PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON THE PHYSICS OF SEMICONDUCTORS, KYOTO, 1966
JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 21, SUPPLEMENT, 1966

XIX-10. Magneto-Acoustic Resonances in Antimony

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Spike-like sharp absorption peaks have been observed in an ultrasonic attenuation coefficient of antimony in a weak magnetic field range. This type of resonance is not caused by the electron in an extremal orbit as has been observed in many cases of geometric resonance, but it is probably due to the particular kind of selective coupling predicted by Kaner *et al.* On applying the Kaner's formula, we can get the cyclotron mass which is in agreement with the value derived from the Shoenberg's results within 13 per cent.

§ 1. Introduction

Magneto-acoustic geometric resonances are well understood both experimentally and theoretically[1] for the case of transverse propagation of ultrasonic waves. Such resonance, however, for another geometrical configuration of the magnetic field and the ultrasonic wave has not been clarified completely. Magneto-acoustic resonance in this case was investigated by several authors[2-7] and it was proposed by Kaner *et al.*,[2] Kaner,[3] and by Daniel and Mackinnon[4] that the resonant oscillation of ultrasonic attenuation should occur when the electrons in the nonextremal orbit drifted in the direction of sound propagation. Eckstein *et al.*[8] reported the magneto-acoustic geometric resonances in antimony for an arbitrary direction of the magnetic field. They did not observe, however, the resonant oscillation with spike-like absorption peaks, but only the sinusoidal attenuation curves of a nonresonant character. This peculiar type of resonant oscillations in antimony was reported in our preliminary communication.[9] In this paper the cause of the resonant oscillations are discussed in terms of the Kaner's formula.

§ 2. Experimental Procedure

A specimen with the resistivity ratio of $\rho_{300°K}/\rho_{4.2°K}=2500$ was cut with a Servomet spark machine in the dimension of 8.73 mm$\times$10.35 mm $\times$9.21 mm, each side being parallel to binary, bisectrix and trigonal axis, respectively. The mean free path was estimated to be about 0.5 mm at least from the number of oscillations observed. The ultrasonic measurements were made by a usual pulse technique using 20–180 Mc/sec longitudinal sound waves at the temperatures between 1.2°K and 4.2°K. The echo signals were led to a logarithmic amplifier through a selective gate and were registered on an *X-Y* recorder. A sound wave was transmitted along the trigonal axis of the antimony crystal. The magnetic field was rotated in the trigonal-bisectrix plane. The angle between the direction of the propagation vector of sound wave $\boldsymbol{q}$ and the direction of the magnetic field $\boldsymbol{H}$ was determined exactly from the absorption anisotropy similar to the fork effect found in bismuth. The angle θ between $\boldsymbol{q}$ and $\boldsymbol{H}$ was measured from the trigonal axis.

§ 3. Experimental Results

A recorder trace of the oscillation pattern for 180 Mc/sec sound wave at 1.2°K is shown in Fig. 1.

Two different kinds of oscillations were observed: spike-like oscillations and sinusoidal oscillations. The characteristic features of the pattern are as follows.

1) The absorption peaks took the spike-like character similar to that in giant quantum oscillations, when the field made the angles 20°$\pm$5°, 45°$\pm$20°, and $-60°\pm25°$ with respect to the trigonal axis in the trigonal-bisectrix plane. The energy corresponding to the half width of the peak was nearly equal to $kT/100$ at the most favorable direction of the magnetic field. The half width of these peaks became the narrower for the lower temperatures and/or the higher frequencies. In Fig. 2 the half width is plotted against temperatures.

2) For some directions of the magnetic field, these peaks split into two, and for some directions these split into four.

The reciprocal of the period of these oscillations was proportional to the ultrasonic frequency. If these spikes are numbered by integers n, the

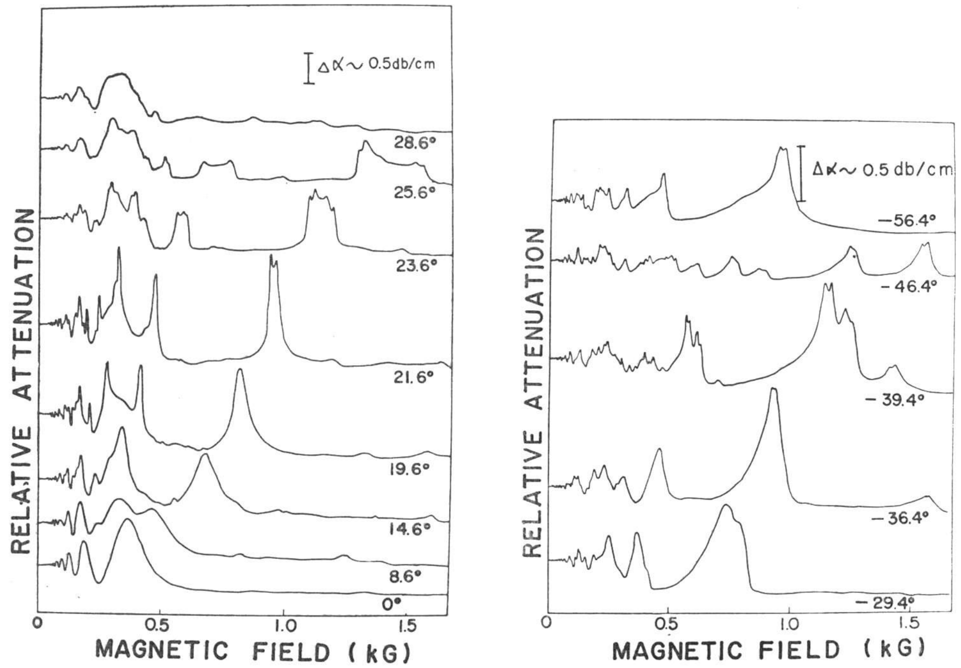

Fig. 1. Recorder trace of the longitudinal sound absorption of antimony at 1.2°K vs. magnetic field intensity. Sound wave frequency is 180 Mc/sec. $\boldsymbol{q}$ along the trigonal axis, $\boldsymbol{H}$ is rotated in the trigonal-bisectrix plane.

value of $1/H$ at each peak was proportional to n.

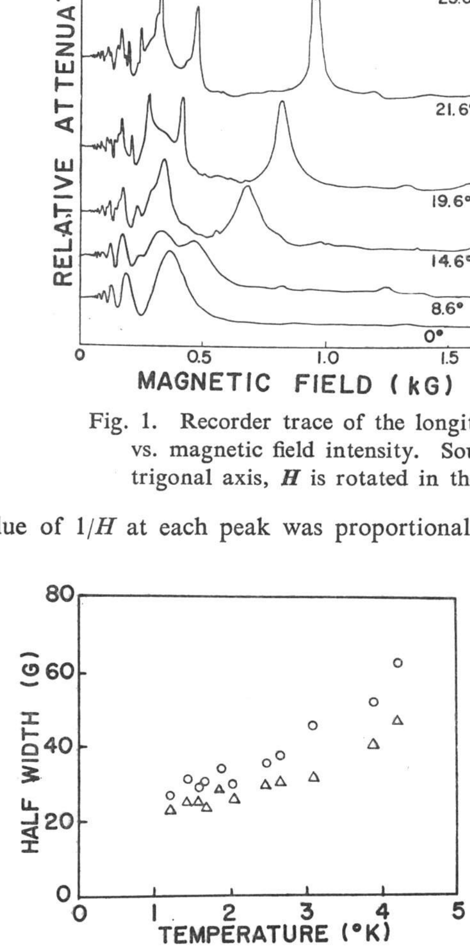

Fig. 2. Temperature variation of the half width of the absorption peaks at 180 Mc/sec, q along the trigonal axis, $\theta=21.6°$ from the trigonal axis. ○ denotes the first peak at 915G and △ the second peak at 720G.

The period $\varDelta(1/H)$ as a function of angle θ of the magnetic field in the trigonal-bisectrix plane is shown in Fig. 3. The points are experimental results, and the solid curve is drawn by the calculation discussed later.

§ 4. Discussion

The next two interpretations, (A) and (B), were considered for these oscillations.

(A) If the oscillations are due to the electrons in an extremal orbit, the resonance will take place with the condition that the extremal dimension of the electron orbit in real space is equal to the integral multiples of the wave length. This resonance condition is identical to the geometric resonance in the case of transverse propagation. Using eq. (1) of ref. 6), the periods for oscillations of the present experimental geometry are given by

$$\varDelta_1(1/H)=\frac{e\lambda}{2c}\frac{1}{(2mE_{\mathrm{F}})^{1/2}}\left[\frac{\alpha_{11}(\alpha_{33}\sin^2\theta+\alpha_{22}\cos^2\theta-2\alpha_{23}\sin\theta\cos\theta)^2}{(\alpha_{33}\sin\theta-\alpha_{23}\cos\theta)^2}\right]^{1/2}, \tag{1a}$$

$$\varDelta_{2,3}(1/H)=\frac{e\lambda}{2c}\frac{1}{(2mE_{\mathrm{F}})^{1/2}}\left[\frac{\{(3\gamma+\alpha_{11}\alpha_{33})\sin^2\theta+4\alpha_{11}\alpha_{22}\cos^2\theta+4\alpha_{11}\alpha_{33}\sin\theta\cos\theta\}^2}{4\alpha_{11}(\alpha_{33}\sin\theta+2\alpha_{23}\cos\theta)^2+12\alpha_{33}\gamma\sin^2\theta}\right]^{1/2}, \tag{1b}$$

where $\gamma=\alpha_{22}\alpha_{33}-\alpha^2_{23}$, $-e$ is the electronic charge, c the light velocity, λ the sound wave length, m the electron mass, E_{F} the Fermi energy, α_{ij} the reciprocal effective mass tensor elements defined by $2mE_{\mathrm{F}}=\sum_{ij}\boldsymbol{P}_i\cdot\alpha_{ij}\cdot\boldsymbol{P}_j$. The quantities $\alpha_{11}\alpha_{22}$, $\alpha_{11}\alpha_{33}$ and $\alpha_{11}\alpha_{23}$ were obtained experimentally

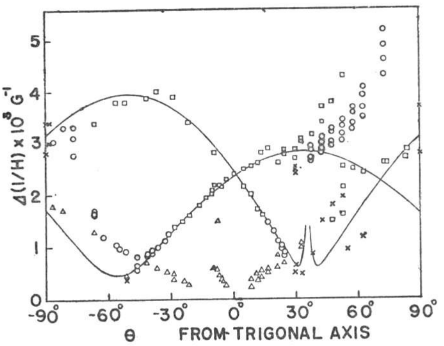

Fig. 3. Periods of oscillations vs. the angle θ between the magnetic field and the sound propagation vector $\boldsymbol{q}$. Sound frequency is 180 Mc/sec, $\boldsymbol{q}$ along trigonal axis, $\boldsymbol{H}$ is rotated in the trigonal-bisectrix plane. ○ denotes the period of large spike-like oscillation, △ small but spike-like oscillation, □ large but sinusoidal oscillation, and × the period of minor oscillation.

from the periods of the quantum oscillation of ultrasonic attenuation in the higher field region. α_{ij} were evaluated by the substitution of these values and the sound wave velocity at 4.2°K ($v_3=2.30\times10^5$ cm/sec), at $\theta=0°$ into eqs. (1a) and (1b). The numerical values thus obtained are listed in Table I. Two branches of the experimental data which were considered to be hole branches, are in fair agreement with the calculated curves. The above interpretation, however, can hardly explain the following two characteristic features of the present experiment.

1) Spike-like absorption peaks:

If these oscillations were due to the electron in the extremal orbit, the width should be independent of temperatures and the absorption curve should take a nonresonant character. The observed oscillations obviously contradict with the above presumption.

Table I. Reciprocal effective mass tensor elements.
(in units $10^{14}\,E_F=1$)

	Present experiment	Shoenberg[10] (dH-vA)	Saito[11] (dH-vA)	Eckstein *et al.*[8] (geometric resonance)
α_{11}	1.17	1.07	0.569	1.011
α_{22}	0.278	0.286	1.63	0.330
α_{33}	0.504	0.554	2.86	0.557
α_{23}	0.369	0.359	2.11	0.348

2) Splitting of the absorption peaks:

Provided that we try to explain the splitting of the peak at $H=910$ gauss and $\theta=21.6°$ in terms of the geometric resonance due to the electrons in an extremal orbit, the observed splitting implies that there exist two extremes in the orbit which are very close to each other in their geometry in momentum space. This implication does not seem to be the case according to our understanding of the band structure of antimony.

(B) The existence of sharp absorption maxima was predicted by Kaner *et al.*[1] and by Kaner[2] who claimed that the absorption was due to the electron in a nonextremal orbit. The resonance condition in this case is that the mean electron displacement during one period of motion is not equal zero, but

$$\boldsymbol{q}\cdot\boldsymbol{v}t=2\pi n\,(n=1,\,2,\,3,\,\cdots), \tag{2}$$

when $\Lambda\ll2\pi r\ll l$, where $\boldsymbol{v}$ denotes the mean electron velocity during one period, t the period of motion, r the cyclotron radius, l the mean free path and Λ equal to $1/q$. If the Fermi surface is closed, this condition is satisfied when $\boldsymbol{q}$ is not perpendicular to $\boldsymbol{H}$. According to Kaner,[2] the resonance peak may split, if $\omega\tau\gg1$, where τ denotes the relaxation time, and give two different sets of resonance peaks. The peak positions and their periods differ by $\pm\omega$, so that the period for each series of resonance is given by,

$$\Delta(1/H)_\pm=\frac{e}{cm_c\,|\,\boldsymbol{q}\cdot\boldsymbol{v}\pm\omega\,|}, \tag{3}$$

where m_c denotes the cyclotron mass at the limiting orbit which has a extreme value of $\boldsymbol{q}\cdot\boldsymbol{v}$ on the Fermi surface, and ω the angular frequency of sound wave. From the splitting of the resonances it is possible to determine directly m_c and $\boldsymbol{q}\cdot\boldsymbol{v}$ at the limiting orbit for a given orientation of the $\boldsymbol{q}$ and $\boldsymbol{H}$.

The splitting of the peak at 910G of attenuation curve at $\theta=21.6°$ is 18.8G. On substituting this value to eq. (3) one obtains $m_c/m_0=0.147$. This value is in reasonable agreement with that of 0.166, which is derived from the Shoenberg's mass tensor.[10] $\boldsymbol{v}\cdot\boldsymbol{q}/q$ is found to be 2.21×10^7 cm/sec at the same time.

From these observations it seems that the spike-like absorption peaks observed here are not due to the electron in an extremal orbit like the one observed by Eckstein *et al.*,[6] but due to a non-extremal orbit as predicted by Kaner *et al.*[2]. The reason is not clear why such resonance occurs only in a certain range of the field orientation, and why one spike-like peak splits into four subsidiary peaks only for a particular range of the field orientation. The possibility of the contribution from the electrons of a small effective mass would be ruled out on account of our present knowledge on the band structure. The reason is now under investigation.

Acknowledgements

The authors wish to thank Dr. Y. Muto, Dr. S. Mase and Dr. Y. Nishina for many helpful discussions, and Dr. K. Tanaka and Mr. M. Hongo for preparing the single crystal of antimony. Thanks are also due to Prof. S. Tanuma for helpful guidance in using the spark machine for preparing the specimen.

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COMMENT BY THE AUTHORS

It is suggested that some portions, at least, in the Fermi surface of antimony have a negative curvature, which perturbs an ideal set of ellipsoidal surfaces. The reasons are the following:

1) Orientation dependence of the cyclotron mass derived from the present experiment showed some deviation from the lines which were drawn with the assumption of the ellipsoidal Fermi surface.
2) Spike like oscillation occurred only in the certain range of the field orientation, and one peak split into four subsidiary peaks for a particular range of the field orientation.
3) The line shape of our absorption peaks could be explained fairly well by the formula for an extreme orbit in ref. 3) which had the extreme value of $\boldsymbol{vq}$.