JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN VOL. 34, SUPPLEMENT, 1973
PROCEEDINGS OF THE INTERNATIONAL CONFERENCE ON NUCLEAR MOMENTS AND NUCLEAR STRUCTURE, 1972

IV.g. Quadrupole Moments and Coexistence in Calcium Nuclei

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(Presented by D. CLINE)

At Rochester we[1] have measured the static electric quadrupole moments of first excited 2^+ states of the even isotopes of nickel, iron, chromium, titanium and calcium in order to investigate the onset of deformation within the shell-model picture. We find that the moments in the nuclei with $N \geq 28$ can be reproduced by $(1f\text{-}2p)^n$ shell-model calculations even though the moments vary both in sign and magnitude. However, as may be seen in Fig. 1, our measured moments become more and more negative as we approach the $N = 20$ shell closure, in complete disagreement with the (fp) shell-model calculations. This discrepancy indicates that appreciable core excitation is occurring in these isotones, the admixture increasing with decrease in neutron number.

We[11] made a shell-model calculation in the $(fd)^n$ space for ^{42}Ca to see if we could re-

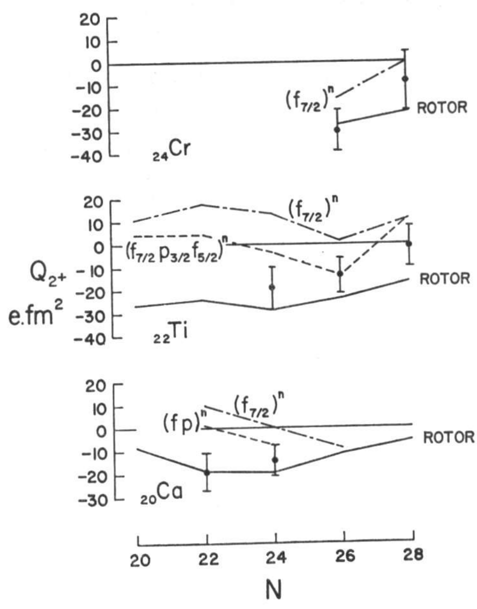

Fig. 1. Static electric quadrupole moments for the 2_1^+ states of the even isotopes of calcium,[11] titanium,[12,15] and chromium.[16] The dot-dashed lines are the predictions of the $(f_{7/2})^n$ shell model whereas the dashed lines result from $(fp)^n$ shell model calculations. In both cases, effective charges $e_p = 1.5e$ and $e_n = e$ were used.

* Supported by the National Science Foundation.

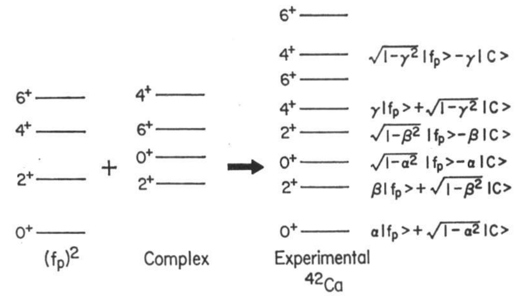

Fig. 2. States included in mixed model of ^{42}Ca. Four lowlying complex states were assumed to exist. However, no assumptions were made regarding the wavefunctions of these other than they were orthogonal to the $(fp)^2$ states. Kuo-Brown matrix elements were used to calculate the E2 properties, uncertain to an effective charge. No assumptions were made regarding the energies of the levels.

produce this core excitation. Although we could reproduce the low level spectrum, we calculate $Q_{2_1^+} = +16\ e \cdot fm^2$ which is more positive by an amount 32 $e \cdot fm^2$ than the experimental value. This is not surprising since the E2 operator does not couple the f and d orbits and thus it is not possible to produce sufficient collective enhancement.

Obviously we must extend the valence space to include the 2p and 2s orbits in order to build up appreciable collective enhancement. Shell-model calculations in such a large space are difficult to perform and not particularly useful. Instead, it is more useful to follow the Gerace and Green[2)] approach and postulate that the low-lying spectra can be described as a mixture of $(fp)^n$ states and some other complex states—presumably deformed ones. For ^{42}Ca, Gerace and Green calculated the interaction between deformed and spherical $(fp)^2$ states using Nilsson wavefunctions from which they predicted the wavefunctions and E2 matrix elements for ^{42}Ca. However, we now have enough E2 information for ^{42}Ca to reverse this procedure; that is, calculate the wavefunctions from the experimental matrix elements.

As illustrated in Fig. 2, we assume that in a sufficiently complete vector space, we can find for each J value two low-lying orthogonal states, one of which can be described as an $(fp)^2$ state, the other as some complex configuration of core-excited states. These two states are mixed together by some residual interaction to produce the physically observed pair of states for that J value. We assume further that all E2 matrix elements between $(fp)^2$ states and the complex states "C" are zero. Then we want to find properties of the "C" states, in particular the matrix elements $\langle C0^+|\,|E2|\,|C2^+\rangle$, $\langle C2^+|\,|E2|\,|C4^+\rangle$ and $\langle C2^+|\,|E2|\,|C4^+\rangle$, such that the properties of the mixed states agree with the experimentally observed values. There are nine experimental E2 matrix elements which couple the pairs of 0^+, 2^+ and 4^+ levels in ^{42}Ca. These data were used to determine the seven unknowns, that is, the neutron effective charge, the mixing parameters for the wavefunctions of the three pairs of levels, and the three E2 matrix elements between the complex states "C". The wavefunction admixtures and experimental excitation energies in ^{42}Ca were then used to determine the energies for the $(fp)^2$ and complex states before the mixing interaction was applied. The experimental and unperturbed energies are shown in Fig. 3. The unperturbed excitation energies of the complex states have a rotational character in that they fit the relation $E(J) = [(1.345 \pm 0.009) +$

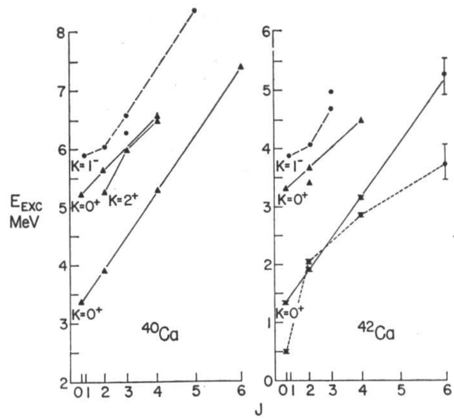

Fig. 3. Energy levels in ^{40}Ca and ^{42}Ca plotted versus $J(J+1)$ to illustrate the rotational bands in these nuclei. The lowest band in ^{42}Ca corresponds to the unperturbed energies of the complex states. The lowlying levels in ^{42}Ca, connected by the dashed line, are the unperturbed $(fp)^2$ spectrum.

$(0.0906 \pm 0.0008)J(J+1)]$ MeV, as illustrated in Fig. 3 by the solid line. The lowest rotational band in ^{40}Ca, also shown in Fig. 3, obeys the relation $E(J) = [(3.345 \pm 0.009) + (0.096 \pm 0.0009)J(J+1)]$ MeV, that is, the moment of inertia differs by 6% from the value for the unmixed ^{42}Ca levels. Furthermore, in both nuclei a second $K = 0^+$ band and a $K = 1^-$ band appear to occur at the same excitation energies above the lowest $K = 0^+$ band. An additional 2^+ level is known in ^{42}Ca at an energy corresponding to the first member of the $K = 2^+$ band in ^{40}Ca. Thus, ^{40}Ca and ^{42}Ca appear to have identical rotational band structures, that is, the moments of inertia and relative spacing of bands are the same in both nuclei.

The observation of enhanced in-band E2 transitions is the only sure identification of a rotational band. Several E2 transitions are known[3)] in ^{40}Ca which correspond to the intrinsic quadrupole moments listed in Table I. The E2 matrix elements for the complex states in

Table I. Intrinsic electric quadrupole moments for ^{40}Ca and the complex states of ^{42}Ca, in units of e fm^2, derived from the BE2's and static quadrupole moments assuming a rigid spheroidal rotor.

	^{40}Ca	^{42}Ca
Lowest $K = 0^+$ Band		
$0^+ \leftrightarrow 2^+$	$\|112 \pm 8\|$	$\|127.8 \pm 1.8\|$
$2^+ \leftrightarrow 2^+$		$+112 \pm 33$
$2^+ \leftrightarrow 4^+$	$\|138 \pm 14\|$	$\|110 \pm 73\|$
	—	—
Mean	$\|118 \pm 7\|$	$+127.7 \pm 1.8$
Second $K = 0^+$		
$4^+ \leftrightarrow 2^+$	$\|170 {+42 \atop -32}\|$	
First $K = 2$		
$3^+ \leftrightarrow 2^+$	$\|144 \pm 28\|$	
$4^+ \leftrightarrow 2^+$	$\|130 {+42 \atop -21}\|$	

Table II. Spectroscopic factors for the $^{41}Ca(d, p)^{42}Ca$ reaction.

J^{π}	Theory[a]	Experiment[b]
0_1^+	0.96 ± 0.015	1.0
0_2^+	0.35 ± 0.02	0.2
2_1^+	0.56 ± 0.03	0.6
2_2^+	0.78 ± 0.03	0.7
4_1^+	1.14 ± 0.08	1.1
4_2^+	0.29 ± 0.08	
6_1^+	1.2 ± 0.2	1.4
6_2^+	0.3 ± 0.2	

a) Present work.
b) Ref. 4.

^{42}Ca determined from the mixed model correspond to the intrinsic quadrupole moments also listed in Table I. Note that within the experimental errors, the intrinsic quadrupole moments in ^{42}Ca and ^{40}Ca have the same value. However, our static moment measurement shows conclusively that the intrinsic moment is positive corresponding to a prolate rotor. The extracted neutron effective charge $e_n = (0.646 \pm 0.039)$e and a constant intrinsic quadrupole moment yield E2 matrix elements in excellent agreement with the available experimental data.

The essentially identical moments of inertia, relative spacing of bands and intrinsic quadrupole moments in ^{40}Ca and ^{42}Ca are convincing evidence that the band structures in these nuclei are similar in nature. Thus our measurement of a positive sign for the deformation of the rotational band in ^{42}Ca almost certainly implies that the lowest $K = 0^+$ rotational band in ^{40}Ca has a prolate deformation.

The reliability of the mixed model is further supported by the measured ^{41}Ca(d, p) ^{42}Ca spectroscopic factors.[4] If we make the simplest possible assumption, *i.e.* that only the $(f_{7/2})^n$ components in ^{41}Ca and ^{42}Ca contribute to the $l = 3$ cross sections and assume that ^{41}Ca has a 76% $(f_{7/2})^1$ admixture,[5] then we predict spectroscopic factors shown in Table II. Excellent agreement is obtained with experimental results of Ellegaard *et al.*.[4] Unfortunately the spectroscopic factor to the presumed upper 6^+ level is not yet known.

An interesting prediction of the mixed model is that the $B(\mathrm{E2}; 6_1^+ \to 4_1^+)$ in the analogue nucleus ^{42}Ti will be smaller than for ^{42}Ca since the E2 matrix elements due to the $(fp)^2$ and deformed components are out of phase and the former is smaller than the latter. Thus the large effective charge for the proton should result in a smaller net matrix element. If ^{42}Ti and ^{42}Ca have the same deformed admixtures then we would predict that in ^{42}Ti the $B(\mathrm{E2}; 6_1^+ \to 4_1^+) \leq 46\ e^2\mathrm{fm}^4$. A measure of this B(E2) in ^{42}Ti would be a sensitive measure of the deformed tsate admixture for the 6^+ levels.

The model independent sum-rule method of Cline and Flaum[6] which is based on an idea of Kumar[7] allows a direct model-independent determination of the centroids, vibrational amplitudes, etc. for the intrinsic quadrupole moments for any level. This method is outlined in more detail in an[1] invited talk at this Conference. The intrinsic quadrupole moments can be expressed in terms of two other parameters Q and δ, analogous to the (β, γ) representation of the deformation tensor. That is,

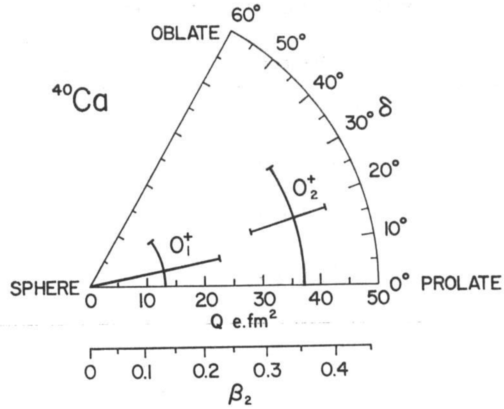

Fig. 4. The centroids and distribution widths (*i.e.* vibrational amplitudes) of the intrinsic quadrupole moment, in the Q, δ plane, of the ground state, 0_1^+ and lowest excited 0_2^+ state in ^{40}Ca.

$$Q_{20} \equiv \sqrt{\frac{16\pi}{5}}\, Q \cos \delta$$

$$Q_{22} \equiv \sqrt{\frac{16\pi}{10}}\, Q \sin \delta$$

where Q is positive and $0 \leq \delta \leq 60°$. A sum-rule analysis was made of the available E2 data from ^{40}Ca to determine the centroids $\langle Q^2 \rangle$, $\langle \cos 3\delta \rangle$, and vibrational amplitudes, $\sigma(Q^2)$, $\sigma(\cos 3\delta)$, for the intrinsic quadrupole moments of the ground state 0_1^+ and first excited state 0_2^+ of ^{40}Ca. The static moment for the 2_1^+ state in ^{40}Ca was assumed to be given by the measured value for the complex 2_1^+ state in ^{42}Ca normalized by the ratio of $\langle Q^2 \rangle$ in these two nuclei. The results are presented in Fig. 4 on a $Q\delta$ vector plot analogous to the usual $\beta\gamma$ vector plot. The coexistence of a soft prolate deformed excited 0_2^+ state and a spherical 0_1^+ ground state is clearly shown. The 0_2^+ state asymmetry centroid, δ, equals 18° with a vibrational amplitude $\sigma(\delta) = {}^{+13°}_{-30°}$. Using the adiabatic approximation and assuming some transition charge density distribution allows the distribution moments of the intrinsic quadrupole moment parameters Q, δ to be directly related to the centroids and vibrational amplitudes from the shape parameters β, γ. If we assume that the radial shape of the charge density scales with some radial parameter r, then it is not necessary to know the exact charge distribution, only the expectation value of $\langle \rho r^2 \rangle$ which has been accurately determined by electron scattering. This assumption results in effective β_2 values of 0.375 ± 0.006 and 0.355 ± 0.026 for ^{42}Ca and ^{40}Ca respectively, also it means that $\gamma \approx \delta$ and gives the β_2 scale shown in Fig. 4. A reasonable Fermi charge distribution would give a similar result.

The second excited 2^+ states in both ^{40}Ca and the uncoupled complex band in ^{42}Ca both could be interpreted as the 2^+ member of the $K = 2^+$ rotational band of an ellipsoidal rotor with γ ranging from 16° to 20° depending on the assumed magnitude of the rotation-vibration interaction.[8] The odd-even staggering of the 2^+, 3^+, 4^+ members of the $K = 2^+$ band in ^{40}Ca is as expected[9] for a potential well with only a small energy difference between the prolate and oblate minima, that is, a shape which is soft to γ vibrations. Thus, both the level positions and electric quadrupole properties of the deformed bands are consistent with a macroscopic shape which is very soft to γ vibrations with a γ centroid of about 18°. This is another example[1,10] of the strong correlation between the low-lying level spectrum and E2 properties due to the dominance of the two collective centroids $\langle\beta^2\rangle$ and $\langle\cos 3\delta\rangle$ in determining the low-lying spectral properties. This is further evidence that a collective model interpretation is meaningful.

Figure 4 very clearly shows the coexistence of spherical and deformed bands in ^{40}Ca. We think it is remarkable that within the experimental errors, the E2 properties and level positions in these bands are accurately described by a phenomenological collective model in such a light nucleus. It is equally surprising that the same strongly deformed band structure is seen in ^{42}Ca and appears to exist in ^{44}Ca. Johnstone[13] has found similar deformed bands, with $\beta_2 = 0.31$, in ^{43}Sc and we[14] have evidence they may exist in ^{43}Ca. It would be interesting to search for these deformed states in neighboring nuclei.

References

1) D. Cline, presented at this conference V.f. and University of Rochester report NSRL—59, and references contained therein.
2) W. J. Gerace and A. M. Green: Nuclear Phys. **A93** (1967) 110.
3) J. R. MacDonald, D. H. Wilkinson and D. E. Alburger: Phys. Rev. **C3** (1971) 219.
4) C. Ellegaard, J. Lien, O. Nathan, G. Sletten, F. Ingebretsen, E. Osnes, P. O. Tjøm, O. Hansen and R. Stock: Phys. Letters **40B** (1972) 641.
5) K. K. Seth, J. Picard and G. R. Satchler: Nuclear Phys. **A140** (1970) 577.
6) D. Cline and C. Flaum: BAPS II **17** (1972) 36, *Proc. Intern. Conf. on Electron Scattering and Nuclear Structure, Sept. 1972, Sendai.*
7) K. Kumar: Phys. Rev. Letters **28** (1972) 249.
8) A. S. Davydov and A. A. Chaban: Nuclear Phys. **20** (1960) 499.
9) G. Gneuss, W. Greiner and U. Mosel: Phys. Letters **32B** (1970) 161.
10) D. Cline, *Proc. of the Colloquium on Intermediate Nuclei,* ed. Foucher, Perrin and Veneroni (Orsay 1971) p. 4. University of Rochester Report -NSRL -40A.
11) C. W. Towsley, D. Cline and R. N. Horoshko: Phys. Rev. Letters **28** (1972) 368; Nuclear Phys.: submitted for publication (1972).
12) P. M. S. Lesser, D. Cline, P. Goode and R. N. Horoshko: Nuclear Phys. **A190** (1972) 597.
13) I. P. Johnstone: Nuclear Phys. **A110** (1968) 429.
14) R. N. Horoshko, D. Cline and C. W. Towsley: in preparation.
15) O. Hausser, D. Pelte, T. K. Alexander and H. C. Evans: Nuclear Phys. **A150** (1971) 417.
16) C. W. Towsley, D. Cline and R. N. Horoshko: presented at this conference V–15.