JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 34, SUPPLEMENT, 1973
PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON NUCLEAR MOMENTS AND NUCLEAR STRUCTURE, 1972

III.g. Summary of Magnetic Moments of High-Spin States in the Pb Region

SHOJI NAGAMIYA

Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo, Japan

The g-factors of several shell-model states in the Pb region are summarized from the viewpoint of the nuclear shell model. Almost all the g-factors of many-particle states in this region satisfy the shell-model additivity relation of the g-factor. Empirical values of δg_s and g_p are compared with the calculated values. Blocking effect of the $(\pi h_{11/2}^{-1}\pi h_{9/2})$ type core polarization on the g-factors of the $\pi h_{9/2}^n$ states has been observed in the experimental data, in agreement with the theoretical expectation. The g-factors of the several states, which involve the $(\pi h_{9/2}^2)_{8^+}$ and $\nu i_{13/2}^n$ configurations, are discussed in the last part.

§1. Introduction

In these past few years a lot of data on the magnetic moments in the lead region have been accumulated, and they have provided important new information on nuclear physics, such as the anomalous g_l factor of the nucleon. In this paper we summarize recent experimental data, in particular, the data presented at this Conference, and try to extract some systematics from the data.*)

A compiled data-table is shown in Table IV in the Appendix. The table is divided into four parts according to the character of the state of interest. Single-particle states are listed in the first part, and many-particle states are listed in the subsequent three parts. In making the table attention has been paid to the corrections for the Knight shift and chemical shift, because different values of such corrections were used so far, the values being dependent on the choice of each author. For non-metallic samples we have corrected the chemical-shift by $-1.4 \pm 0.4\%$,[3] and for metallic samples the chemical-shift plus Knight-shift by $0.0 \pm 1.0\%$.[3,4]

With regard to the single-particle states, the $\nu f_{5/2}^{-1}$ moment was remeasured by Schroeder and Toschinsky[5] in a very strong magnetic field. The Munich group[6] determined the magnetic moment of the $15/2^-$ state of ^{211}Po and demonstrated that this state has configurations of $\nu j_{15/2}$ and also of 3^- (in ^{208}Pb) $\otimes$ $\nu g_{9/2}$. The g-factor of the $\nu i_{13/2}^{-1}$ state of ^{207}Po was determined by the Berlin group.[7] These interesting data have been contribute to this Conference.

§2. Additivity of g-Factor

One of the characteristic features of the lead region is the presence of a large amount of high-spin many-particle isomeric states whose configurations are relatively simple in view of the shell model. The half-lives of these high-spin states usually range from a few 10 ns to a few μs, and these half-lives are suitably long for an accurate determination of the

* The mesonic exchange effect revealed in this lead region has been discussed by Yamazaki[1] and Arima[2] in I.c. and III.b., respectively, so that we do not enter into this problem in this paper.

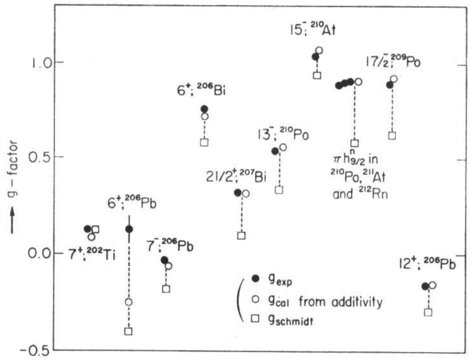

Fig. 1. Additivity of the g-factor. Experimental values of many-particle states are indicated by black-closed circles. Open circles show the calculated values obtained from the observed single-particle moments by using the additivity relation. Open squares are the Schmidt values. An interesting datum presented in this Conference is the core-excited 13^- state of ^{210}Po whose configuration is 5^-(in ^{208}Pb) $\otimes$ $(\pi h_{9/2}^2)_{8^+}$.[8] Even for such a core-excited state the additivity relation holds well.

magnetic moment by the application of ordinary experimental methods. Therefore, the lead region is one of the best regions to test the additivity of the g-factor. In Fig. 1 the observed g-factors of such many-particle states (indicated by black-closed circles) are compared with the calculated g-factors (indicated by open circles) which are obtained from the observed single-part`cle moments by using the shell-model additivity relation,

$$g(J) = \frac{1}{2} \cdot \{g(j_1) + g(j_2)\} + \frac{j_1(j_1+1) - j_2(j_2+1)}{2J(J+1)} \cdot \{g(j_1) - g(j_2)\} ,$$

for $(j_1 \otimes j_2; J)$ state.

The Schmidt values, that is, the single-particle estimates for these many-particle states calculated by using the free nucleon g_s and g_l values, are indicated by open squares in Fig. 1. Although the experimental values often disagree with the Schmidt values, the additivity relation holds roughly for all the cases listed in the figure. This fact is significant, because an experimental value involves many corrections whereas the additivity relation is based on the pure single-particle picture. If almost all the corrections to the g-factor of a single-particle state could approximately be renormalized into the one-body effective operator, with some state-independent parameters, then we would expect the additivity relation to hold for the experimental values. In what follows we shall discuss this point.

For the g-factor of a single-particle state the following corrections to the Schmidt magnetic moment are to be noted:

1. Configuration mixing in nuclear wavefunction
   (first order ····· core polarization
   higher order ····· g_R mixing)
2. Mesonic exchange effect
3. Effect of two-body $\boldsymbol{L} \cdot \boldsymbol{S}$ force
4. Other corrections
   (relativistic correction, heavy boson mixing, etc.)

Among these, the first two are relatively important in view of the magnitude of the correction.

Suppose that the magnetic-moment operator of a single particle is written as

$$\boldsymbol{\mu} = g_s \boldsymbol{s} + g_l \boldsymbol{l} + \sqrt{2\pi}\, g_p (\boldsymbol{\sigma} \cdot \mathbf{Y}_2)^1,$$

where

$$g_s = g_s^{\text{free}} + \delta g_s$$

$$g_l = g_l^{\text{free}} + \delta g_l \,.$$

Then, if we use the microscopic theory of Arima and Horie,[9] we can prove[10] that the first-order configuration mixing can effectively be renormalized into two state-independent parameters δg_s and g_p, to the extent that we can ignore the exchange term. The main term of the one-pion exchange current can also be renormalized into δg_l and δg_s. Therefore, the main part of the correction can effectively be described by the state-independent three parameters in the one-body effective operator. Thus we would expect the additivity relation to hold.

In Table I the experimental values of δg_s and g_p are compared with the calculated values, where the calculated values have been obtained with the use of the δ-function type interaction applied by Arima and Horie.[9] Because of the fact that the experimental values include not only the core polarization part but also the mesonic exchange part and several other corrections, the magnitudes of the empirical values are expected to be even larger than the calculated values. A rather good agreement between the theory and experiment thus indicates that the main contribution to δg_s and g_p is due to the core polarization. The isovector character of δg_s and g_p is somewhat peculiar to the lead region, since in this region the quantity $(g_s - g_l) \cdot n$, where n is the number of particles which can be polarized, has almost the same absolute value but has the opposite sign between the proton h-orbital and neutron i-orbital.

A useful application of the additivity is an extraction of a single-particle magnetic mo-

Table I. Comparison of empirical values of δg_s and g_p with the calculated values.

Experimental

		δg_s	g_p	$\delta g_s/g_p$
Maier *et al.* (ref. 19)*	proton state	−3.43	0.91	−3.8
	neutron state	3.43	−0 91	
Nakai *et al.* (ref. 20)	neutron state	2.77	−0.66	−4.2
Present analysis*	proton state	−2.5	0.5	−4.1
	neutron state	2.5	−0.5	

* Isovector character of δg_s and g_p is assumed.

Theoretical (Arima-Horie theory)

	δg_s a)			g_p	$\delta g_s/g_p$ b)
	$\pi h_{11/2}^{-1} \pi h_{9/2}$	$+ \nu i_{13/2}^{-1} \nu i_{11/2}$	= Sum		
proton state	−2.00	+ −0.50	= −2.50	0.62	−4
neutron state	1.98	+ 0.50	= 2.48	−0.62	

a) Contributions from $\pi h_{11/2}^{-1} \pi h_{9/2}$ and $\nu i_{13/2}^{-1} \nu i_{11/2}$ are listed separately.
b) According to the Arima-Horie theory the ratio $\delta g_s/g_p$ is exactly −4.

Table II. The g-factors of single-particle states deduced from the g-factors of many-particle states.

Orbital	g-factor	Origins for the deduction[a)]
$\pi i_{13/2}$	$\left\{\begin{array}{l}1.27 \pm 0.04^{\text{b)}}\\ 1.37 \pm 0.04^{\text{c)}}\end{array}\right.$	$g(11^-;\ ^{210}\text{Po})$, $g(9/2^-;\ ^{209}\text{Bi})$
$\nu i_{13/2}^{-1}$	-0.155 ± 0.004	$g(12^+;\ ^{206}\text{Pb})$
$\nu g_{9/2}$	-0.30 ± 0.01	$g(7^-$ and $5^-;\ ^{210}\text{Bi})$, $g(9/2^-;\ ^{209}\text{Bi})$

a) Number of each g-factor is given in Table IV in the Appendix.
b) Deduced by assuming the wavefunction of the 11^- state as
$|11^-) = 95\%\cdot|\pi h_{9/2}\pi i_{13/2}) + 5\%\cdot|(\pi h_{9/2}^2)_{8^+} \otimes 3^-)$.
c) Deduced by assuming the wavefunction of the 11^- state as
$|11^-) = 76\%\cdot|\pi h_{9/2}\pi i_{13/2}) + 20\%\cdot|\pi h_{9/2}\pi f_{7/2} \otimes 3^-) + 4\%\cdot|(\pi h_{9/2}^2)_{8^+} \otimes 3^-)$.

ment from the observed g-factor of a many-particle state. The g-factors of the $\pi i_{13/2}$, $\nu i_{13/2}^{-1}$, and $\nu g_{9/2}$ states thus extracted are listed in Table II.

§3. Breakdown of the Additivity

So far, we have stressed the fact that the additivity relation holds well in the lead region. However, a more careful study reveals a slight breakdown of the additivity. Investigation of such a slight breakdown often provides some new information which otherwise would not be obtained. In this section we shall discuss this point.

3.1 The $\pi h_{9/2}^n$ states

First we discuss the $\pi h_{9/2}^n$ states. In this Conference several g-factors of these states have been reported. The g-factor of the $(\pi h_{9/2}^2)8^+$ state of ^{210}Po has been remeasured by the Munich group[11] and also by our group.[12)]* An interesting result for the $(\pi h_{9/2}^3)$ $21/2^-$ state of ^{211}At was obtained by the Erlangen group.[13] In addition to these contributions, the g-factor of the $(\pi h_{9/2}^4)$ 8^+ state was already determined by Maier *et al.*[14] in Berkeley. The data are summarized in Fig. 2, from which we see that the observed g-factor slowly decreases as the number of particles in the $\pi h_{9/2}$ orbital increases.

The origin of such a decrease is schematically illustrated in Fig. 3. For the proton state, the largest contribution from the first-order configuration mixing is due to the $\pi h_{11/2}^{-1}\pi h_{9/2}$ particle-hole excitation. The probability of making such particle-hole pairs is proportional to the number of particles in the $h_{11/2}$ orbital and also proportional to the number of vacancy-sites in the $h_{9/2}$ orbital. For the $h_{9/2}^n$ state the number of vacancy sites is $10 - n$. Hence, if we increase the particle number in the $h_{9/2}$ orbital, then we would expect that such particle-hole excitation will become blocked with a probability proportional to $10 - n$. In other words, the magnitude of the correction to the g-factor due to the $\pi h_{11/2}^{-1}\pi h_{9/2}$ excitation will become smaller by increasing the number of particles in the $h_{9/2}$ orbital.

A calculated blocking effect is shown in Fig. 2, in which three different parameters so far reported[9,15,16] are used. The gradient of each line in the figure is proportional to the mag-

* The g-factor of the $(\pi h_{9/2}^2)$ 6^+ state of ^{210}Po was also determined by the Munich group.[11] The difference in the g-factor between the 8^+ and 6^+ states is of interest from the viewpoint of the state-dependence of the blocking effect. This point is discussed in III.h. and VI.c. of this Conference.

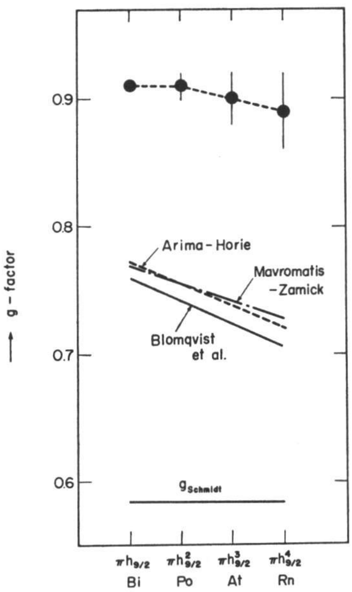

Fig. 2. Summary of the g-factors of the $\pi h_{9/2}^{n}$ states. Shown are the experimental values (black-closed circles) together with the calculated values, where the latter were obtained within the framework of the first-order configuration-mixing theory by taking the blocking effect into account.

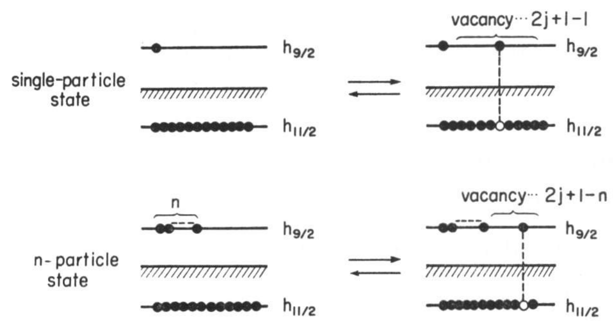

Fig. 3. Mechanism of the blocking effect of the $\pi h_{11/2}^{-1}\pi h_{9/2}$ particle-hole excitation.

nitude of the correction to the ^{209}Bi moment due to the $\pi h_{11/2}^{-1}\pi h_{9/2}$ excitation. The experimental gradient is a little bit smaller than the calculated gradient but roughly agrees with the calculated one. This fact further indicates the following. If we assume that the large deviation of the ^{209}Bi moment from the Schmidt value should be due to the $\pi h_{11/2}^{-1}\pi h_{9/2}$ excitation only, then the relevant calculated-gradient would become too sharp to explain the experimental variation, and therefore, the experimental results indicate that the $\pi h_{11/2}^{-1}\pi h_{9/2}$ core polarization is not the sole contributor to the large deviation of the ^{209}Bi moment but a part (nearly the half) of the observed deviation should be ascribed to other mechanisms, such as δg_l.

3.2 *The high-spin states in light Po-isotopes*

In this Conference several g-factors of neutron-deficient Po-isotopes are reported. The g-factors of the $(\pi h_{9/2}^{2})$ 8^+ states of ^{206,204}Po were determined by the Berlin group.[17] The g-factors of the $[(\pi h_{9/2}^{2})_{8^+} \otimes \nu p_{1/2}^{-1}]$ $17/2^-$ state of ^{209}Po and the g-factors of the $(\pi h_{9/2}^{2})8^+$ states of ^{208,206,204}Po were reported by our Tokyo group.[12] These are summarized in Fig. 4.

With regard to the 8^+ states, the g-factors are almost constant, although there still remains a problem for the 8^+ state of ^{204}Po. Why are the g-factors almost the same for such neutron-deficient nuclei, being insensitive to the neutron number? For neutron-deficient

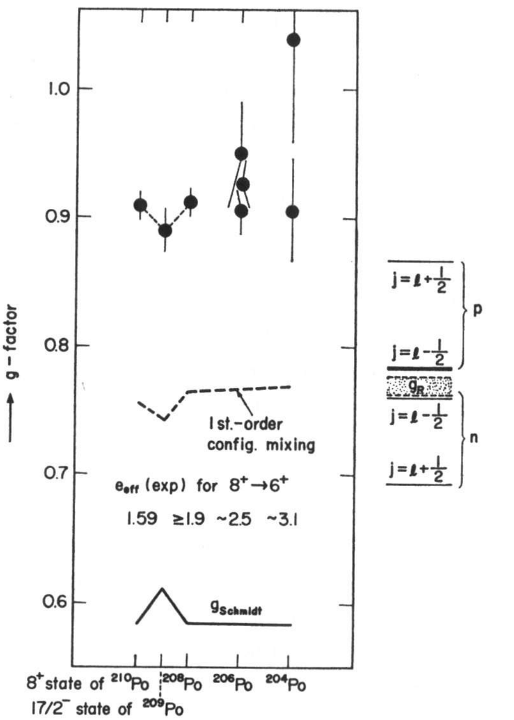

Fig. 4. Summary of the g-factors of the high-spin states in light Po-isotopes. Calculated values for the $(\pi h_{9/2}^2)$ 8^+ states of ^{210,208,206,204}Po were obtained from the formula of Arima and Horie[9] with use of the calculated V_j^2-values[21] for the neutron orbitals. The wavefunction listed in ref. 18 was used for the evaluation of the g-factor of the $[(\pi h_{9/2}^2)_{8^+} \otimes \nu p_{1/2}^{-1}]$ $17/2^-$ state of ^{209}Po.

Po-isotopes we should consider two effects on the g-factor. The first one is the correction due to the neutron core polarization. The situation for possible particle-hole excitation is different in each of these isotopes. As shown in Fig. 5, the $\nu p_{1/2}$ orbital is empty in ^{208}Po so that the $\nu p_{3/2}^{-1}\nu p_{1/2}$ excitation will take place, and in ^{206}Po the $\nu f_{7/2}^{-1}\nu f_{5/2}$ excitation comes

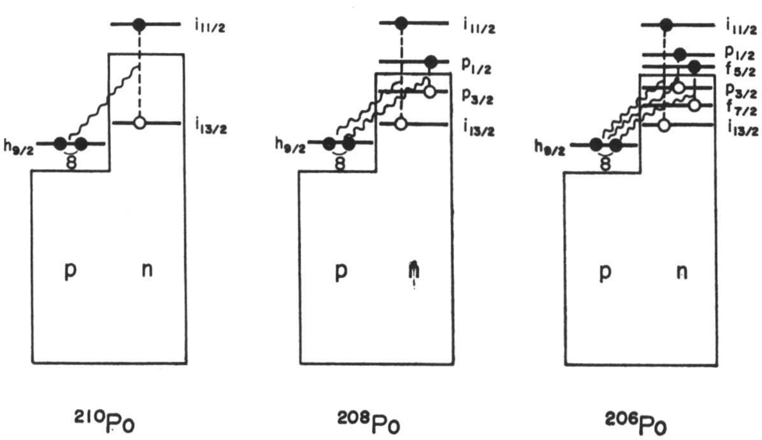

Fig. 5. Isotope dependence of the M1 core polarization for Po-isotopes.

in. According to Arima and Horie,[9] a slight increase of the g-factor is expected in going from ^{210}Po to ^{204}Po (see Fig. 4). The variation is not large because the effect of neutron particle-hole excitations on the g-factor of a proton state is expectedly small. The second effect which we should consider is the correction due to the mixing of collective states. This effect will make the g-factor closer to g_R; in this case $g_R = 0.3$. Because of the fact that the effective charges for the transition between 8^+ and 6^+ in light Po are rather larger than that in ^{210}Po, the collective-state mixing will induce a slight decrease of the g-factor in going from ^{210}Po to ^{204}Po. Thus, we expect almost the same value for the g-factors of the 8^+ states.

A kink is observed in Fig. 4 at the $17/2^-$ state of ^{209}Po. The direction of the observed kink is opposite to that of the kink of the Schmidt value. If we consider a small mixing of the $[(\pi h_{9/2}^2)_{8^+} \otimes \nu p_{3/2}^{-1}]\ 17/2^-$ component into the main component of $[(\pi h_{9/2}^2)_{8^+} \otimes \nu p_{1/2}^{-1}]\ 17/2^-$,[18] then the observed kink can be well explained as we show by the calculated line in Fig. 4.

3.3 *The $\nu i_{13/2}^{-n}$ states*

The variation of the g-factor of the $\nu i_{13/2}^{-n}$ states is shown in Fig. 6. In this case, however, the situation for the correction coming from the core polarization and the collective-state mixing is more complicated. We show in the figure the calculated values obtained from the first-order configuration mixing, but we cannot find a detailed explanation for the observed variation, so that this is open to further investigations.

§4. Concluding Remarks

In conclusion we shall make a comment. The accuracy of experimental data has recently become higher, and as a result, differences between several data have become appreciably noticeable. Of course, some data still lack accuracy and have to be remeasured, but one of the important problems left in the future is not only to understand the gross structure of the whole magnetic moments but also to evaluate their fine structures.

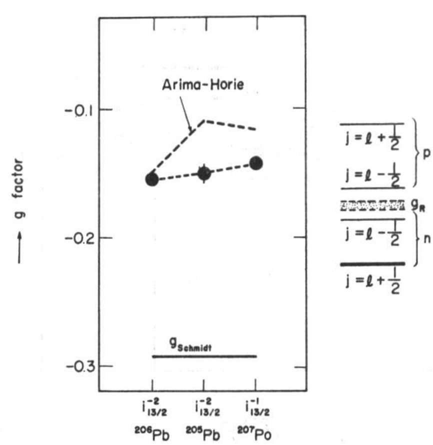

Fig. 6. Summary of the g-factors of the $\nu i_{13/2}^{-n}$ states.

Acknowledgment

I would like to thank Professors T. Yamazaki and K. Nakai for various stimulating discussions. Financial support by Sakkokai Foundation is gratefully acknowledged.

References

1) T. Yamazaki, presented at this conference I.c.
2) A. Arima, presented at this conference III.b.
3) F. D. Feiock and W. R. Johnson: Phys. Rev. Letters **21** (1968) 785.
4) L. E. Drain: Met. Rev. **12** (1967) 195.
5) F. J. Schroeder and H. Toshinsky: presented at this conference III–15.
6) T. Fasetermann, F. Feilitzsch, K. E. G. Löbner, C. Signorini, T. Yamazaki, C. V. K. Baba and D. B. Fossan, presented at this conference III–24.
7) B. Focke, A. Goldmann, M. v. Hartrott, K. Nishiyama and D. Riegel, presented at this conference III–20.
8) Y. Yamazaki, S. Nagamiya, T. Nomura, K. Nakai and T. Yamazaki, presented at this conference III–23.
9) A. Arima and H. Horie: Progr. theor. Phys. **11** (1954) 509.
10) S. Nagamiya: Ph. D. Thesis (1972), Sci. Papers IPCR **66** (1972) 39. See also ref. 2.
11) C. V. K. Baba, D. B. Fossan, T. Faestermann, F. Feilitzsch, P. Kienle and C. Signorini, presented at this conference III.h.
12) S. Nagamiya, Y. Yamazaki, O. Hashimoto, T. Nomura, K. Nakai and T. Yamazaki, presented at this conference III–21.
13) H. Ingwersen, W. Klinger, G. Schatz, W. Witthuhn and R. Maschuw, presented at this conference III–25.
14) K. H. Maier, J. R. Leigh, R. M. Diamond and F. S. Stephens: to be published (1972).
15) J. Blomqvist, N. Freed and H. O. Zetterström: Phys. Letters **18** (1965) 47.
16) H. A. Mavromatis and L. Zamick: Nuclear Phys. **A104** (1967) 17.
17) N. Bräuer, A. Goldmann, J. Hadijuana, M. v. Hartrott, K. Nishiyama, D. Quitmann, D. Riegel and W. Zeitz, presented at this conference III–19.
18) W. Baldridge, N. Freed and J. Gibbons: Phys. Letters **36B** (1971) 179.
19) K. H. Maier, K. Nakai, J. R. Leigh, R. M. Diamond and F. S. Stephens: Nuclear Phys. **A186** (1972) 97.
20) K. Nakai, B. Herskind, I. Bergström, J. Blomqvist, A. Filevich, K.-G. Rensfeld, J. Sztarkier and S. Nagamiya: Nuclear Phys. **A189** (1972) 526; presented at this conference III–16.
21) L. S. Kisslinger and R. A. Sorensen: K. Danske Vidensk. Selsk. mat. -fys. Medd. **32** (1960) No. 9.

Discussion

L. ZAMICK (Rutgers): You showed that as one adds protons to ^{209}Bi the g factors lie on a straight line. But the slope of the line as calculated by Arima and Horie, Mavromatis and Zamick, or Blomqvist is too steep compared to experiment. Is that correct?

NAGAMIYA: Because of the relatively large errors we cannot say conclusively whether it is correct or not. But, it may be correct.

ZAMICK: I would like to note that one pion exchange is not involved. As you point out, the slope is due to a blocking effect: protons interacting with protons. But two protons can only exchange a neutral pion and hence there is no contribution from the exchange current. Probably the slope is determined mainly by configuration mixing.

A. ARIMA (Stony Brook and Tokyo): I agree with Dr. Zamick. However, there is one interesting point, namely, the g factors of the 6^+ and 8^+ states in ^{210}Po are even larger than that of ^{209}Bi. The core polarization reduces the g-factor because of the blocking effect. This is something which we do not understand yet. Possibly there is some contamination of the configuration which we are not aware of at this moment.