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# XIX-11. Magneto-Acoustic Effects in $n$-InSb at 9 GHz

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The propagation of 9 GHz longitudinal ultrasonic waves has been studied in $n$-InSb at temperatures between 4.2°K and 50°K and in magnetic fields up to 25 kG for several crystal orientations and electron concentrations, with particular emphasis on the electronic contribution to the attenuation and velocity of the ultrasonic wave. These measurements have yielded a value of $0.06\pm0.005\ C/m^2$ for the magnitude of the piezoelectric constant $e_{14}$ and $4.5\pm0.5$ eV for the magnitude of the conduction band deformation potential $C_1$ for InSb. The experiments also verify several aspects of the theory of ultrasonic propagation in semiconductors for $ql>1$.

## § 1. Theory

The effect of conduction electrons on the propagation of longitudinal ultrasonic waves may be described in terms of the total longitudinal dielectric constant $\varepsilon_l=\kappa\varepsilon_0-\sigma(\omega, q, B)/i\omega$, which includes the lattice dielectric constant $\kappa\varepsilon_0$ and the electron polarizability $-\sigma/i\omega$. The change in the attenuation $\Delta\alpha$ and the (normalized) change in velocity $\Delta v_s/v_s$ due to a magnetic field are

$$\Delta\alpha=-K^2 q\,\mathrm{Im}\left[\frac{\kappa\varepsilon_0}{\varepsilon_l(B)}-\frac{\kappa\varepsilon_0}{\varepsilon_l(0)}\right];\qquad \Delta v_s/v_s=\frac{K^2}{2}\mathrm{Re}\left[\frac{\kappa\varepsilon_0}{\varepsilon_l(B)}-\frac{\kappa\varepsilon_0}{\varepsilon_l(0)}\right]. \tag{1}$$

The coupling constant $K^2=K_p^2+K_d^2$ where

$$K_p^2=\frac{e_p^2}{\kappa\varepsilon_0 c};\qquad K_d^2=\frac{q^2\kappa\varepsilon_0}{c}\left(\frac{C_1}{e}\right)^2. \tag{2}$$

Here $q$ is the ultrasonic wavevector, $c$ is the elastic constant and $e_p$ the piezoelectric constant appropriate to the wave ($e_p=2e_{14}/\sqrt{3}$ for $q\|\langle111\rangle$). These expressions are considerably simpler than those obtained for metals[1] since it is an excellent approximation to consider only the longitudinal component of the self-consistent electric field and hence only the longitudinal component of the conductivity tensor.[2] It has been necessary to extend previous quantum density-matrix calculations[2] of $\sigma$ to include the case of nondegenerate statistics with collisions. The collisions were treated phenomenologically by the introduction of a constant (velocity independent) relaxation time $\tau$, and the quantum mechanical Einstein relation was used to combine the diffusion and conduction components of the current into the effective conductivity $\sigma$. (Note that the complications in introducing $\tau$ that are discussed by Tosima *et al.*[3] do not arise when the $E$ field is longitudinal.) For nondegenerate statistics the resulting expression for $\sigma$ in terms of the carrier density $n$, effective mass $m$, sound velocity $v_s$, and temperature $T$ is[4]

$$\sigma(\omega, q, B)=\frac{2ne^2}{m\omega}\left(\frac{v_s}{v_0}\right)^2(1-i\omega\tau)\left[\frac{x}{1+ix}\right], \tag{3}$$

where

$$x=\frac{4}{\beta\omega\tau}\times\sum_{p=0}^{\infty}\frac{\{\exp(-\xi\coth\beta/2)\}p\sinh(p\beta/2)I_p(\xi\,\mathrm{csch}\,\beta/2)}{p^2+[(1-i\omega\tau)/\omega_c\tau]^2} \tag{4}$$

with $\omega_c=eB/m$, $\beta=\hbar\omega_c/kT$, $\xi=\hbar q^2/2m\omega_c$, $v_0=\sqrt{2kT/m}$, and $I_p$ is the hyperbolic Bessel function. The quantum treatment has been necessary since electron recoil is quite significant. Although this effect has been negligible in previous ultrasonic experiments, here it is so large that the electron-phonon interaction should be viewed as Bragg scattering rather than the usual picture of a spatial resonance or Landau damping.[5] The electrons responsible for the damping travel against the wave and backscatter on absorption of a phonon. Also in contrast to the usual Boltzmann equation treatment, the quantum calculation including recoil predicts

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that the attenuation for $B=0$ is collision independent only for $\omega\tau\gg1$ instead of the more easily obtainable condition of $ql=(v_0/v_s)\omega\tau\gg1$, where $l=v_0\tau$ is the electron mean free path.

## § 2. Experimental Results

Samples were prepared in the shape of rods about 8 mm in length and 3 mm in diameter from high-purity undoped ingots of *n*-InSb supplied by the Monsanto Chemical Company. Hall measurements were used to determine the *dc* mobility and carrier concentrations between 4.2°K and 77°K. The samples used had a *dc* mobility greater than $5\times10^5$ and $2\times10^5$ cm$^2$/V sec at 77°K and 4.2°K respectively. The measured carrier concentration was $1.75\times10^{14}$ cm$^{-3}$ and was magnetic field dependent above 2kG at the lowest temperature. Data were taken at 19.7°K in order to avoid this magnetic "freezeout" as well as the complications of partially degenerate electron statistics. Higher temperatures were avoided because of the sharp increase in the attenuation due to phonon-phonon scattering, which shows a $T^{5.8}$ dependence below 35°K with 3 *dB*/cm occurring at about 25°K. The data were taken using a sensitive microwave receiver in an interferometer circuit[6] which could measure a velocity change of about 1 part in $10^7$ and a total insertion loss (transducer and path attenuation) as high as 170 *dB*. Transducers were thin films of CdS ranging from 1 to 2 microns in thickness which were deposited on the samples using an electron bombardment evaporation technique.[7] There was no attempt to produce resonant thicknesses of the films.

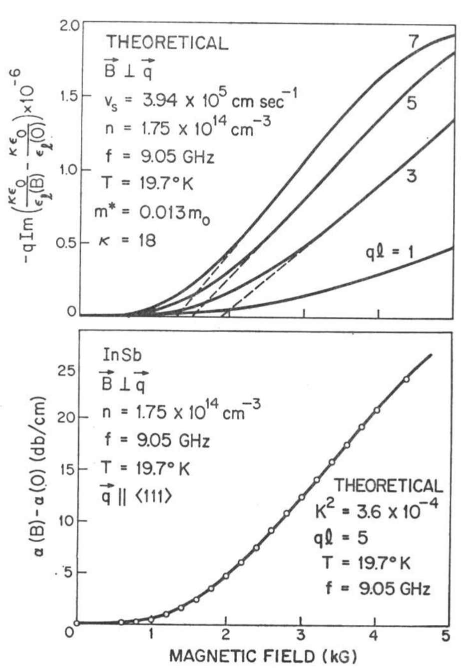


Fig. 1. Comparison of theory with experimental attenuation data on ⟨111⟩ sample.

Measurement of the magnetic field dependence of the acoustic attenuation in different crystallographic directions has allowed the determination of $C_1$ and $e_{14}$ independently since ⟨111⟩ longitudinal waves are piezoelectrically coupled to the electrons whereas ⟨110⟩ and ⟨100⟩ longitudinal waves are coupled via the deformation potential only. In Fig. 1 the result of machine computations of the change in attenuation $\varDelta\alpha$ from eq. (1) is given for several values of *ql*, normalized to the coupling constant $K$.[2] The shape, and, in particular, the low field intercept, determines *ql*; $K^2$ is then determined by scaling. The value of *ql* obtained by curve fitting to the experimental data (below) is 5. This corresponds to a $\tau$ about 15% lower than that obtained from the *dc* Hall measurements of the mobility. The value of $K^2=3.6\times10^{-4}$ determined by scaling is rather insensitive to the choice of *ql*. Fitting the data at the high field (4.5 kG) point with theoretical curves for a range of *ql* of 3 to 6 produces a variation of $K^2$ of only about $\pm10\%$. The magnitude of the piezoelectric constant is found from these data to be $|e_{14}|=0.06\pm.005\ C/m^2$. This value is also consistent with the velocity change to 4 kG. Similar data for a ⟨100⟩ sample are shown in Fig. 2 from which a value is obtained for the magnitude of the conduction band deformation potential of $|C_1|=4.5\pm0.5$ eV. The deformation potential coupling accounts for about 2% of the observed attenuation in the ⟨111⟩ sample. The amount of piezoelectric coupling for a plane wave pro-

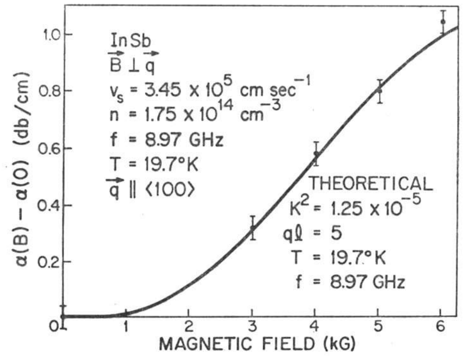


Fig. 2. Comparison of theory with experimental attenuation data on ⟨100⟩ sample.

pagating at a small angle $\theta$ to $\langle 100 \rangle$ was calculated perturbatively in order to determine the effect of sample misorientation on the measured value of $|C_1|$. This calculation indicates a $\theta^4$ dependence of the coupling about $\langle 100 \rangle$ and that $\theta \geq 10°$ would be necessary to produce the observed coupling. Since the measured sample orientation was within 2 degrees of the $\langle 100 \rangle$ direction, the piezoelectric coupling was negligible.

Velocity and attenuation data for propagation along the magnetic field at 4.2°K and 19.7°K are shown in Fig. 3 with a smooth curve drawn through the data points. The initial decrease in attenuation observed at 4.2°K for magnetic fields below 1 kG is also present in the data taken for propagation across the field at 4.2°K as shown in Fig. 4. This anomalous behavior occurs at the same fields and temperature as an observed negative magnetoresistance,[8,9] which has been ascribed to scattering by electrons localized in magnetic states around donor impurities. The high-field behavior (above about 5 kG) has not been satisfactorily explained for propagation either along or across the field. Although the assumption of a constant collision time $\tau$ appears adequate to explain the low field data used to determine $C_1$ and $e_{14}$, the generalization to a velocity-dependent $\tau$ may be necessary to account for the high field observations. Also, as noted above, the interpretation of the data for the samples at 4.2°K is complicated by the magnetic field dependence of the electron concentration.

The temperature dependence of the ultrasonic velocity and attenuation is shown in Fig. 5 for a low concentration $\langle 111 \rangle$ sample which is nondegenerate over the temperature range studied. The change in velocity relative to the maximum velocity at 9°K is shown by the solid curve. The increase in velocity between 4.2°K and 9°K is the result of the electron-phonon coupling. From measurements of the temperature dependence of the velocity in the $\langle 100 \rangle$ and $\langle 110 \rangle$ directions, the temperature dependence of the

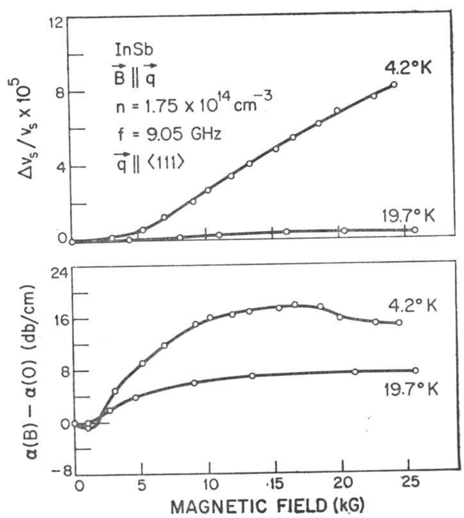


Fig. 3. Velocity and attenuation data on $\langle 111 \rangle$ sample for propagation along B.

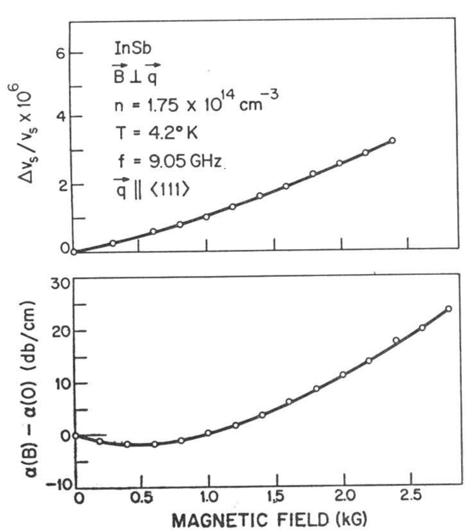


Fig. 4. Velocity and attenuation data on $\langle 111 \rangle$ sample for propagation across B.

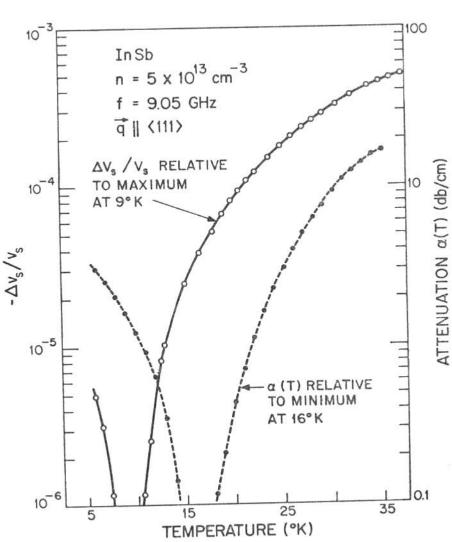


Fig. 5. Temperature dependence of the velocity relative to maximum velocity at 9°K (solid curve) and attenuation relative to minimum attenuation at 16°K (dashed curve).

velocity in the $\langle 111 \rangle$ direction due to phonon-phonon coupling alone can be deduced. This is possible since the electron-phonon coupling is quite small for $\langle 110 \rangle$ and $\langle 100 \rangle$ propagation and the elastic constant appropriate the $\langle 111 \rangle$ direction is determined by the $\langle 110 \rangle$ and $\langle 100 \rangle$ velocities. After subtracting the phonon-phonon contribution, the observed increase between 4.2°K and 9°K is in good quantitave agreement with theory using the value of $K^2$ deduced above. The dashed curve in Fig. 5 is the measured attenuation, which reaches a minimum value at about 16°K. The decrease in attenuation between 4.2°K and 16°K follows quantitatively the predictions of the theory for the electron-phonon coupling if we assume a negligible phonon-phonon attenuation below 16°K. Above 16°K the attenuation rises due to the anharmonic phonon-phonon coupling.

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**References**

1) M. H. Cohen, M. J. Harrison and W. A. Harrison: Phys. Rev. **117** (1960) 937.
2) H. N. Spector: Phys. Rev. **134** (1964) A507; *ibid.* **137** (1965), A311.
3) S. Tosima, J. J. Quinn and M. A. Lampert: Phys. Rev. **137** (1965) A883.
4) K. W. Nill: Ph. D. Thesis, (Massachusetts Institute of Technology, May 13, 1966).
5) N. Takimoto: Progr. theor. Phys. **25** (1961) 327.
6) K. W. Nill and A. L. McWhorter: 1965 Ultrasonics Symposium, Boston, Massachusetts, December 1–4, 1965.
7) R. Weber: to be published.
8) Y. Katayama and S. Tanaka: Phys. Rev. Letters **16** (1966) 129.
9) R. P. Khosla and R. J. Sladek: Phys. Rev. Letters **15** (1965) 521.

## DISCUSSION

**Herring, C.:** One of the most recent determinations of the deformation potential constant is that of Puri, who obtained it from the magnitude of the phonon-drag thermoelectric power in the temperature range where it is limited by boundary scattering. As I recall, he obtained a value around 8 eV, and it is hard for me to see why this method should not yield quite a reliable value. Do you have any comment on the difference between his value and yours?

**Nill, K. W.:** Puri obtained his value of 8.25 eV for the deformation potential from measurements on a sample of very low concentration. The agreement between his theory and experiment is poor for magnetic fields both above and below the range around 30 or 40 kilogauss which he used to determine the deformation potential. We believe that Puri's determination may be in error because of the magnetic freezeout of the carriers and the attendant change in the scattering processes which occur for such low-concentration samples at the low temperature and high magnetic field employed in his experiments. In addition, with the value of the piezoelectric constant which we obtain, Puri's neglect of the piezoelectric scattering in the interpretation of his data may not be justified. The high field behavior which he used to justify the neglect of the piezoelectric scattering is, in fact, not explained by his assumption of only deformation potential scattering.

**Lax, B.:** Did Puri take into account the effect of high magneıic field upon the scattering in determining his deformation potential?

**Herring, C.:** Puri took full accouut of the modification of the scattering due to quantization of the electrons in Landau levels.

**Gurevich, V. L.:** If the mean electron velocity is much larger than the sound velocity, the criteria $\omega\tau \gg 1$ and $ql \gg 1$ differ essentially. So in such a case a region should exist where $\omega\tau \ll 1$ and $ql \gg 1$. In such a region the quantum mechanical approach and the classical approach give different results. Will you tell me what is the difference or just say what quantum mechanical parameter is responsible for the difference?

**Nill, K. W.:** The fundamental difference between the quantum mechanical and the classical predictions in the region $ql > 1$ and $\omega\tau < 1$ is indeed the point we wish to make. In this region the quantum density-matrix treatment yields a different result from the Boltzmann equation treatment at low magnetic fields. As mentioned in the talk, the effect is due to electron recoil. The quantum mechanical parameter of significance is $\hbar q^2/2m\omega$ which is about 10 for our experiments.