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V.a. Nuclear Spectroscopic Quadrupole Moments from Muonic Atoms and Related Topics

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(Presented by H. J. Leisi)

Hfs splittings of "highly excited" states in exotic atoms are considered. It is shown that the quadrupole splittings of the corresponding X-ray transitions yield precise values for the spectroscopic quadrupole moment of the nuclear ground state. These quantities are model independent, in contrast to the quadrupole moments deduced from X-ray transitions between lowlying states. Experimental results on the 5g–4f and 4f–3d X-ray transitions in μ ^{175}Lu and μ ^{235}U are discussed. The hfs components in pionic atoms show, apart from their electromagnetic splitting, a shift and a width due to the strong interaction of the pion, which is different for different angular momentum states. The effect of this phenomenon on the level positions may be described by an additional quadrupole coupling constant. An experiment on the quadrupole splitting of the 5g–4f transition in π ^{175}Lu is described.

§1. Introduction

A great wealth of nuclear information has been accumulated in recent years from the analysis of the X-rays from muonic atoms. Practically all these investigations were based on X-ray transitions emitted from low-lying levels of muonic atoms. In this talk I am concerned with "highly excited" levels for which the mean muonic radius $\langle r_\mu \rangle$ is much larger than the nuclear radius but still small compared to the size of the electronic cloud of the atom. Under these circumstances it is preferable to describe the energy levels of the systems by an approximation which is different from the usual one used in calculating muonic atoms.[1] The appropriate approach is basically the same as the one used in calculating the hyperfine interaction for electronic atoms.[2] We determine the energy levels of the systems by perturbation theory and define as the zeroth-order approximation the muonic states in a stationary point-nucleus Coulomb field (solutions of the Dirac equation) and the nucleus in its ground state. This approximation implies neglecting the muon-nucleus interaction while the muon is inside the nucleus. For each muonic state characterized by the angular momentum j, we have then $(2j+1)(2I+1)$ degenerate states, where I is the spin of the nuclear ground state. This degeneracy is removed by the perturbation

$$H' = H - \frac{-Ze^2}{r_\mu}, \tag{1}$$

where H is the true total interaction Hamiltonian of the nucleus and the muon.

§2. Quadrupole Interaction in Muonic Atoms

The dominant part of H in eq. (1) is the electric interaction between the nucleus and the

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muon:[3)]

$$H = \int \mathrm{d}r_\mu^3 \, \mathrm{d}r_n^3 \, \frac{\rho_\mu(\boldsymbol{r}_\mu)\rho_n(\boldsymbol{r}_n)}{|\boldsymbol{r}_\mu - \boldsymbol{r}_n|}, \tag{2}$$

where $\rho_\mu(\boldsymbol{r}_\mu)$ and $\rho_n(\boldsymbol{r}_n)$ are the muonic and nuclear charge-density operators, respectively. In writing eq. (2) we have neglected the small magnetic hyperfine interaction. In as much as we can neglect nuclear polarization effects, *i.e.* admixtures of excited nuclear states to the ground state wave function due to the interaction H', the quadrupole part of interaction (2) only contributes to the energy splittings of the degenerate levels:

$$H'_{\mathrm{E2}} = \frac{4\pi}{5} \sum_\nu \int \mathrm{d}r_\mu^3 \, \mathrm{d}r_n^3 \rho_\mu(\boldsymbol{r}_\mu)\rho_n(\boldsymbol{r}_n) \frac{r_<^2}{r_>^3} \, Y_{2\nu}(\theta_n, \phi_n) Y_{2\nu}^*(\theta_\mu, \phi_\mu) \,. \tag{3}$$

In first-order perturbation theory the energy levels are determined by the matrix elements of the total angular momentum states $|FMIj\rangle$ ($\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{j}$). We assume for the moment that the level spacings of the unperturbed muonic levels are much larger than the matrix elements of H'_{E2}. The perturbed energy levels are then given by[4)]

$$\begin{aligned} E_F^{(nlj)} &= \langle FMIj|H'_{\mathrm{E2}}|FMIj\rangle \\ &= \frac{3C(C+1) - 4I(I+1)j(j+1)}{2I(2I-1)j(2j-1)} [A_2^{(0)}(\mu) + A_2^{(1)}(\mu)] \,, \end{aligned} \tag{4}$$

where

$$C = F(F+1) - I(I+1) - j(j+1) \,. \tag{5}$$

The quantity $A_2^{(0)}(\mu)$ in eq. (4) is the usual quadrupole hfs constant in the point-nucleus approximation:

$$A_2^{(0)}(\mu) = -\frac{1}{2}\sqrt{\frac{4\pi}{5}} e^2 Q \left\langle jj|\frac{1}{r_\mu^3} Y_{20}(\theta_\mu, \phi_\mu)|jj \right\rangle, \tag{6}$$

where Q is the nuclear spectroscopic quadrupole moment and the matrix element is to be evaluated with Dirac wave functions appropriate to a point-nucleus Coulomb potential. The quantity $A_2^{(1)}(\mu)$ in eq. (4) describes the interaction of the muon inside the nucleus:

$$A_2^{(1)}(\mu) = \frac{4\pi}{5} \langle II| \int \mathrm{d}^3 r_n F_\mu(r_n) \rho_n(\boldsymbol{r}_n) Y_{20}(\theta_n, \phi_n)|II\rangle, \tag{7}$$

where

$$F_\mu(r_n) = \left\langle jj| \int_{|\boldsymbol{r}_\mu| \leqslant r_n} \mathrm{d}^3 r_\mu \left(\frac{r_\mu^2}{r_n^3} - \frac{r_n^2}{r_\mu^3} \right) \rho_\mu(\boldsymbol{r}_\mu) Y_{20}(\theta_\mu, \phi_\mu)|jj \right\rangle . \tag{8}$$

The integration in expression (8) is over a sphere with radius r_n. The nuclear model dependence of expression (4) is fully contained in the small quantity $A_2^{(1)}(\mu)$. In the limit of a small nuclear radius, $R_n \ll \langle r_\mu \rangle$, the quantity $A_2^{(1)}(\mu)$ approaches zero and the quadrupole splitting is model-independent; the only nuclear dependence is through the spectroscopic quadrupole moment Q, eq. (6). In contrast to the situation in electronic atoms the expectation value $\langle r_\mu^{-3} \rangle$ in eq.

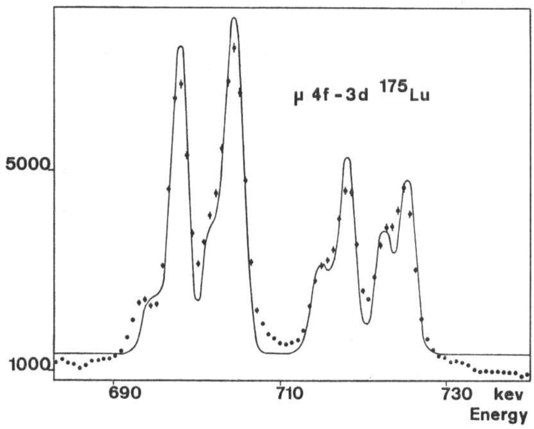

Fig. 1. Energy spectrum of 4f–3d transition in μ ^{175}Lu, taken from the work of ref. 5.

(6) can rigorously be calculated in muonic atoms. Hence observation of the quadrupole splittings of "highly-excited" states should provide a powerful new method for measuring spectroscopic quadrupole moments.

In strongly deformed nuclei, such as ^{175}Lu, the fine structure level spacings are not much greater than the matrix elements of the quadrupole interaction. In this case all appropriate fine structure states have to be included. The corresponding states $|FMInlj\rangle$ and $|FMInl'j'\rangle$ etc. are mixed and we need in general to evaluate the off-diagonal matrix elements with respect to the quantum numbers j and l of the operator H'_{E2} in order to calculate the energy levels E_F.

Figure 1 shows the experimentally observed 4f$-$3d transitions in μ^{175}Lu.[5] The solid line is a fit to the data based on a mixture of the $3d_{3/2}$ and $3d_{5/2}$ levels in the final state and the $4f_{5/2}$ and $5f_{7/2}$ levels in the initial state. According to a selection rule by Ericson and Scheck[6] other fine structure states with the same principal quantum number do not contribute if the muon is described by its non-relativistic wave function. Admixtures of such states may therefore safely be neglected. The following assumptions have been made:

a) the initial F levels are statistically occupied and unperturbed intensity ratios are used;
b) nuclear excitations and polarization effects are neglected;
c) the weak fine structure transition $4d_{3/2}-3p_{1/2}$ (intensity 0.6%) is neglected;
d) the intensity ratio for the dominant $4f_{7/2}-3d_{5/2}$ and the $4f_{5/2}-3d_{3/2}$ transitions is taken from the cascade program of Hüfner;
e) the magnetic hfs is neglected;
f) non-relativistic limits are used for the off-diagonal matrix elements.

The fair agreement between experiment and theory suggests that the neglected effects are not too important. A preliminary value of the quadrupole moment is obtained from this experiment as[7]

$$Q = 3.38 \text{ b.} \tag{9}$$

Figure 2 shows the 5g$-$4f transitions of the same muonic atom.[5] The displayed quadrupole splitting is mainly the splitting of the 4f levels. All corrections in this case are expected to be smaller than for the 3d levels. The assumptions made for the treatment of the data

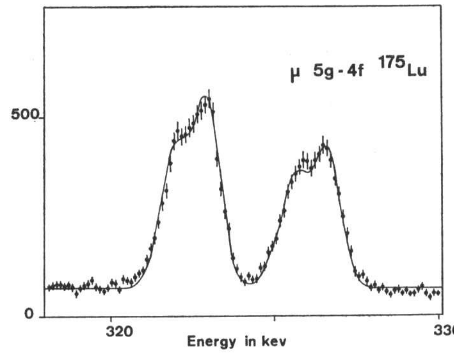

Fig. 2. Energy spectrum of 5g–4f transition in μ ^{175}Lu taken from ref. 5.

are the same as mentioned previously. Mixing of the 4f and 5g fine structure states is included. The fit yields the preliminary value of

$$Q = 3.74 \pm 0.05 \text{ b.} \tag{10}$$

The deviation from the values (9) and (10) is believed to be due mainly to the effects of nuclear excitation and neglected admixtures of fine structure components. A detailed evaluation of all the corrections is in progress. The quadrupole splittings of the same transitions have also been observed in muonic ^{235}U.[5)]

§3. Quadrupole Effects in Pionic Atoms

The electromagnetic interaction between the pion and the nucleus may be treated in exactly the same way as in the case of the muonic atom, except that the solutions of the Klein-Gordon equation have to be substituted for the Dirac wave functions. In addition, the strong interaction of the pion may change in a characteristic way the quadrupole splitting in a pionic atom. Consider an X-ray transition with the critical level of the pionic atom as its final state. For reasons mentioned earlier it is sufficient to consider a single fine structure level. The electromagnetic quadrupole interaction then couples the orbital angular momentum $\boldsymbol{l}$ of the pion to the nuclear spin $\boldsymbol{I}$ as indicated in Fig. 3. States of well-defined total angular momentum F are formed. For each F state, since the pionic charge cloud is oriented with respect to the orbital angular momentum, it also has a well-defined orientation with respect to the nuclear

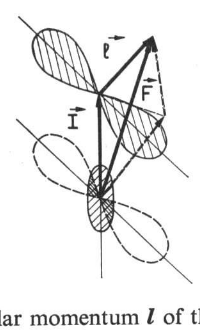

Fig. 3. Coupling between orbital angular momentum $\boldsymbol{l}$ of the pion and angular momentum $\boldsymbol{I}$ of the nucleus. The hatched regions indicate the mass distributions of the pion and the nucleus.

spin axis in the vector model. Therefore the amount of overlap between the nucleus and the pion does depend on F. Hence the width and the shift, due to the strong interaction between the pion and the nucleus, must be different for different F states, provided the nuclear shape deviates from spherical symmetry. Scheck[8)] has shown that the effect of this phenomenon on the level splitting may be described by an additional quadrupole coupling constant ε_2:

$$E_F = \frac{3C(C+1) - 4I(I+1)l(l+1)}{2I(2I-1)l(2l-1)}[A_2(\pi) - \varepsilon_2], \tag{11}$$

where

$$C = F(F+1) - I(I+1) - l(l+1). \tag{12}$$

The electromagnetic quadrupole coupling constant $A_2(\pi)$ in eq. (11) is given, analogously to eq. (4), by

$$A_2(\pi) = A_2^{(0)}(\pi) + A_2^{(1)}(\pi), \tag{13}$$

where again

$$A_2^{(0)}(\pi) = -\frac{1}{2}\sqrt{\frac{4\pi}{5}}\, e^2 Q \left\langle ll \left| \frac{4}{r_\pi^3} Y_{20}(\theta_\pi, \phi_\pi) \right| ll \right\rangle \tag{14}$$

is the quadrupole constant in the point-nucleus approximation and $A_2^{(1)}(\pi)$ is a small term describing the nuclear finite size effect. A similar expression holds for the width due to the strong absorption[8)]

$$\Gamma_F = \Gamma_0 + \frac{3C(C+1) - 4I(I+1)l(l+1)}{2I(2I-1)l(2l-1)}\Gamma_2 . \tag{15}$$

Equations (11) and (15) are derived on the basis of an equivalent optical potential for the pion nucleus interaction. The nucleus is assumed to be of quadrupole shape only. In the case where $R_n \ll \langle r_\pi \rangle$ the ratio $\varepsilon_2/\varepsilon_0$, where ε_0 is the over-all shift of the hyperfine multiplet due to the strong interaction, and also the ratio Γ_2/Γ_0 are independent of the parameters of the optical potential. They thus represent new model-independent nuclear quantities which probe the mass distribution near the nuclear surface for the high angular momentum states under consideration. Based on a Fermi-type charge distribution and the rotational model, Scheck[8)] has calculated numerical values for the strong interaction quadrupole coupling constant ε_2. In the case of the 4f level in π ^{175}Lu the quantity $\varepsilon_2/A_2(\pi)$ is predicted to be -0.032 and $\Gamma_2/\Gamma_0 = -0.21$.[9)] Figure 4 shows the 5g$-$4f transition in π ^{175}Lu.[10)] The theoretical curve is based on a fit involving the 4f and the 5g levels. Assuming no strong interaction quadrupole effect an (effective) quadrupole moment of

$$Q_{\text{eff}} = 3.70 \pm 0.04 \text{ b} \tag{16}$$

results, where

$$Q_{\text{eff}} A_2(\pi) = Q[A_2(\pi) - \varepsilon_2]. \tag{17}$$

Until a careful evaluation of the muonic data is completed no definite conclusion as regards to the strong interaction quadrupole effect can be drawn.

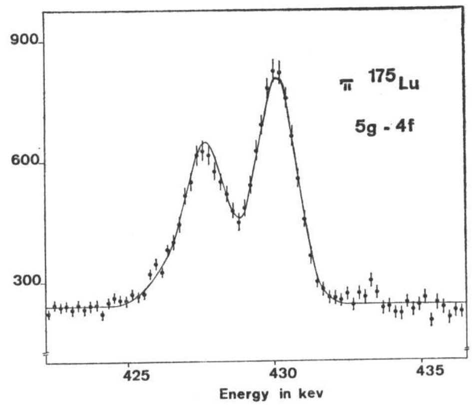

Fig. 4. Energy spectrum of the 5g–4f transition in π ^{175}Lu, taken from ref. 10.

Acknowledgment

We would like to thank Dr. G. H. Eaton for reading the manuscript.

References

1) See *e.g.* S. Devons and I. Duerdoth: *Advances in Nuclear Physics* **2** (1965) 295.
2) H. B. G. Casimir: *On the Interaction between Atomic Nuclei and Electrons* (W. H. Freeman and Company, San Francisco and London, 1963)
3) Weak interactions may safely be neglected.
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5) W. Dey *et al.*: "Nuclear Spectroscopic Quadrupole Moments of ^{175}Lu and ^{235}U: "A New Approach", presented at this conference VII–5.
6) T. E. O. Ericson and F. Scheck: Nuclear Phys. **B19** (1970) 450.
7) An E2 finite size correction of 4% has been applied.
8) F. Scheck: Nuclear Phys. **B42** (1972) 573.
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Discussion

K. ALDER (Univ. of Basel): One of the difficulties in the determination of quadrupole moments by the reorientation effect is the virtual excitation to higher levels. It seems to me that the same difficulties also arise here, and that these polarization effects give a limitation of the accuracy of this new method.

LEISI: Such nuclear polarization effects have been considered for muonic atoms. If I recall correctly the effect in the 2p levels of the deformed nuclei is something between three and five percent. We are certainly going to look into this, but it seems to me it can not be a very big effect.

C. S. WU (Columbia Univ.): In most of these high orbital transitions, the fine structure of the upper level is generally unresolved. On the other hand, it is known that the distributions of the sublevels in muonic atoms may not always follow the statistical law. What precautions do you take to ascertain that the statistical distribution is valid in the case which you studied?

LEISI: I shall explain exactly how we calculated the intensities. We have taken the intensity ratios of the two dominant fine structure lines from the cascade program of Hüfner. We have distributed these intensities among the various hyperfine structure components. As you have seen, the two groups are nicely separated and hyperfine quadrupole structure is really resolved.

E. -W. OTTEN (Univ. of Mainz): First, I want to refer to professor Alder's remark concerning nuclear excitation: Since the nuclear excitation by the muon is a second order effect in the quadrupole interaction H_Q, it can not contribute more than $\langle H_Q \rangle^2$ divided by the nuclear excitation energy. This is a small number for the higher excited mesonic states. I guess that this circumstance was one of the ideas leading to the experiment, besides avoiding the quadrupole form factors.

Second, I wish to remark on the question of population numbers: The population numbers would become irrelevant in the case of complete resolution of the hyperfine structure. Do you have a chance to achieve that at SIN with a crystal spectrometer?

LEISI: It would be difficult because the crystal spectrometer is superior only at low energies. As you go to higher states, however, the quadrupole splittings decrease quite fast.

L. GRODZINS (MIT): I would like to add a word to Professor Wu's remark. Hüfner's calculations of distributions were not - a few years ago - in close accord with experiment in many cases. Under the circumstances it seems best to be most cautious in fitting resolved fine structure components which are composed of a large number of unresolved hyperfine lines.

E. KANKELEIT (ITK, Darmstedt): Hüfner's cascade program has been considerably improved by Dr. Backe by starting the cascade with a statistical population at $n = 20$ instead of $n = 14$. Excellent agreement was thus obtained with experimental intensity measurements. I can't make a quantitative statement right now, but it is my impression that little uncertainty results from the intensities in the determination of the quadrupole moment.

WU: Did you analyze the 2p $\rightarrow$ 1s dynamic E2 spectral lines in ^{153}Eu to see whether the mixing in 3d states can be completely neglected?

LEISI: No, this has not been done yet. We have also many more data on the higher transitions which are not yet analyzed. I probably should also mention that the spectroscopic quadrupole moment of ^{175}Lu obtained from atomic hfs strongly disagrees with our value.