorption.

The detailed calculations have been published in Zh. eksper. teor. Fiz. **51** (1966) N1(7), 2(8); Fiz. tverdogo Tela **8** (1966) N12.

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* The expression, similar to (4), was independently derived by A. E. Junovich and A. B. Ormont.

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COMMENT

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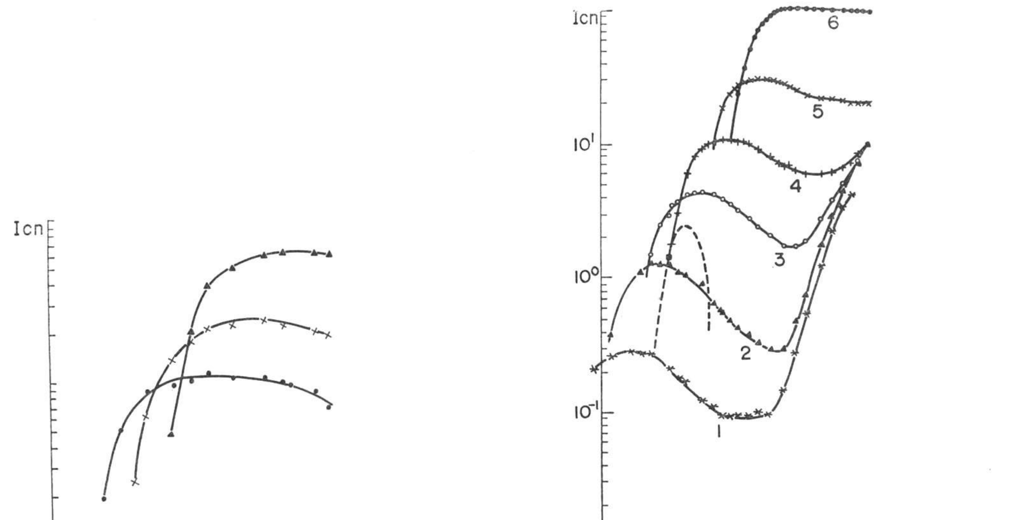
My comment deals with the theoretical and experimental study of the radiative recombination, connected with the tunnel effect in *p-n* junctions in GaAs, InP, GaSb, which is in progress in our laboratory.

We considered theoretically the interband radiative recombination in the high electric field in *p-n* junction degenerated on both sides. The model of such a "diagonal" tunneling has been suggested and qualitatively described by Leite *et al.*[1] In the quantitative analysis the probability of the direct optical transition of an electron with quasimomentum p in the constant electric field ε was taken in the form, given by Keldysh.[2] The probability of the spontaneous transition is proportional to the Fermi-functions $f_c(E)$ and $1-f_v(E-\hbar\omega)$, respectively for electrons and holes, and also to the number of quanta in the interval $d(\hbar\omega)$. The number of emitted quanta per sec per cm^2 of the junction is found by integration over all permitted components of the quasi-momentum $p_\perp$, perpendicular to the field ε, and over all permitted energies E. The analysis is similar to that of the *I-V* characteristics of tunnel diodes.[3]

The results of the experimental investigation of the spectral band connected with tunnel



(a) (b)

Fig. 1. Dependence of the spectral intensity of radiation $I(U)$ on the applied voltage.
(a) GaSb; $T=9$°K; $1/\hbar\omega=0.62$ eV; $2/\hbar\omega=0.64$ eV; $3/\hbar\omega=0.66$ eV; the measurements at higher voltages are not shown because of heating of the sample.
(b) InP; 77°K; $1/\hbar\omega=0.95$ eV; $2/\hbar\omega=1.00$ eV; $3/\hbar\omega=1.05$ eV; $4/\hbar\omega=1.10$ eV; $5/\hbar\omega=1.15$ eV; $6/\hbar\omega=1.20$ eV; the dashed curve is the calculated one.

effect showed that the position of the maximum of the spectral band in thin (300–600 Å) *p-n* junctions changes in wide intervals (1.25–1.47 eV in GaAs, 0.95–1.35 eV in InP; 0.61–0.74 eV in GaSb at 77°K). The value $\hbar\omega_M$ linearly depends on voltage V, in accordance with the model. The theory describes the high-energy side of the spectrum rather well. The form of the low-energy side of the band is similar to the theoretical, but is shifted to lower $\hbar\omega$.

It was especially important to find out whether one observes the decrease of the intensity $I(V)_{\hbar\omega=\text{const.}}$ with the increase of the applied voltage. This was observed for GaSb and InP. The curves $I(V)$ for different energies $\hbar\omega$ are shown in Fig. 1 for GaSb (9°K) and InP (77°K) samples. There is a maximum on the curves, its position corresponding to a voltage a little bit greater than $\hbar\omega$. The lower is the energy $\hbar\omega$, the more distinctly is expressed the maximum. If the voltage was increased further the intensity passed through the minimum, and then increased.

The observed curves are quite similar to the current-voltage characteristics of tunnel diodes. At low voltages the calculated curves coincide with the experimental ones.

The discrepancy between experimental data and the theory in the region of low energies or in the region of high voltages we connect with participation of the "tails" of density of states in the recombination and tunnel radiative recombination via impurity centers. This explanation is analogous to that of excess currents in the tunnel diodes.

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DISCUSSION

Lax, B.: We have carried out both theory and experiment on photon assisted crossed field tunneling. The experiments show that the effect is small but confirms the *H*-square dependence of the crossed field Franz-Keldysh effect.

Pikus, G. E.: We agree that this effect is very small close to the intrinsic absorption edge but it is increasing with decreasing frequencies. It results from the fact that the *H*-square terms in the Taylor expansion of the exponent according to our calculations are proportional to $(E_g-\hbar\omega)^{5/2}$, while *H*-independent terms according to Franz-Keldysh formula are proportional to $(E_g-\hbar\omega)^{3/2}$.