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III.a. Nuclear Moments of Conjugate Nuclei

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Magnetic moments of conjugate nuclei are analyzed together with the Gamow-Teller moments in the framework of a one-body operator and charge symmetry. The information on the isoscalar parts is sensitive to the small correction due to the relativistic effect. Anomalies of the effective g-factors for nucleons in nuclei are also discussed.

§1. Introduction

Owing to the symmetry of the nuclear interaction with respect to protons and neutrons, light isobaric nuclei are grouped in pairs of conjugate nuclei with equal and opposite neutron excesses. Since the respective roles of protons and neutrons are simply interchanged in two such conjugate nuclei, for which the asymmetry due to the Coulomb force can in first approximation be neglected, pairs of nuclear moments, if both of them are known, constitute valuable sources of information on nuclear structure. In the early fifties there were many attempts to correlate magnetic moments, especially of mirror nuclei, with other information on nuclear structure. Although it was known that such correlations could give much information,[1] there were hardly any measured moments of mirror nuclei at that time.

Owing to the recent advance in experimental techniques which can be applied to electromagnetic-moment measurements of short-lived states, a number of nuclear magnetic moments of conjugate nuclei are known presently:

a) 9 self-conjugate cases ($T = 0$), in which 6 are in the ground states and 3 in excited states,

b) 13 mirror pairs of isospin doublets ($T = 1/2$), in which 11 are in the ground states and 2 in excited states,

c) 4 mirror pairs of isospin triplets ($T = 1$), for which $A = 8$, 12, 20 and 36.

Recently, interesting attempts were made to find regularities among these moments,[2–7] and to interpret and correlate them with the information obtained from β decay. In the present work, the data of conjugate nuclei were again analyzed by taking into acount the small but important correction due to the relativistic effect[8] and by including the recent data and also refined ft values.[9]

The relations which are employed in the present analysis, in the framework of a one-body magnetic-moment operator, are as follows. The magnetic moment of a nuclear state is the expectation value for the state $M = J$ and is assumed to be

$$\mu = \frac{1}{2}[g_l^s\langle\sum_i l_z^{(i)}\rangle_J + \mu_N^s\langle\sum_i \sigma_z^{(i)}\rangle_J + g_l^v\langle\sum_i \tau_3^{(i)} l_z^{(i)}\rangle_J + \mu_N^s\langle\sum_i \tau_3^{(i)}\sigma_z^{(i)}\rangle_J], \tag{1}$$

where the isoscalar and isovector parts of the orbital g-factors, g_l^s and g_l^v, as well as of the intrinsic nucleon magnetic moments, μ_N^s and μ_N^v are assumed constant. The first two terms are isospin scalars and have the same values for both members of the mirror pair, while the latter two terms comprise the 3-components of isovectors and have the same absolute values but opposite signs for a mirror pair.

The sum moment of a mirror pair is

$$\mu(T_3 = +T) + \mu(T_3 = -T) = g_l^s\langle\sum_i l_z^{(i)}\rangle_J + \mu_N^s\langle\sum_i \sigma_z^{(i)}\rangle_J\,. \tag{2}$$

For a self-conjugate nucleus, the magnetic moment is

$$\mu(T = 0) = \frac{1}{2}[g_l^s\langle\sum_i l_z^{(i)}\rangle_J + \mu_N^s\langle\sum_i \sigma_z^{(i)}\rangle_J]\,. \tag{3}$$

The difference moment of a mirror pair is

$$\mu(T_3 = +T) - \mu(T_3 = -T) = g_l^v\langle\sum_i \tau_3^{(i)} l_z^{(i)}\rangle_J + \mu_N^v\langle\sum_i \tau_3^{(i)}\sigma_z^{(i)}\rangle_J\,. \tag{4}$$

For the case of $T = 1/2$, the Gamow-Teller matrix element of the mirror decay can be related to $\langle\sum \tau\sigma\rangle$ in eq. (4), *i.e.*

$$\left|\langle\sum_i \tau_3^{(i)}\sigma_z^{(i)}\rangle_J\right|^2 = \frac{J}{J+1}\left|\int\sigma\right|^2. \tag{5}$$

The relation between the Gamow-Teller matrix element and the ft value of a mirror decay is given by

$$B/ft = |\int 1|^2 + R|\int\sigma|^2, \tag{6}$$

where the constants are given as $B = (6146 \pm 10)$sec and $R = 1.54 \pm 0.02$. In a mirror decay, to the extent that charge symmetry holds, the Fermi matrix element is expected to be $|\int 1|^2 = 1$. Thus, the value of $\langle\sum \tau\sigma\rangle$, apart from its sign, can be determined from the ft values.

§2. Sum Moments of Conjugate Nuclei

The information about the isoscalar part of the moments is given by the sum moment of a mirror pair. Since $J = \langle\sum l\rangle + \frac{1}{2}\langle\sum \sigma\rangle$, and under the assumption that the g-factors for nucleons in nuclei are equal to the free nucleon values, *i.e.* $g_l^s = 1$ and $\mu_N^s = \mu_p + \mu_n$, the sum moment can be expressed as

$$\mu(T_3 = +T) + \mu(T_3 = -T) = J + \left(\mu_p + \mu_n - \frac{1}{2}\right)\langle\sum_i \sigma_z^{(i)}\rangle_J\,. \tag{7}$$

Since the coefficient of the second term is small, equal to 0.38, the sum moment is relatively insensitive to the selection of the wave function, *i.e.* only a small fraction of the value of the sum moment has a dominant effect in selecting a suitable wave function. This is the reason why even a simple *jj* model predicts a good value for the sum moment itself. It is therefore necessary to take into account even a small correction, such as the relativistic correction, in order to obtain reliable information from the sum moment.

The relativistic correction is small, at most equal to a few percent,[8] but it is asymmetric with respect to protons and neutrons and is therefore important in the analysis of the isoscalar part of the moment.

In Table I, the known sum moments are listed, including the selfconjugate ones for

Table I. Sum moments of conjugate nuclei.

a) $T = 0$ (μ: magnetic moments of self-conjugate nuclei).

Nuclide	J^π	μ	$\Delta\mu_{rel.}$	$\langle\Sigma\sigma\rangle$	
				Exp.	jj†
^{2}D	1^+	0.857	—	1.88	2
^{6}Li	1^+	0.822	—	1.70	0.67
^{10}B	3^+	1.801	— 0.040	1.58 1.79	2
^{14}N	1^+	0.404	— 0.013	−0.51 −0.44	−0.67
^{18}F*	5^+	2.86(3)	— 0.084	1.89(16) 2.34(16)	2
^{22}Na	3^+	1.746(3)	— 0.050	1.30(2) 1.56(2)	1.2
^{38}K	3^+	1.374	— 0.037	−0.67 −0.47	−1.2

† $\langle\Sigma\sigma\rangle_{JJ} = \begin{cases} J/j & : j = l + \frac{1}{2}, \text{ for } |(j)^n, J> \\ -J/(j+1) & : j = l - \frac{1}{2}, \end{cases}$

b) $T = 1/2$.

A	J^π	μ^Σ	$\Delta^\Sigma_{rel.}$	$\langle\Sigma\sigma\rangle$	
				Exp.	jj
3	$1/2^+$	0.851	—	0.93	1
11	$3/2^-$	1.66(1)	— 0.040	0.43(3) 0.53(3)	1
13	$1/2^-$	0.380	— 0.012	−0.32 −0.27	−0.33
15	$1/2^-$	0.436	— 0.013	−0.17 −0.13	−0.33
17	$5/2^+$	2.829	— 0.084	0.87 1.09	1
19	$1/2^+$	0.742	— 0.017	0.64 0.68	0.2
19*	$5/2^+$	2.865(8)	— 0.084	0.96(2) 1.18(2)	1
21	$3/2^+$	1.724	— 0.070	0.59 0.72	0.6
29	$1/2^+$	0.680	— 0.018	0.47 0.52	1
35	$3/2^+$	1.454	— 0.039	−0.12 −0.02	−0.6
37	$3/2^+$	1.15(20)	— 0.036	−0.9(5) −0.8(5)	−0.6
37*	$7/2^-$	3.5(2)	— 0.123	0.0(5) 0.3(5)	1
41[a)]	$7/2^-$	3.84(2)	— 0.123	0.88(5) 1.21(5)	1

* Excited state.

a) ^{41}Sc: K. Sugimoto, A. Mizobuchi, T. Minamisono and Y. Nojiri, presented at this conference II–4.

Table I. Continued.

c) $T = 1$.

Nuclide	J^{π}	μ^{Σ}	$\Delta\mu^{\Sigma}_{\mathrm{rel.}}$	$\langle\sum\sigma\rangle$ Exp.	jj	Kurath[d)]
^{8}Li – ^{8}B[a)]	2^+	2.689	—	1.81	1.33	1.71
			0.054	1.96		
^{12}B – ^{12}N	1^+	1.460	—	1.21	1[c)]	0.98
^{20}F – ^{20}Na	2^+	2.463	—	1.22	0.8	—
			0.068	1.40		
^{36}Cl – ^{36}K[b)]	2^+	1.832	—	−0.44	−0.8	—
			0.038	−0.34		

a) ^{8}B: T. Minamisono, Y. Nojiri, A. Mizobuchi and K. Sugimoto, presented at this conference II–3.
b) ^{36}K: H. Schweickert, J. Dietrich, R. Neugart and E. W. Otten, presented at this conference II–6.
c) For $|(p_{3/2})^{-1}(p_{1/2})^1, 1^+\rangle$.
d) D. Kurath: private communication: $\mu^{\Sigma}(A=8) = 2.651$ nm with (6–16)2B interaction, $\mu^{\Sigma}(A=12) = 1.372$ nm with (8–16) interaction.

which the magnetic moments themselves are given, and the values of $\langle\sum\sigma\rangle$ are listed with and without the relativistic correction. For comparison, the values predicted by the simple $(j)^n$ configurations in the jj-coupling scheme are also given, together with some other theoretical predictions.

The values of $\langle\sum\sigma\rangle$ for $T = 1/2$ obtained with the relativistic correction are plotted in Fig. 1(a) as a function of the mass number A. The values for $T = 0$, divided in half, are also included when the states are thought to have the stretched configurations. It is noted that the values of $\langle\sum\sigma\rangle$ with relativistic correction are found to be larger than the single-particle values at the begining of the sd and f shells, *i.e.* at $A = 17$, 18 and 41, while the values without

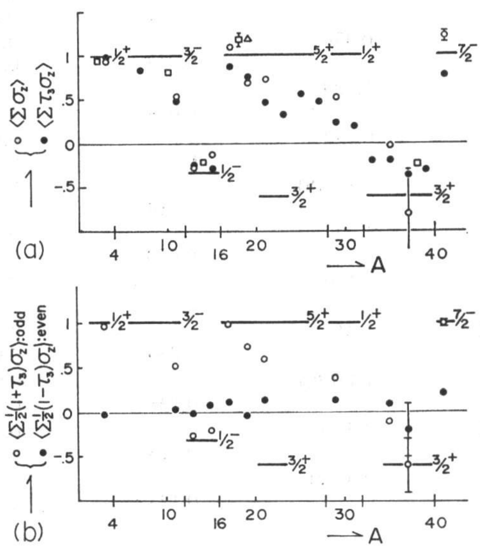

Fig. 1. Spin angular momentum distribution in conjugate nuclei as a function of mass number A. The values are denoted by circles for isospin doublets, by triangle for the excited states of $A = 19$ and by squares for self-conjugate nuclei.

Table II.

A	J^π	$\langle\Sigma\sigma\rangle$	$\langle\Sigma\tau\sigma\rangle$	$\langle\Sigma\frac{1+\tau}{2}\sigma\rangle$ odd	$\langle\Sigma\frac{1-\tau}{2}\sigma\rangle$ even
3	$1/2^+$	0.93	0.98	0.96	-0.03
7	$3/2^-$		(0.83)		
11	$3/2^-$	0.53(3)	0.48	0.51(2)	0.03(2)
13	$1/2^-$	-0.27	$(-)0.26$	-0.27	-0.01
15	$1/2^-$	-0.13	$(-)0.28$	-0.21	0.08
17	$5/2^+$	1.09	0.87	0.98	0.11
19	$1/2^+$	0.68	0.75	0.72	-0.04
21	$3/2^+$	0.72	0.46	0.59	0.13
23	$3/2^+$		0.33		
25	$5/2^+$		0.55		
27	$5/2^+$		0.47		
29	$1/2^+$	0.52	0.24	0.38	0.14
31	$1/2^+$		0.20		
33	$3/2^+$		$(-)0.20$		
35	$3/2^+$	-0.02	$(-)0.19$	-0.11	0.09
37	$3/2^+$	$-0.8(5)$	$(-)0.36$	$-0.6(3)$	$-0.2(3)$
39	$3/2^+$		$(-)0.30$		
41	$7/2^-$	1.21(5)	0.77	0.99(3)	0.22(3)

relativistic correction are always smaller than the single-particle values.

The values of $\langle\sum\sigma\rangle$ which were deduced from the Gamow-Teller moments are also plotted in the same figure. Using the values of $\langle\sum\sigma\rangle$ and $\langle\sum\tau\sigma\rangle_{Z=\mathrm{odd}}$, one can form the sums and the differences. The expression $\langle\sum\frac{1}{2}(1+\tau)\sigma\rangle_{Z=\mathrm{odd}}$ gives the component of the total intrinsic spin of the protons in the direction of J in a nucleus with odd proton number. On the other hand, $\langle\sum\frac{1}{2}(1-\tau)\sigma\rangle_{Z=\mathrm{odd}}$ gives the same component for the neutrons which form an even group in the same nucleus with odd proton number. These values are listed in Table II and plotted in Fig. 1(b). The odd-group values at $A=17$ and 41 are close to the single particle values, while the even-group values deviate from zero indicating that excitations of the unlike nucleons in the core are responsible for the spin-expectation values in these nuclei. It is also noted that the even-group values for $A\geqslant 15$ always have definite positive values except for the cases of $A=19$ and 37.

For completeness in discussing the isoscalar parts of the moments, it is necessary to consider corrections other than the relativistic one. The effect due to the spin-orbit force of the two-body interaction may produce a correction of the isoscalar type of about 0.02 nm in the same direction as the relativistic correction.[10] However, the origin of this effect and its relation to the relativistic and mesonic effects is not clear. Thus, the effect was not considered in the present work.

As for the mesonic effect, its contribution to the isoscalar parts is believed to be small because of the Sachs theorem. In this connection, a trial calculation was made to estimate the effective isoscalar g-factor for nucleons in nuclei. For nuclei differing by one particle or hole from a doubly closed shell, the values of $\langle\sum\sigma\rangle$ are given by the single particle values in the jj-coupling shell model in which no tensor interaction is taken into account.[11] By inserting the single-particle values in eq. (2) and treating g_l^s and μ_N^s as free parameters, a

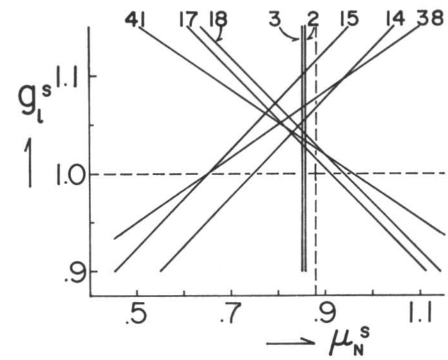

Fig. 2. Effective isoscalar nucleon g-factors, g_l^s vs μ_N^s. The broken lines indicate the free nucleon values.

sum moment determines a line in the $g_l^s - \mu_N^s$ plane. The results are shown in Fig. 2 in which the lines for the self-conjugate states of $A = 14$, 18 and 38 are included. All lines thus determined cross each other in a region where μ_N^s is about 0.05 nm smaller than, and g_l^s about 0.05 nm larger than the free nucleon values.

Concerning our understanding of the isoscalar moments, especially the moments around doubly closed shells, it is unknown whether the interpretation which relates to the wave functions or that of the anomalies of the effective nucleon g-factors is the major concern.

§3. Difference Moments of Isospin Doublets and Gamow-Teller Moments

A difference moment together with the Gamow-Teller moment for a pair of isospin doublets can yield information on $\langle\sum \tau l\rangle$, if in eq. (4) it is assumed that the g-factors for nucleons in nuclei are equal to the free nucleon values.

This assumption for the nucleon g-factors is not well justified, however, especially for the isovector parts, because of the mesonic effect. In this respect, a trial calculation was made to estimate the effective g-factors for nucleons in nuclei.[6]

The following relation is assumed,

$$\langle\sum \tau_3 j_z\rangle^0 = \langle\sum\tau_3 l_z\rangle + \frac{1}{2}\langle\sum \tau_3\sigma_z\rangle, \tag{8}$$

where, in the jj-coupling scheme, $\langle\sum \tau j\rangle^0$ is the zero-order term of $\langle\sum \tau j\rangle$ which will change only in the second order because there are no matrix elements connecting different configurations. The values deduced from the Gamow-Teller moments can be used for $\langle\sum \tau\sigma\rangle$. Thus, eq. (4) for the difference moment can be used to determine the isovector parts of the effective g-factors in the present approximation, in which g_l^v and μ_N^v are treated as free parameters. A difference moment determines a line in the $g_l^v - \mu_N^v$ plane as shown in Fig. 3(a). All lines thus determined cross each other in a region where μ_N^v is about 1 nm larger than the free nucleon value. In view of the smallness of the anomaly in the coupling constant of the Gamow-Teller moment, the values of $\langle\sum \tau\sigma\rangle$ are expected to be close to the real values. On the other hand, the higher-order corrections to the values of $\langle\sum \tau j\rangle^0$ will change the slopes of the lines. Using the available values of the second-order corrections for $A = 15$, 17 and 41,[11] we can

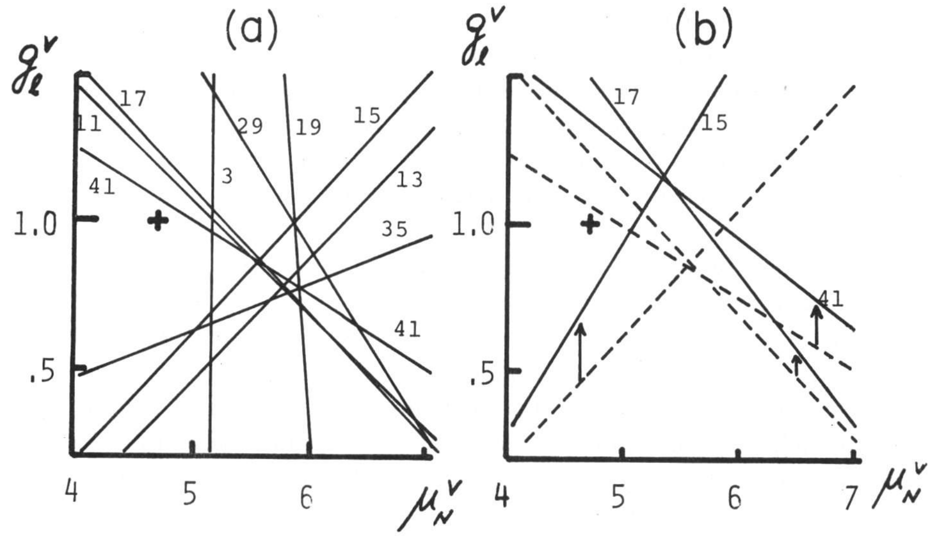

Fig. 3. Effective isovector nucleon g-factors, g_l^v vs μ_N^v. The free nucleon values are denoted by crosses.
(a) The zero-order terms are assumed for $\langle\sum\tau j\rangle$.
(b) The second-order terms of $\langle\sum\tau j\rangle$ are included in the solid line.

correct the corresponding lines as shown in Fig. 3(b). Not only μ_N^v but also g_l^v appears to be larger than the free nucleon values, provided that the state dependence of the effective g-factors is not appreciably large.

Our present analyses are simple minded trials. It is therefore necessary to study further to draw definite conclusions about the details of nuclear wave functions and mesonic effects. However the usefulness of the nuclear moments of conjugate nuclei is emphasized for the study of nuclear structure.

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Discussion

H. Miyazawa (Tokyo): In a nucleus the magnetic moment has three parts: orbital, spin and a third of tensor type which you have not included. This latter term is not very important, but may have some effect on your analysis.

L. ZAMICK (Rutgers): In a case such as the $J=5$, $T=0$ state in ^{18}F, configuration mixing can give an isoscalar contribution provided that a tensor force has been used. Did you take this into account in order to get an integrated result for the isoscalar effect?

SUGIMOTO: Our calculation are very simple. We did not take into account the tensor interaction.

A. ARIMA (Stony Brook and Tokyo): The tensor force seems to break the symmetry and this is particularly important in the matrix element of $\tau\sigma$. The tensor force reduces the value of $\tau\sigma$ very much. There are many operators in the magnetic moment, and every operator can be changed in many ways: by the mesonic correction, the relativistic correction, and furthermore by configuration mixing. But $\tau\sigma$ is not much changed, I believe, by mesonic corrections, but might be changed by the tensor force which causes second-order perturbation.

G. BROWN (Stony Brook): Mesonic corrections do change $\langle\tau\sigma\rangle$. The change is about one third of the $\langle\tau\sigma\rangle$ term in the magnetic moment.