PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

XIX-1. Observation of Quantum, Spin, and High Field Damping Effects in Microwave Helicon Propagation in Degenerate Semiconductor Plasmas

J. K. FURDYNA*

*National Magnet Laboratory,** Massachusetts Institute of Technology Cambridge, Massachusetts, U.S.A.*

Microwave helicon transmission is investigated in *n*-type InSb and InAs in the quantum region. Experiments are performed at 4.2°K on highly degenerate samples in fields up to 150 kG using a 35 Gc microwave interference technique. Quantum oscillations of the helicon amplitude are observed, and the effects of spin-splitting are resolved. Slight quantum oscillations are also observed in the helicon dispersion. Helicon damping seen beyond the oscillatory range is related to the behavior of the diagonal conductivity in the presence of ionized impurity scattering in strongly degenerate plasmas.

§ 1. Theoretical

The problem of helicon wave propagation is conveniently formulated in terms of the conductivity tensor. For propagation along an external magnetic field B (taken in the z-direction) in materials with spherical constant energy surfaces, we have[1)]

$$\alpha=(\mu_0\omega\sigma'_{xy})^{1/2}\ , \qquad (1)$$

$$\beta=\tfrac{1}{2}\alpha\frac{\sigma'_{xx}}{\sigma'_{xy}}\ , \qquad (2)$$

where α and β are the dispersive and absorptive parts of the propagation vector, μ_0 is the permeability of free space, ω the frequency, and the single prime indicates the real part of the complex components σ_{ij} of the conductivity tensor.

It was pointed out in a recent publication[1)] that in the parameter range corresponding to the helicon limit, the σ'_{ij} components in eqs. (1) and (2) can be, as a good approximation, replaced by their *dc* counterparts. This formulation is particularly useful in cases for which the explicit general high frequency model is not known or is too cumbersome for direct application, as in the case of degenerate quantum electron gas involving collisions. The above approximation will be used throughout this paper, and the prime on σ_{ij} will henceforth be dropped.

It is well known from the *dc* theory that in strong magnetic fields σ_{xy} is given by ne/B, and is independent of quantum contributions as well as of the form of the distribution function.[2)] On the other hand σ_{xx}, with its explicit dependence on carrier scattering, will be drastically affected by both degeneracy and quantization. In the case at hand, *i.e.* in the strongly degenerate quantum region in InSb and InAs, helicon damping will then display an oscillatory character. The condition for maxima in σ_{xx}, which correspond to minima in helicon transmission, can be obtained on the basis of the theory for the spin-dependent Shubnikov-de Haas effect[3)] by eqs. (6) and (7) of ref. 1).

§ 2. Experimental

Helicon wave transmission in *n*-type InSb and InAs in the degenerate quantum region was investigated at 35 Gc. A series of samples with concentration n ranging from 10^{16} to $10^{18}\,\mathrm{cm}^{-3}$ was studied. Experiments were performed in fields up to 150 kG, *i.e.* extending well into the extreme quantum limit for the lower concentrations. The Rayleigh-type microwave interferometer technique, described earlier,[4)] was adapted to measurements at liquid helium temperatures.

a) Microwave Shubnikov-de Haas effect

Figure 1 shows typical *X-Y* recorder data of a set of Rayleigh interferograms, taken at 4 phase settings of the interferometer bridge, as indicated. The period of the patterns yields α (and thus n), and the envelope displays the behavior of β (and thus σ_{xx}) as a function of the field.[4)] The dashed lines indicate the inter-

* Present address: Department of Physics, Purdue University, Lafayette, Indiana, U. S. A.

** Supported by the U. S. Air Force Office of Scientific Research.

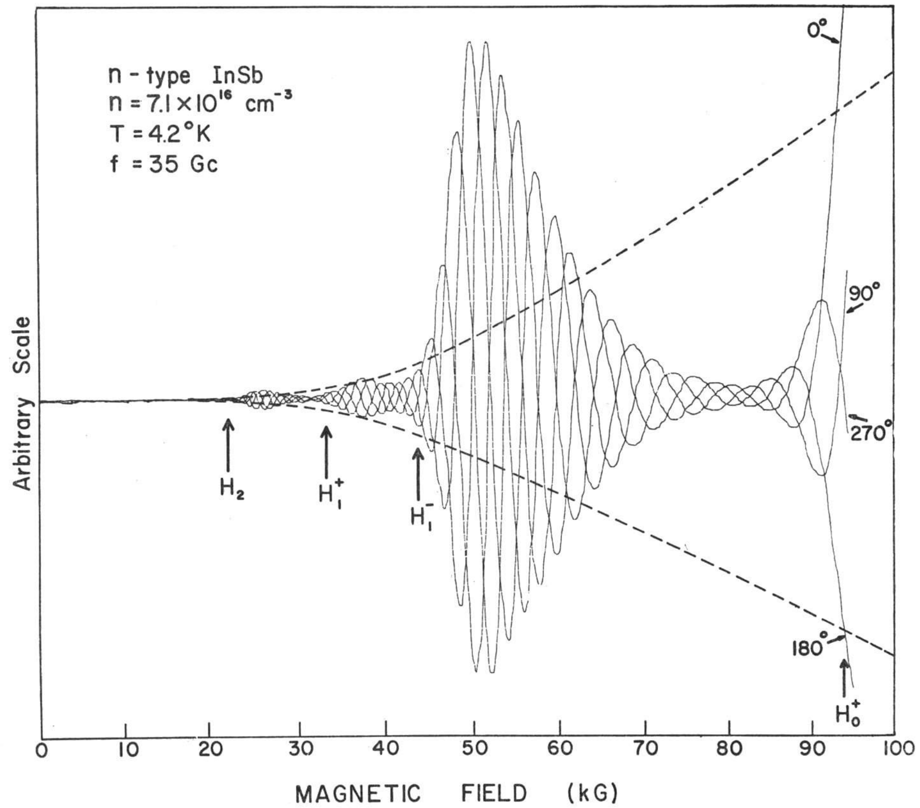

Fig. 1. *X-Y* recorder data showing Rayleigh interference patterns obtained at 4 phase shifter settings 90° apart as a function of magnetic field. Sample thickness is 4.0 mm. The dashed lines indicate the envelope of helicon transmission obtained when the sample was warmed up to 77°K. The arrows indicate theoretical positions for transmission minima.

ferogram envelope obtained under identical conditions at 77°K. The fields satisfying eqs. (6) and (7) of ref. 1) for the concentration 7.1×10^{16} cm^{-3}, obtained from the interference period, are indicated by the arrows. Note the spin splitting of the $N=1$ level. Note also that the last transmission minimum occurs at a field appreciably lower than the corresponding theoretical value. It was recently pointed out[5] that there exist several mechanisms, notably carrier collisions and a possible departure of n from a constant value in the vicinity of the last singularity, which will tend to lower the value of B_0^+. It is important at this point to indicate the distinction between the helicon data and the *dc* galvanomagnetic measurement. It is easily seen that the helicon extinction coefficient varies as $\sigma_{xx}\sigma_{xy}^{-1/2}=\sigma_{xx}B^{1/2}/(ne)^{1/2}$, while transverse *dc* magnetoresistance equals $\sigma_{xx}\sigma_{xy}^{-2}=\sigma_{xx}B^2/n^2e^2$. Thus, in the case of broadening of the σ_{xx} extrema (as by collisions) the helicon β is expected to yield an amplitude minimum closer to the field corresponding to the actual singularity. On the other hand, it must be born in mind that the helicon amplitude varies as $B^{1/2}\exp(-\beta z)$, and the presence of the $B^{1/2}$ term provides additional background tending to shift the minima to lower fields. The latter effect becomes relatively insignificant when thick samples are involved.

The advantage of taking data with multiple phase settings lies not only in a precise definition of the envelope (as illustrated in Fig. 1), but in addition offers closely spaced crossover points which permit one to study the details of dispersion variation as a function of magnetic field, e.g. within the span of a single quantum oscillation. Dispersion data obtained in this manner reveal a small but systematically reproducible oscillation in α as a function of the field. This oscillation, superimposed on the $B^{-1/2}$ variation, is typically of the order of 1% or less in the magnetic field region below B_1, but rises to about 5% in the vicinity of B_0^+. The behavior corresponds closely to that observed for the *dc*

Hall effect.[5] It can be shown that in a more exact form eq. (1) will contain a factor $(1+\frac{1}{4}\sigma_{xx}^2/\sigma_{xy}^2)^{1/2}$, which makes a small oscillatory contribution to the helicon dispersion.[6] However, the phase of the oscillation in α with respect to the β-oscillation, as well as its magnitude, show that the above correction is insufficient to explain the observed results. As suggested in connection with the *dc* Hall effect data,[5] it is possible that the total carrier density fluctuates somewhat near B_0^+ and is thus responsible for the observed effects in helicon dispersion. An alternate mechanism which can affect the phase relationship between the respective quantum oscillations in σ_{xx} and σ_{xy} was recently discussed by Antcliffe and Stradling,[7] also in connection with *dc* measurements. Because of their complimentary nature, it should be interesting to compare the *dc* and the helicon data in detail.

Figure 2 summarizes the helicon amplitude measurements obtained on *n*-type InSb crystals. The theoretical lines representing $B_N\pm$ vs. n were obtained through eqs. (6) and (7) of ref. 1). Non-parabolicity of the conduction band was taken into account in the *g*-factor[8] and the effective mass[9] involved in the calculation. The

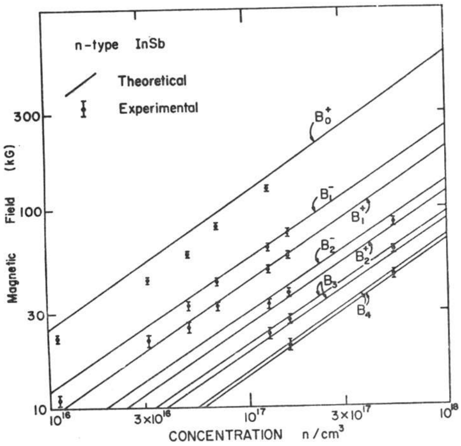

Fig. 2. Summary of InSb absorption data. The lines represent fields corresponding to singularities in the density of states calculated using the Gurevich-Efros expressions as a function of carrier concentration. Concentration for the experimental points was obtained from the corresponding helicon dispersion data. Note the systematic departure of theory and experiment in the case of $B_0{}^+$. Experimental data for other extrema are also slightly shifted to lower fields, though much less appreciably than in the $B_0{}^+$ case.

experimental points indicate positions of the observed helicon transmission minima. The concentration for each sample was determined from corresponding helicon dispersion data. Spin splitting of the $N=1$ level is unambiguously resolved in all samples with concentration above $5\times10^{16}\,\mathrm{cm}^{-3}$. Significant broadening of the minima corresponding to higher levels was also observed. The observed position of the $B_0{}^+$ minimum occurs consistently at lower fields than the corresponding calculated value for all samples studied. A slight but systematic shift to lower fields is also present in the minima corresponding to higher Landau levels.

Absorption data obtained on the InAs samples at 4.2°K are summarized in Fig. 3. Theoretical lines were calculated using $m^*=0.024\,m_0$ and $g=-16$ in eqs. (6) and (7) of ref. 1). The shift of the experimental points to lower fields is somewhat more pronounced than in the InSb case. No spin splitting of the $N=1$ level is resolved, although the presence of the $B_0{}^+$ extremum indicates, of course, the splitting of the $N=0$ level.

b) *Extreme quantum limit*

Figure 4 shows helicon transmission data for InSb with relatively low doping. The theoretical fields for the σ_{xx} maxima are indicated by the arrows. The envelope was obtained by continuous phase-shifter variation as the field was swept.[4] The small "dents" in the envelope

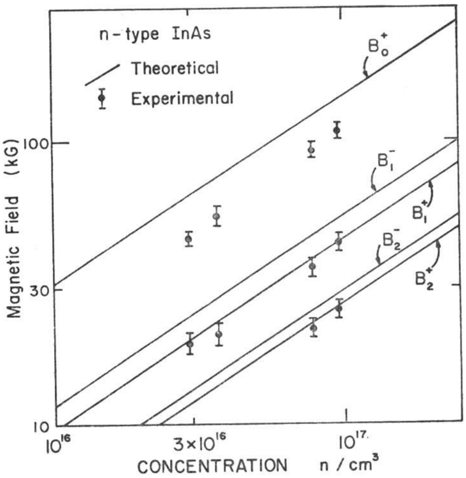

Fig. 3. Summary of InAs absorption data. The lines are calculated using the Gurevich-Efros expressions with $m^*=0.024\,m_0$ and $g=-16$. The shift of the experimental minima to lower fields with respect to the theoretical values is somewhat more pronounced than in the InSb case.

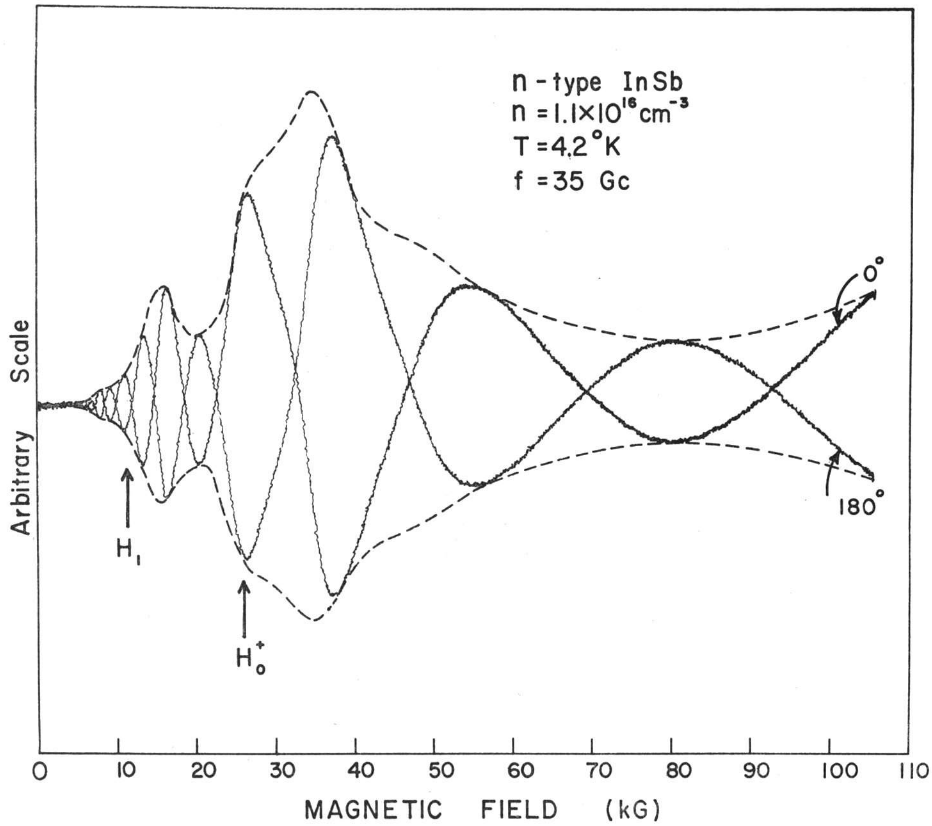

Fig. 4. *X-Y* recorder data showing Rayleigh interference patterns for two phase positions. Sample thickness was 1.6 mm. Theoretical fields corresponding to σ_{xx} maxima are shown by the arrows. The dashed lines indicate the envelope of the interference pattern obtained on the basis of additional data. The "dents" in the envelope near maximum transmission are caused by multiple reflection effects. Note the damping out of the waves at high fields, and the final tendency to increasing amplitude above 80 kG.

arise due to dimensional resonances detectable in the vicinity of the large transmission peak. Note the damping out of the helicon amplitude beyond the last quantum oscillation. This is consistent with the behavior of σ_{xx} in the extreme quantum limit in the presence of ionized impurity scattering.[2] The increased damping of helicon transmission in this field limit was reported in ref. 1), and independently in megacycle range experiments by Libchaber,[10] who observed a gradual decrease in the Q of the dimensional resonances as a function of increasing B. The interesting feature illustrated by the data in Fig. 4 is the tendency of the transmission to increase again at highest fields (beyond about 80 kG). This effect is clearly observable because of the relatively high transmission of the present sample (1.6 mm thick). The mechanism for this behavior is not clear. It should be born in mind in this connection that the degree of degeneracy for this concentration is relatively weak, and may be further modified by the presence of the field, while the high-field increase in σ_{xx} is characteristic of strongly degenerate media. Our measurements on a sample with $n=2.5\times10^{15}\,\mathrm{cm}^{-3}$ at 4.2°K indicate in fact a rapidly increasing transmission in the extreme quantum limit. It should also be remembered that the transmitted helicon amplitude is actually proportional to[4] $B^{1/2}\exp(-\beta z)$, and the continuously rising background due to the $B^{1/2}$ factor competes with the effect of increasing σ_{xx} in β. Further experiments involving higher fields, as well as measurements as a function of varying thickness, should resolve this interesting point.

Acknowledgements

The author is grateful to Drs. J. Zak, R. W. Arndt, L. G. Rubin and W. Zawadzki for suggestions and stimulating discussions; to Dr. A. J. Strauss and Prof. R. J. Sladek for some of the samples used in this investigation; and to Mr. P. Coluccio for his highly skilled assistance in all phases of the experimental work.

References

1) J. K. Furdyna: Phys. Rev. Letters **16** (1966) 646.
2) E. N. Adams and T. D. Holstein: J. Phys. Chem. Solids **10** (1959) 254.
3) L. E. Gurevich and A. L. Efros: Zh. eksper. teor. Fiz. **43** (1962) 561. translation: Soviet Physics-JETP **16** (1963) 402.
4) J. K. Furdyna: Rev. sci. Instrum. **37** (1966) 462.
5) S. T. Pavlov, R. V. Parfen'ev, Yu. A. Firsov, and S. S. Shalyt: Zh. eksper. teor. Fiz. **48** (1965) 1565. translation: Soviet Physics-JETP **21** (1965) 1049.
6) J. K. Furdyna: unpublished.
7) G. A. Antcliffe and R. A. Stradling: Phys. Letters **20** (1966) 119.
8) W. Zawadzki: Phys. Letters **4** (1963) 190; Phys. Status solidi **3** (1963) 1421.
9) J. Kolodziejczak: Acta phys. Polon. **20** (1961) 289.
10) A. Libchaber: *Doctorat d' Etat* (University of Paris, 1965).

DISCUSSION

Nagata, S.: What do you think about the phase difference between the σ_{xx} maxima and transmission minima of helicon wave?

Furdyna, J. K.: The helicon transmission minima should occur very close to the σ_{xx} maxima. They should in fact occur closer than *dc* magnetoresistance maxima. The reason for this is that *dc* magnetoresistance varies essentially as $\sigma_{xx}B^2/n^2e^2$, while helicon attenuation coefficient varies like $\sigma_{xx}B^{1/2}/(ne)^{1/2}$. Thus in the presence of broadening of the σ_{xx} singularity (as by collisions), helicon transmission minimum will correspond much closer to the σ_{xx} maximum than will the magnetoresistance maximum, because of the B^2 term in the latter.

Neuringer, L. J.: The theoretical expressions which you have used to calculate the positions, with respect to magnetic field, of the σ_{xx} maxima were derived under the assumption of a parabolic conduction band, which is not the case.

Furdyna, J. K.: We have in fact taken non-parabolicity into account in the expressions to which you refer, by using values of the effective mass and the *g*-factor appropriate to each concentration. It turns out that, because the above quantities appear as a product m^*g, non-parabolicity introduces only a very minor correction to fields calculated on the assumption of a parabolic band.

Neuringer, L. J.: We have compared our *dc* magnetoresistance data with the recent theory of Pidgeon and Brown, which includes the effect of non-parabolicity. We find no systematic deviation of the B_0^+ data from the theoretical values as you do.

Furdyna, J. K.: The value of B_0^+ will depend strongly on the variation of the Fermi level with magnetic field. This is of course not included in the theory of Pidgeon and Brown, and consequently their theory cannot be used self-consistently to determine B_0^+. It may of course give somewhat better values for fields corresponding to higher quantum numbers.

Mitchell, D. L.: I would like to suggest a possible physical origin for the small and unexplained effects you observe in the real part of the propagation through $\sigma_{xy}^{(\prime)}$. If band-population effects are included, then there can be quite large interband contributions to $\sigma_{xy}^{(\prime)}$ for materials with large spin-orbit interactions. These contributions, which have been calculated for PbS, have de Haas-van Alphen oscillatory as well as non-oscillatory terms. The calculation is being extended to InSb and InAs so that a direct check should be possible in the near future.

Furdyna, J. K.: Although in principle the mechanism which you suggest may contribute to the total dispersion, I suspect that this contribution will be exceedingly small compared with the free carrier term. The free carrier term in the dispersion does itself contain higher order oscillatory terms in the *dc* Hall effect, and I suspect that these terms are primarily responsible for the small dispersion oscillations.