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XVIII-4. Magnetic Field Effects on the Static Shielding of an Impurity Charge by an Electron Plasma*

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The results of our calculations on magnetic field effects on static shielding are discussed. The quantum analogue of the Debye-Thomas-Fermi (DTF) shielding law bears quantum corrections with concomitant spatial anisotropy. Improvements over earlier nondegenerate DTF calculations are discussed, and the role of de Haas-van Alphen (DHVA) oscillatory terms in the degenerate limit of the DTF law is discussed in relation to measuring $g(m/m_0)$. In the quantum strong field limit the long-range character of the Friedel-Kohn "wiggle" is destroyed, (appropriate experimental conditions are realizable in some semiconductors). Finally, in the degenerate-DHVA limit, the long-range character of the Friedel-Kohn "wiggle" is restored, and it is shown that DHVA oscillatory correction terms are "washed out".

§ 1. Introduction

An account of the results of our calculations on the influence of a magnetic field on the static shielding of a point charge is presented here. The calculations were carried out using a thermodynamic Green's function formulation of the random phase approximation (RPA) description of the inverse dielectric function of the electron gas plasma. A closed-form thermal Green's function for electrons experiencing Landau quantization in a magnetic field is used in conjunction with the Green's function formulation of the RPA, and this results in a relatively tractable analytical form for the inverse dielectric function upon which our calculations are based. We shall not dwell upon the details of the calculations here, but merely sketch the principal theoretical ideas and the basic formulae. A fuller exposition will be published elesewhere.[1)]

What we do wish to accomplish here is to present a comprehensive picture of shielding phenomena in a magnetic field by exhibiting the results of our calculations in the nondegenerate, degenerate and quantum strong field limits. (It has been possible to cover such a broad range of interests by taking advantage of the flexibility of the thermodynamic Green's function method insofar as the choice of statistical regimes is concerned. In this regard it was also important to treat the magnetic field in a non-perturbative manner, using a closed form Green's function for Landau electrons). Magnetic field corrections in the quantum analogue of the Debye-Thomas-Fermi law will be exhibited. Also the effect of the magnetic field on the Friedel-Kohn "wiggle" will be described. In the degenerate case the magnetic field effects arise through terms monotonic in magnetic field strength, and in addition through de Haas-van Alphen (DHVA) oscillatory terms; both types of magnetic field effects will be discussed in conjunction with both the Debye-Thomas-Fermi part of the shielding law and the Friedel-Kohn "wiggle" part of the shielding law. Finally, one may well expect that maximal influence of the magnetic field is realized in the quantum strong field limit, and we shall discuss the fate of the Friedel-Kohn "wiggle" in this case too. Spatial anisotropy of the shielding law exists whenever magnetic field effects are nontrivial, and this will be exhibited in context.

§ 2. Static Shielding in a Magnetic Field; Low Wavenumber Description

In opening our discussion of magnetic field effects on the static shielding of an impurity charge by an electron plasma, it is important to note that so long as one is concerned with semiclassical or classical plasmas the magnetic

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field cannot affect the static shielding law at all, no matter how strong it may be. The reason for this is simply that the magnetic field can do no work so long as the plasma electron dynamics are classical, and therefore the application of a magnetic field of arbitrary strength cannot provide any energy which would be necessary for a redistribution of the shielding charges. However the magnetic field is no longer insignificant in regard to the static shielding law when the quantized nature of plasma electron dynamics is felt. One can expect substantial magnetic field effects on shielding when the applied magnetic field causes changes in the single particle energy spectrum. As was mentioned in the Introduction, the calculations were carried out using a thermodynamic Green's function formulation of the RPA description of the inverse dielectric function of the plasma, together with a closed-form thermal Green's function for Landau electrons in a magnetic field. The resulting frequency-wavenumber dependent inverse dielectric function is relatively tractable analytically and can be applied to the nondegenerate, degenerate, and quantum strong field limits. Since the inverse dielectric function relates an impressed potential at one space-time point with the effective potential at another space-time point in the plasma, the long-time static shielding of a Coulomb impurity charge may be obtained from the inverse dielectric function specialized to zero frequency. The wavenumber dependence of the inverse dielectric function then yields the details of the shielding of the Coulomb impurity charge according to the result below which is exact within the scope of the RPA (and is exact with respect to magnetic field dependence),

$$V(\boldsymbol{r},\infty)=\int\frac{d\boldsymbol{p}}{(2\pi)^3}e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\times\frac{4\pi e/p^2}{1-(4\pi e^2/p^2)\,\mathrm{Im}\,I(\boldsymbol{p},\omega=0+i\varepsilon)},\tag{1}$$

($U(\boldsymbol{p})=4\pi e/p^2$ for the Coulomb potential of an impurity). Here, the denominator is given by

$$1-\frac{4\pi e^2}{p^2}\mathrm{Im}\,I(\boldsymbol{p},\omega=0+i\varepsilon)=1-\frac{4\pi e^2}{p^2}\int_0^\infty d\omega\frac{f_0(\omega)}{\hbar^3}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{ds}{2\pi i}e^{s\omega}\times\frac{\pi^{3/2}}{(2\pi)^3}\sqrt{\frac{2m}{s}}\frac{m\hbar\omega_c}{\tanh\hbar(\omega_c/2)s}\mathscr{H}_0,\tag{2}$$

where

$$\mathscr{H}_0=-s\int_{-1}^{1}dT\,e^{(\hbar^2p_z^2/8m)s(T^2-1)}\times\exp\left\{\frac{\hbar\bar{p}^2}{2m\omega_c}\frac{\cosh\hbar(\omega_c/2)sT-\cosh\hbar(\omega_c/2)s}{\sinh\hbar(\omega_c/2)s}\right\}.\tag{3}$$

The anisotropic quantum analogue of the Debye-Thomas-Fermi static shielding law may be derived as the low wavenumber contribution to $V(\boldsymbol{r},\infty)$. To obtain this one expands $\mathscr{H}_0$ to order p^2, and inverts the Fourier transform (1). The result has the form

$$V(\boldsymbol{r},\infty)\ \exp=\frac{e\exp\{-r\sqrt{(4\pi e^2/Q_1)(\partial\rho/\partial\zeta)}\sqrt{1+\delta\cos^2\varphi}\}}{r\sqrt{Q_1Q_2}\sqrt{1+\delta\cos^2\varphi}},\tag{4}$$

(φ=angle between $\boldsymbol{H}$ and $\boldsymbol{r}$)

where the quantum anisotropy parameter δ is defined by

$$\delta=Q_1/Q_2-1,\tag{5}$$

and the quantum corrected anisotropic effective Debye length may be identified as

$$r_D=\left[\frac{4\pi e^2}{Q_1}\frac{\partial\rho}{\partial\zeta}(1+\delta\cos^2\varphi)\right]^{-1/2}.\tag{6}$$

Specific evaluations of $\partial\rho/\partial\zeta$, Q_1, Q_2, and δ based on eqs. (2) and (3) have been carried out, and the results are given below for the nondegenerate, degenerate, and quantum strong field limits:

(a) Nondegenerate Limit

$$4\pi e^2\partial\rho/\partial\zeta=m\omega_p{}^2\beta,\tag{7a}$$

$$Q_1=1+\omega_p{}^2/\omega_c{}^2\{1-\hbar(\omega_c/2)\beta/\tanh\hbar(\omega_c/2)\beta\},\tag{7b}$$

$$Q_2=1-(\hbar\omega_p\beta)^2/12,\tag{7c}$$

$$\delta\cdot Q_2=(\hbar\omega_p\beta)^2/12+(\omega_p{}^2/\omega_c{}^2)\times\{1-\hbar(\omega_c/2)\beta/\tanh\hbar(\omega_c/2)\beta\},\tag{7d}$$

(b) Degenerate Limit

$$4\pi e^2\partial\rho/\partial\zeta\sim m\omega_p{}^2/\zeta+(m\omega_p{}^2/\zeta)(\hbar\omega_c/\zeta)^{1/2}\times[\text{DHVA Osc.}],\tag{8a}$$

$$Q_1\sim1-(\hbar\omega_p/\zeta)^2+(\omega_p{}^2/\omega_c{}^2)(\hbar\omega_c/\zeta)^{1/2}\times[\text{DHVA Osc.}],\tag{8b}$$

$$Q_2\sim1-(\hbar\omega_p/\zeta)^2+(\hbar\omega_p/\zeta)^2(\zeta/\hbar\omega_c)^{1/2}\times[\text{DHVA Osc.}],\tag{8c}$$

$$\delta\cdot Q_2\sim(\hbar\omega_p/\zeta)^2(\hbar\omega_c/\zeta)^2+(\omega_p{}^2/\omega_c{}^2)(\hbar\omega_c/\zeta)^{1/2}\times[\text{DHVA Osc.}],\tag{8d}$$

(c) Quantum Strong Field Limit

$$4\pi e^2 \partial\rho/\partial\zeta = m\omega_p^2/2\,\zeta\,, \tag{9a}$$

$$Q_1 = 1-(\omega_p^2/\omega_c^2)[\hbar\omega_c/4\,\zeta-1]\,, \tag{9b}$$

$$Q_2 = 1+(1/48)(\hbar\omega_p/\zeta)^2\,, \tag{9c}$$

$$\delta\cdot Q_2 = -(1/48)(\hbar\omega_p/\zeta)^2-(\omega_p^2/\omega_c^2)[\hbar\omega_c/4\zeta-1]\,. \tag{9d}$$

In the nondegenerate limit (7), quantum corrections arise through the parameter $\hbar\omega_c\beta$ as well as $\hbar\omega_p\beta$. This result bears out the general statement that magnetic field effects are nontrivial only when quantum corrections are nontrivial. On the other hand quantum corrections persist even in the limit of zero magnetic field, in which case the quantum corrections as well as the entire shielding law must obviously become isotropic. Indeed the results (7) behave in this manner, whereas the earlier nondegenerate shielding results of Bonch-Bruevich and Mironov[2)] suffer from a spurious zero-field anisotropy, despite their conceptually valid approach to the limited nondegenerate case. The results for the degenerate limit (8) are given for intermediate field strengths, $\hbar\omega_c<\zeta$ and $\hbar\omega_c\beta\gg 1$. Here we have simply given "order of magnitude" estimates since the exact formulae which are represented by (8) are long and cumbersome. The quantities indicated by [DHVA Osc.] are of the form $\sum_{n=1}^{\infty} n^{-p}\cos[2\pi n\zeta/\hbar\omega_c - \text{phase}]$, and it should be noted that such DHVA oscillatory terms are important in the anisotropy parameter δ. It can be shown that the spectral composition of the DHVA oscillatory terms depends sensitively on $g(m/m_0)$, through the insertion of factors such as $(-1)^n\cos[\pi n g(m/m_0)]$ in the summand of $\sum_{n=1}^{\infty}$ above. This remark is made in connection with the recognition that the effective mass m associated with the orbital part of the electronic motion differs from the ordinary electronic mass m_0 and allowance is made for an anomalous electronic g-factor. This "spin effect" phenomenon in the DHVA terms offers the possibility of determining $g(m/m_0)$ through observations of the quantum corrections and anisotropy involved in the quantum analogue of the Debye-Thomas-Fermi shielding law. Finally it should be noted that the results for the quantum strong field limit (9) cannot be expected to be meaningful as either $\hbar\to 0$ or $\omega_c\to 0$, since the quantum strong field limit is defined by $\hbar\omega_c>\zeta$ and $\hbar\omega_c\beta\gg 1$, and these preconditions were used in deriving (9). We shall discuss other aspects of the shielding law in the quantum strong field limit in greater detail below.

§3. High Wavenumber Contributions ($p\sim 2p_F$) to the Static Shielding Law in a Magnetic Field: Magnetic Field Effects on the Friedel-Kohn "Wiggle"

It is well known that the low-wavenumber description (which is tantamount to the quantum analogue of the Debye-Thomas-Fermi static shielding law discussed above) is severely limited in the degenerate case, and thus is unable to describe higher wavenumber phenomena such as the long range Friedel-Kohn "wiggle" part of the static shielding law. Qualitatively, one might expect that the strong anisotropy induced by the magnetic field in the quantum strong field limit would result in significant changes in the long range Friedel-Kohn "wiggle." In order to investigate this, one must use the quantum strong field limit of the exact RPA result for $V(\boldsymbol{r},\infty)$ given earlier. Evaluating eqs. (2) and (3) under conditions appropriate to the quantum strong field limit, $\hbar\omega_c\gtrsim\zeta$ and $\hbar\omega_c\beta\gg 1$, one obtains $V(\boldsymbol{r},\infty)$ as*,

* It should be noted that the logarithmic singularities in the denominator of the integrand of $V(\boldsymbol{r},\infty)$ are highly anisotropic since they refer only to the direction parallel to $\boldsymbol{H}$, whereas reference to the plane perpendicular to $\boldsymbol{H}$ enters in a very different way. This is to be compared with the occurrence of the logarithmic singularity in the field-free case, which is completely isotropic. One can expect the r_z-dependence of the static shielding law in the quantum strong field limit to be similar to the Friedel-Kohn "wiggle"; however it is clear that a very different type of $\bar{r}$-dependence is to be expected. Unfortunately the techniques of Lighthill[3)] for Fourier transforming functions having log-singularities are not useful here, [although they are successful in the field-free case[4)]], since they fail to provide an asymptotic series in the case of eq. (10). It has recently come to our attention that another discussion of this problem was undertaken by J. Durkan, J. E. Hebborn and N. H. March.[5)] However this discussion fails to yield agreement with our eq. (10), and it does not include an evaluation of the Fourier transform which is necessary to provide information on the part of the static shielding law, $V'(\boldsymbol{r},\infty)$, which arises from high wave-number contributions in the neighborhood of the logarithmic singularity, $p\sim 2p_F$. (Our result for $V'(\boldsymbol{r},\infty)$ is presented in eq. (11).) The lack of agreement with our eq. (10) is manifest in several ways: The one which is most significant seems to be the neglect of Pauli spin terms (which are vital to a correct representation of the quantum strong field limit) in the considerations of Durkan. Hebborn and March.

Note added in proof; Prof. N. H. March informed me a few months ago that he and coworkers were working on numerical evaluation of the shielding law in the quantum strong field limit, and this should be of interest to compare with the analytical results presented here.

$$V(\boldsymbol{r},\infty)=\frac{4\pi e}{(2\pi)^3}\int d\boldsymbol{p}\, e^{i\boldsymbol{p}\cdot\boldsymbol{r}}\Bigg[p^2+p_D^{\,2}\frac{p_F}{|p_z|}$$
$$\times\exp(-\zeta\bar{p}^2/\hbar\omega_c p_F^{\,2})\Bigg\{\log\left|\frac{|p_z|+2p_F}{|p_z|-2p_F}\right|$$
$$+\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\zeta\bar{p}^2}{\hbar\omega_c p_F^{\,2}}\right)^n$$
$$\times\log\left|\frac{(|p_z|+p_F[1+ia_n])(|p_z|+p_F[1-ia_n])}{(|p_z|-p_F[1-ia_n])(|p_z|-p_F[1+ia_n])}\right|\Bigg\}\Bigg]^{-1}. \tag{10}$$

The branch point of the first log-term in braces is real ($|p_z|=2p_F$), whereas the complex nature of the branch points for $n\geq 1$ has been made explicit by setting $(1-n\hbar\omega_c/\zeta)^{1/2}=ia_n$ where a_n is real. Insofar as the high wavenumber contribution to the static shielding law is concerned, the most significant feature of (10) is the log-singularity at $|p_z|=2p_F$. We have explored techniques for evaluating (10), but cannot discuss them in detail here. The results are rather long and cumbersome, and we shall be content to present an approximate asymptotic formula which should be understood in an "order of magnitude" sense. With this qualification, the high wavenumber contribution to the static shielding law in the quantum strong field limit may be written as, ($p_F|\bar{r}|, p_F|r_z|\gg 1$),

$$V'(\boldsymbol{r},\infty)\cong -e(p_D/p_F r)e^{(2\zeta/\hbar\omega_c)}$$
$$\times\exp[-(p_D^{\,2}|r_z|/8p_F)|\sin\varphi\cos\varphi|e^{(4\zeta/\hbar\omega_c)}]$$
$$\times\sin(2p_F r_z-\varphi)\exp(-2p_F|\bar{r}|)\,. \tag{11}$$

This evaluation of the contribution from the neighborhood of the logarithmic singularity was obtained subject to the condition that $p_F\gg p_D$, and the accuracy of this evaluation improves as magnetic field strength increases, so that its best accuracy corresponds to an extreme quantum strong field limit, $\hbar\omega_c\gg\zeta$. Equation (11) shows that the Friedel-Kohn "wiggle" is still discernable through the r_z-dependent oscillatory factor $\sin(2p_F r_z-\varphi)$, with an angle-dependent phase. However the strong anisotropy of the quantum strong field limit results in an $\bar{r}$-dependent factor which confines the "wiggle" within a decaying exponential envelope $e^{-2p_F|\bar{r}|}$. Thus the long-range character of the Friedel-Kohn "wiggle" is destroyed in the quantum strong field limit. Moreover, it should be noted that there is also a relatively slowly decaying r_z-dependent exponential envelope factor$\sim\exp[-p_D^{\,2}|r_z|/8p_F]$.

Finally, we shall discuss the results of our recent investigation of magnetic field effects on the Friedel-Kohn "wiggle" under intermediate field strength conditions, $\hbar\omega_c\ll\zeta$ and $\hbar\omega_c\beta\gg 1$. It is under these conditions that DHVA oscillatory terms are most prominent, and we have explored the possibility of DHVA oscillatory terms occurring in conjunction with the Friedel-Kohn "wiggle". At the outset it is appropriate to point out that under conditions of intermediate field strength, the long-range character of the Friedel-Kohn "wiggle" (which is destroyed in the quantum strong field limit) is restored. In fact the high wavenumber contribution to the static shielding law may be represented as

$$V'(\boldsymbol{r},\infty)=V'(\boldsymbol{r},\infty,\{\text{F.K.}-(H=0)\})$$
$$+V'(\boldsymbol{r},\infty,\{\text{F.K.}-\text{DHVA}\})\,, \tag{12}$$

where $V'(\boldsymbol{r},\infty,\{\text{F.K.}-(H=0)\})$ is the ordinary zero-field long-range Friedel-Kohn "wiggle" part of the static shielding law, and $V'(\boldsymbol{r},\infty,\{\text{F.K.}-\text{DHVA}\})$ represents the effect of DHVA oscillatory terms on the high wavenumber part of the shielding law. The calculation of $V'(\boldsymbol{r},\infty,\{\text{F.K.}-\text{DHVA}\})$ must begin with an evaluation of the DHVA isolated singularity terms of the inverse Laplace transform $\int_{-i\infty+\delta}^{i\infty+\delta}ds$ in (2) and (3), and one must take account of the essentially singular character of these isolated singularities in order to treat high wavenumber shielding phenomena properly. This evaluation has been carried out, and after partially inverting the ensuing Fourier transform (1), we obtain a quasi-final result given in part by (13), ($P=p_z-2p_F$; I_0=modified Bessel function),

$$V'(\boldsymbol{r},\infty,\{\text{F.K.}-\text{DHVA}\})\sim(e/8\sqrt{\pi})(\hbar\omega_c/\zeta)^{1/2}$$
$$\times p_D^{\,2}|\bar{r}|^{1/2}e^{i2p_F r_z}e^{-2p_F|\bar{r}|}\sum_{n=1}^{\infty}(\pm in)^{-1/2}$$
$$\times\int_0^{\infty}dP e^{iPr_z}e^{-P|\bar{r}|}(P+2p_F)^{-3/2}$$
$$\times\exp\{\pm i(2\pi\zeta n/\hbar\omega_c)(1-[(P/2p_F)+1]^2)\}$$
$$\times P^{1/2}(P+4p_F)^{1/2}/(P+2p_F)\int_0^1 dT'$$
$$\times\exp\left[\pm i\frac{2\pi\zeta n}{\hbar\omega_c}\left(\frac{P}{2p_F}+1\right)^2\frac{P(P+4p_F)}{(P+2p_F)^2}T'^2\right]$$
$$\times I_0\Bigg\{(P+2p_F)(4\zeta/m\omega_c^{\,2})^{1/2}$$
$$\times\left[1+\left(\frac{P}{2p_F}+1\right)^2\left(\frac{P(P+4p_F)}{(P+2p_F)^2}T'^2-1\right)\right]^{1/2}$$
$$\times\left[(-1)^n-\cos\left(\pi n\frac{P^{1/2}(P+4p_F)^{1/2}}{P+2p_F}T'\right)\right]^{1/2}\Bigg\}$$
$$+(r_z\to -r_z)\,. \tag{13}$$

Actually there are also other terms in (13) which will not be exhibited here since the essential character of the result can be understood from what is exhibited. An anisotropic Friedel-Kohn "wiggle" is evident in the factor $e^{i2p_F r_z}$, and its long range character is destroyed by the factor $e^{-2p_F|\bar{r}|}$. The sum $\sum_{n=1}^{\infty}$ is over the index of the isolated essential singularities mentioned above, which are associated with DHVA oscillatory terms. However, the DHVA oscillatory terms occur with effective frequencies which depend on the wavenumber variable P, for example the factor $\exp\{\pm i(2\pi\zeta n/\hbar\omega_c)(1-[(P/2p_F)+1]^2)\}$ in (13). Within the context of the P-integral, which has the form $\int_0^\infty dP\cdots=\int_0^\infty dPe^{iPr_z}e^{-P|\bar{r}|}\sqrt{P}\Phi(P)$, it is clear that the effective DHVA oscillation frequency must be taken as $\{(2\pi\zeta n/\hbar\omega_c)[(2p_F r)^{-1}$ or $(2p_F r)^{-2}]\}$. The function $\Phi(P)$ does not involve r, and it can be expanded in a power series about $P=0$ with a finite radius of convergence: Such a power series in P^n generates an asymptotic series appropriate for large r within the context of the P-integral, and the leading term is given by $\int_0^\infty dP\cdots=\sqrt{\pi/2}(2p_F)^{-2}(|\bar{r}|-ir_z)^{-3/2}+0(r^{-5/2})$. Thus the DHVA oscillatory terms are "washed out", because the asymptotic expansion in powers of $(1/2p_F r)$ corresponds in part to an expansion in powers of the effective DHVA oscillation frequency $\{(2\pi\zeta n/\hbar\omega_c)[(2p_F r)^{-1,2}]\}$. For this reason DHVA oscillatory terms cannot occur in conjunction with the Friedel-Kohn "wiggle".

In conclusion, we see that in the case of intermediate field strength, ($\hbar\omega_c\ll\zeta$, $\hbar\omega_c\beta\gg1$), the ordinary zero-field long-range Friedel-Kohn "wiggle" is still the principal high wavenumber static shielding phenomenon, and corrections to it involving the DHVA oscillation frequency are short range terms in which anisotropic Friedel-Kohn "wiggle" behavior is discernable. The corrections involving the DHVA oscillation frequency have an effective DHVA oscillation frequency given by $\{(2\pi\zeta n/\hbar\omega_c)[(2p_F r)^{-1,2}]\}$, and therefore DHVA oscillatory terms are "washed out" in asymptotic considerations for the shielding law at large distances.

In the quantum strong field limit, $\hbar\omega_c\gtrsim\zeta$ and $\hbar\omega_c\beta\gg1$, the Friedel-Kohn "wiggle" itself is profoundly modified, (11). A "wiggle" is still discernible through the r_z-dependent oscillatory factor $\sin(2p_F r_z-\varphi)$, with an angle-dependent phase; and the long-range character of the "wiggle" is destroyed by an $\bar{r}$-dependent decaying exponential envelope factor $e^{-2p_F|\bar{r}|}$. (There is also a relatively slowly decaying r_z-dependent exponential envelope factor). This result has been derived subject to the restriction $p_D/p_F=\hbar\omega_p/2\zeta\ll1$. Since $\hbar\omega_c/\zeta>1$, one requires experimental conditions such that $\omega_c\gg\omega_p$ in order to observe the forementioned change in character of the Friedel-Kohn "wiggle". Such experimental conditions are realizable in some semiconducting materials, InSb for example.

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References

1) A fuller exposition of the calculations discussed here will soon be transmitted to the Royal Society (London) for publication in either the Proceedings or the Transactions. The theoretical background of this work is established in two earlier papers, (a) N. J. Horing: Ann. Phys. **31** (1965) 1; (b) N. J. Horing: Phys. Rev. **136** (1964) A 494, and the notation of these references will be maintained here. We briefly review the notation: $\omega_p=(4\pi e^2\cdot\rho/m)^{1/2}$; ρ=density; m= effective mass; m_0=ordinary mass; ω_c=cyclotron frequency=eH/mc; g=anomalous g-factor; $\boldsymbol{p}$= wave vector=$(p_z, \bar{p})$; $\boldsymbol{r}$=position vector from impurity=$(r_z, \bar{r})$; magnetic field=$\boldsymbol{H}||z$-axis; β= $(kT)^{-1}$; k=Boltzmann constant; T=absolute temperature; ζ=chemical potential=Fermi energy E_F in degenerate limit; $f_0(\omega)$=Fermi-Dirac distribution function=$[1+e^{(\omega-\zeta)\beta}]^{-1}$; p_F= Fermi wave number=$(2m\zeta)^{1/2}/\hbar$, p_D=Debye wave number=$(4\pi e^2\partial\rho/\partial\zeta)^{1/2}$.
2) V. L. Bonch-Bruevich and A. G. Mironov: Soviet Physics-Solid State **2** (1960) 454 ff., eqs. (32), (33) and (36).
3) M. J. Lighthill: *Introduction to Fourier Analysis and Generalized Functions* (Cambridge University Press 1958).
4) J. S. Langer and S. H. Vosko: J. Phys. Chem. Solids **12** (1959) 196.
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COMMENT BY THE AUTHOR

The behaviour of the high wavenumber ($p \sim 2p_F$) Friedel-Kohn "wiggle" contribution to shielding in the case of intermediate field strength is actually more complicated than I have described here. The monotonic magnetic field corrections as well as the DHVA magnetic field corrections are fully analyzed and discussed in detail in my contribution to the Proceedings of the International Symposium on Lattice Defects in Semiconductors, Tokyo, 1966, to be published soon.