JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 34, SUPPLEMENT, 1973
PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON NUCLEAR MOMENTS AND NUCLEAR STRUCTURE, 1972

I.a. What Magnetic Moments Tell Us about Nuclear Forces and Nuclear Structure

G. E. BROWN

*Nordita and State University of New York, Stony Brook, New York, USA**

§1. Introduction

For many years, the description of nuclear magnetic moments was pursued in a rather empirical way. Calculations of effects from configuration mixing were carried out using δ-function forces[1)] and various effective interactions of finite range, such as the Rosenfeld mixture. Even effects from exchange currents were calculated in an empirical way; deducing the amount of Majorana exchange in the effective interaction used, one employed the Sachs procedure,[2)] which roughly amounts to the following:

Knowing that the effect of the Majorana exchange operator is

$$P_{12}^{M}\,\phi(\boldsymbol{x}_1-\boldsymbol{x}_2)=\phi(\boldsymbol{x}_2-\boldsymbol{x}_1)\ , \tag{1}$$

one can them obtain $\phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$ back by making the Taylor series

$$\phi(\boldsymbol{x}_1-\boldsymbol{x}_2)=e^{\boldsymbol{r}_{12}\cdot\nabla}\,\phi(\boldsymbol{x}_2-\boldsymbol{x}_1)\ , \tag{1.1}$$

where $\boldsymbol{r}_{12}$ is to be treated as a parameter, not a variable. Using minimal substitution,

$$\frac{\hbar}{i}\nabla\rightarrow\frac{\hbar}{i}\nabla-\frac{e}{c}A \tag{1.2}$$

is the presence of an electromagnetic field. Keeping the terms linear in A in the Hamiltonian gives one the so-called Sachs moment. In the case of the one-pion exchange interaction, this procedure leads to identical results for δg_l, the exchange-current correction to the orbital g-factor, as does the introduction of the electromagnetic field in the Lagrangian of interacting mesons and nucleons, the latter procedure being at a higher and less empirical theoretical level.

The surprising results have been that one has achieved a relatively satisfactory description of nuclear moments by such empirical procedures. Most deviations from the Schmidt line could be explained by configuration mixing of the Arima-Horie type, and various groups, using various effective interactions, obtained results in good agreement with each other. The crucial quantities entering the calculation were the energies of admixed states. These have by now been determined quite well experimentally.

Although the case of Bi^{209} eluded the "configuration mixturers", the δg_l of $\sim 0.1\tau_3$ which follows from the one-pion exchange Sachs moment has a particularly large effect here, because of the high value of l, and is sufficient to fix up the remaining discrepancy.

Thus, one can ask, "What remains to be understood from a more basic theory of interac-

* Supported in part by the U.S. Atomic Energy Commission under Contract AT (11–1)–3001.

ting mesons and nucleons?" One can ask the same question in relation to effective interactions and nuclear spectroscopy, where G-matrix elements obtained from two-body interactions, such as the Hamada-Johnston or Reid potentials, obtained from fitting the nucleon-nucleon scattering data, seem to work well. Thus, it might seem that there is no need to inquire where, in terms of meson theory, the two-body interaction comes from.

Granted that rather empirical approaches work well, I would say that there are fundamental questions which can only be answered at a higher level.

1) Why is the M1 capture cross section in $n + p \rightarrow d + \gamma$ about 10% higher than the value calculated with one-body interaction operators?

2) Why does the magnetic moment of the triton lie outside the Schmidt line?

3) Why do we see so few effects on nuclear magnetic moments from N^*_{33}-isobars and from the tensor force which we know to be present in the one-pion-exchange potential?

4) What effects do exchange currents from the exchange of mesons heavier than the pion have on nuclear magnetic moments?

These questions are all interrelated, especially the first three. Considerable progress has been made in understanding these questions within the past year, and I plan to discuss this here.

§2. Exchange Currents Connected with One Pion Exchange

The one-pion-exchange term, aside from producing most of the tensor force, plays a relatively minor role in the nucleon-nucleon interaction. In exchange currents, it is relatively much more important, for two reasons:

1) There are large effects which are non-tensor in nature, which therefore do not tend to go out when averages over direction are carried out, as does the tensor force in the two-body interaction.

2) The most important of the mesons heavier than the π in the nucleon-nucleon force are the $\sigma(T = 0, J = 0)$ and $\omega(T = 0, J = 1)$. Since both of these are neutral, one would expect exchange current effects connected with their exchange to be small.

F. Villars[3] in 1947 wrote down the exchange operators connected with one-pion exchange, beginning from a meson theory of pseudoscalar mesons interacting with nuclei by pseudovector coupling

$$\delta L = \frac{f}{\mu}\bar{\psi}(x)\boldsymbol{\sigma} \cdot \nabla(\boldsymbol{\tau} \cdot \boldsymbol{\phi})\psi(x) \tag{2}$$

and then treating the nucleons in a static approximation. His treatment is quite adequate for our purposes. We note that the δL of eq. (2) would be called, in more modern terminology, a "Chiral Lagrangian" and that predictions from (2) are the same as predictions from PCAC.[4]

For definiteness, let us consider the exchange current corrections[5] to

$$n + p \rightarrow d + \gamma.$$

Those associated with the one pion exchange come from two processes, shown in Fig. 1.

The process, Fig. 1(b), results from the gamma ray coupling with the pion current. The process, Fig. 1(a), results from the guage correction in the Lagrangian (2). If we let

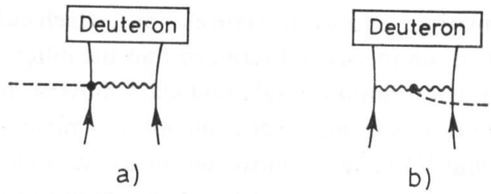

Fig. 1. Exchange currents in the process n + p→d + γ. In (a) the "seagull term" is shown; In (b), the coupling to the pion current.

$$\frac{\hbar}{i}\nabla \rightarrow \frac{\hbar}{i}\nabla - \frac{e}{c}A$$

as in eq. (1.2), then an additional term

$$\delta L' = -\frac{ief}{\hbar c\mu}\bar{\psi}(x)\,(\boldsymbol{\sigma}\cdot\boldsymbol{A})(\boldsymbol{\tau}\cdot\boldsymbol{\phi})\psi(x) \tag{3}$$

appears in the Lagrangian. This guage term turns out to be the largest term. On the other hand, it has been systematically left out in calculations for many years.

The experimental value[6] for the n + p capture cross section for thermal neutrons is 334.2 ± 0.5 mb, whereas the theoretical value calculated using the usual single-particle magnetic moment operators is 302.5 ± 4.0 mb. Thus, there is a 10% discrepancy to explain. The processes, Figs. 1(a) and 1(b), give 2/3 of this; *i.e.* they increase the theoretical value by ~ 7%. We shall return to the remaining 3% later.

Chemtob and Rho[4] found that the processes, Fig. 1, increased the isovector moment in the triton by ~ 0.19 nuclear magnetons, compared with the needed 0.4 nuclear magnetons. However, in calculations exchange-current corrections, they left out the D-state of the triton, and cross terms between S-and D-states are important. Putting in the D-state, Rho later found the correction to be large enough to explain the discrepancy.[7,8]

In Table I, from the work of Arima and Hyuga,[9] I show the corrections from the processes, Fig. 1, to the magnetic moments of the various single-particle and single-hole nuclei. Hyuga's results for the exchange moment tend to be larger than the published ones by Chemtob,[10] because of better account of short-range correlations. The two-body exchange operator for δg_s has the radial dependence

$$2e^{-\mu_\pi r} - e^{-\mu_\pi r}/(\mu_\pi r)$$

Table I. Contribution of one-pion exchange currents to magnetic moments of single-particle and single-hole nuclei.

Nucleus	Configuration	Exchange moment	Schmidt	exp -Schmidt
^{15}N	$1p_{1/2}$	0.03	−0.251	−0.032
^{17}O	$1d_{5/2}$	−0.33	−1.913	0.019
^{39}K	$1d_{3/2}$	0.18	0.148	0.243
^{41}Ca	$1f_{7/2}$	−0.40	−1.913	0.318
^{207}Pb	$3p_{1/2}$	0.01	0.638	−0.048
^{209}Bi	$1h_{9/2}$	0.60	2.657	1.423

with the first term predominating. Short-range correlations, which cut the wave function down for small r_{12} have more effect on the second term, so that the difference is actually increased.

The exchange-moment contribution in O^{17} and Ca^{41} moving the magnetic moment well outside the Schmidt line are very welcome, since configuration mixing-especially that connected with the core-deformed states[11] tends to move the moments well inside the Schmidt line. Thus, the magnetic moment of O^{17} ends up near the Schmidt line as the result of cancellations of rather large terms.

The fact[12] that the one-pion exchange gives a δg_l of $\sim 0.1\tau_3$ is an old story, which I shall not retell.

Using the Sachs procedure, eqs. (1) to (1.2), and the Hamada-Johnston potential Arima and Hyuga have calculated the corrections from parts of the potential other than OPEP, with the results shown in Table II.

Table II. Contributions to δg_l.

Nucleus	δg_l from OPE	from remainder of H. J. potential	Sum
N^{15}	0.11	0.06	0.17
O^{17}	-0.06	-0.03	-0.10
K^{39}	0.11	0.05	0.16
Ca^{41}	-0.07	-0.04	-0.11
Pb^{207}	-0.05	-0.04	-0.09
Bi^{209}	0.10	0.05	0.15

By connecting δg_l with the exchange-current correction to the dipole sum rule. Fujita and Hirata[13] show that the δg_l from meson exchange currents should be

$$(\delta g_l)_{\text{exch}} \simeq 0.2\tau_3.$$

Second-order corrections from configuration mixing[14] tend to give a δg_l of about -0.1, and this produces a

$$(\delta g_l)_{\text{exch}} + (\delta g_l)_{\substack{\text{2nd order}\\ \text{Config.}}} \simeq 0.1, \tag{4}$$

that is, the δg_l is brought back to its OPE value (see discussion in Fujita, Yamaji and Hirata).[15]

§3. The Role of the N_{33}^*-Isobar and the Tensor Force

In the n + p capture process illustrated in Fig. 1, Riska and Brown[5] found the isobar, through the process shown in Fig. 2, to explain the remaining 1/3 of the discrepancy.

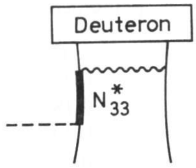

Fig. 2. Participation of the N_{33}^* isobar in $n + p \to \gamma + d$.

The question is, why do we not see more effects from the N_{33}^* on magnetic moments? I believe that recent work of Green and Schucan[16] and of Ichimura, Hyuga and Brown,[17] has provided an answer. Namely, there is a rather complicated interplay of isobar- and tensor-force effects which very nearly cancel each other out. This cancellation would be expected to be a general feature. Let us, however, consider what happens in the triton by way of example.

If we have only symmetric S-state and D-state in the triton, the well-known formula for the isovector magnetic moment is

$$\mu_V = \frac{1}{2}\left(\gamma_p - \gamma_n\right)\left(P_S + \frac{1}{3}P_D\right) - \frac{1}{6}P_D \tag{4}$$

with $\frac{1}{2}(\gamma_p - \gamma_n) = 2.353$ nuclear magnetons, and P_S and P_D are the S-state and D-state probabilities. If this is generalized to a triton containing a component $N^2\ N_{33}^*$, one has[17]

$$\mu_V = \frac{1}{2}\left(\gamma_p - \gamma_n\right)\left(P_S + \frac{1}{3}P_D + \frac{4}{9}P_{N^*}\right) - \frac{1}{6}P_D. \tag{4.1}$$

Now, let us talk, as usual, about the Schmidt value as the value of the magnetic moment if only S-state were present. We see that the presence of both D-state and isobar-component bring the magnetic moment inside the Schmidt line. However, there are exchange moments connecting nucleon and isobar components of the wave function of the type shown in Fig. 3. Note that the spin- and isospin-combinations of the pair of particles involved in the interaction is just the same as in Fig. 2.

To make a long story short, the exchange terms of this type, bring the triton moment back to the Schmidt line.[16] The rhyme and reason for this is most easily seen in the quark model of the triton.[17] In other words, introduction of D-state, N_{33}^*-state and exchange moments between nucleon and isobar components just cancel each other off. This cancellation does not involve any specific features of the triton, and would be expected to occur in heavier nuclei.

The only place where nature shows her hand with respect to N_{33}^*-isobar component is in the $n + p \rightarrow d + \gamma$ process, Fig. 2. In this case, the continuum n and p are normalized to nucleon states at ∞, and one should not renormalize when they come close together. Thus, a sizable effect from the N_{33}^* is left.

It will become clear form Professor Arima's talk that when the nuclear configuration

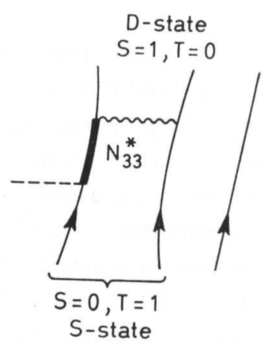

Fig. 3. Role of the N_{33}^* isobar in the triton magnetic moment.

mixing is taken into account, the picture of exchange currents presented here provides quite a good description of the magnetic moments discussed. I do not have time to go into our present understanding of the nucleon-nucleon interaction in terms similar to those used here to discuss exchange moments.

I would like to thank A. Arima and H. Hyuga for suggestions and permission to use their results before publication; also, M. Chemtob, A. M. Green, D. O. Riska and M. Rho for many discussions of the matters treated here.

References

1) A. Arima and H. Horie: Progr. theor. Phys. **11** (1954) 509; R. J. Blin-Stoyle and M. A. Perks: Proc. Phys. Soc. **A67** (1954) 885.
2) R. G. Sachs: Phys. Rev. **74** (1948) 433.
3) F. Villars: Helv. Phys. Acta **20** (1947) 476.
4) M. Chemtob and M. Rho: Nuclear Phys. **A163** (1971) 1.
5) D. O. Riska and G. E. Brown: Phys. Letters **38B** (1972) 193.
6) A. E. Cox, S.A.R. Wynchank and C. H. Collie: Nuclear Phys. **74** (1965) 497.
7) M. Rho: *Nuclear Structure Symposium of the Thousand Lakes, Joutsenlampi, Finland, 1970.*
8) E. P. Harper, Y. E. Kim, A. Tubis and M. Rho: Phys. Letters, to be published.
9) A. Arima and H. Hyuga: to be published.
10) M. Chemtob: Nuclear Phys. **A123** (1969) 449.
11) T. Erikson: to be published.
12) H. Miyazawa: Progr. theor. Phys. **6** (1951) 801; see S. Wahlborn and J. Blomqvist: Nuclear Phys. **A133** (1969) 50, §5.1 for a particularly simple derivation.
13) J. I. Fujita and M. Hirata: Phys. Letters **B37** (1971) 237.
14) M. Ichimura and K. Yazaki: Nuclear Phys. **63** (1965) 401; H. A. Mavromatis, L. Zamick and G. E. Brown: Nuclear Phys. **80** (1966) 545; H. A. Mavromatis and L. Zamick: Nuclear Phys. **A104** (1967) 17; G. F. Bertsch: Phys. Letters **28B** (1968) 302.
15) J. Fujita, S. Yamaji and M. Hirata: J. Phys. Soc. Japan **33** (1972) 541.
16) A. M. Green and T. Schucan: to be published.
17) M. Ichimura, H. Hyuga and G. E. Brown, presented at this conference VI–15.

Discussion

H. MIYAZAWA (Univ. of Tokyo): You said that the isobar effect is not important for the triton. I agree with that, but does this hold in general for heavier nuclei?

BROWN: I didn't really go into that properly, but the point is that if two nucleons collide through the tensor force or if an isobar is formed, they usually start from a relative S state most of the time and through the tensor force which is Y_2 in space their spins are coupled to a state with $J = 2$. Therefore the Fourier transform of the tensor force is actually a second rank tensor of k times the radial function associated with the tensor force. Therefore the Fourier transform starts with k^2 and grows as k grows. The result is that the tensor force in general takes one into an intermediate state of very high excitation energy, something like 300 MeV. If an isobar is formed, one goes to a state at about 600 or 700 MeV. Consequently, these effects involving tensor force and isobar terms are almost completely independent of the motion of the original particles, because that motion is always very slow compared with the motion in the intermediate state. What one learns from the triton one can apply immediately to heavy nuclei.

MIYAZAWA: You have considered the S to D state cross term. In general there could be *e.g.*, a P to P state "sandwiching" isobar. In that case you don't need a tensor force to bring it back to the original state.

BROWN: Such states are not considered. But you see the interactions are all relatively short range and the matrix elements turn out to be quite small unless one starts with an S state.

J. V. NOBLE (Univ. of Virginia): It seems to me that there is a certain double counting which enters when you introduce pions, nucleons and isobars as though they were independent, elementary particles. Has anyone tried to estimate the effects of this?

BROWN: I don't see that there is any double counting. Within the frame work of the quark model, one can work everything out in a deductive fashion.

M. RHO (Saclay): In your discussion, you have emphasized mainly the processes at zero momentum transfer and that in that situation, there is a strong cancellation between Δ ((3, 3) resonance) contributions. Would you expect that such cancellation persists when the momentum transfer is increased?

BROWN: I don't know the answer to that question. The answer depends on the nature of the form factors, for example, whether one introduces $NN^*\pi$ and $NN\pi$ or not.

Y. E. KIM (Purdue Univ.): According to your contributed paper, your quark-model result overestimates the isovector part of the trinuclear magnetic moments by about 0.1 nm, which is about 1/4 of the difference between the single-particle estimate and experiment. Yet you claim that this model gives a very reasonable description. Do you have any comment about that.

BROWN: Well, the comment that I have is that the overestimate comes not only when one uses the quark model, but also when one puts in the additional exchange currents. When one puts in the Sachs terms (note that we have them here) and the Seagull term, my own feeling is that the problem is more in the exchange terms. However, I forgot that we overestimated the isovector part by that much.

RHO: Maybe I can help you. You have added the number that we have obtained for the S to S case to your number. Now in calculating the S to S matrix element there is what we call the normalization correction and the recoil correction. These are probably too large and may even be absent. I think they shouldn't be included in your result. If you take them away, you will get very good agreement.

BROWN: Thank you. Those were the terms I did not want to put in anyway.