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Moments of Inertia for Rotating Deformed Nuclei

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Recently the problem of nuclear rotation has attracted considerable attention with the identification of nuclear states with very high angular momenta. Experiments[1-3] on certain nuclei in the rare-earth region have revealed a singular behaviour in the rotational spectra at high spins in a few cases. A study of the variation of moment of inertia $\mathscr{I}$ with the rotational angular velocity ω in these nuclei showed that there occurs a marked increase in the rate at which $\mathscr{I}$ rises at high ω. In a few cases the rise is so rapid that ω actually decreases as higher spin states are reached resulting in the appearance of backbending in $\mathscr{I}$-ω^2 plot. A few semi-microscopic models[4] have been proposed to explain this feature which is attributed to the breakdown of pairing correlations by the Coriolis force at high rotational frequencies as first suggested by Mottelson and Valatin.[5] We examine this problem within the framework of phenomenological models.

The moment of inertia $\mathscr{I}$, angular velocity ω, and the angular momentum (spin) I of a system (nucleus) are related through the semi-classical relation

$$\hbar\sqrt{I(I+1)} = \mathscr{I}\omega \; . \tag{1}$$

Furthermore ω is defined by the canonical relation appropriate for an axially symmetric rotator

$$\hbar\omega = \frac{\mathrm{d}E}{\mathrm{d}\sqrt{I(I+1)}} \tag{2}$$

which may be rewritten as

$$\hbar^2\omega^2 = 4I(I+1)\left[\frac{\mathrm{d}E}{\mathrm{d}I(I+1)}\right]^2 \; . \tag{3}$$

Combining eqs. (1) and (3) we obtain the following expression for the moment of inertia

$$\frac{\mathscr{I}}{\hbar^2} = \frac{1}{2}\left[\frac{\mathrm{d}E}{\mathrm{d}I(I+1)}\right]^{-1} \; . \tag{4}$$

Assuming that over the interval between I and $I-2$, energy expression is at the most quadratic in $I(I+1)$ one can evaluate the energy derivative appearing in eqs. (3–4) using the experimentally measured quantities ΔE_I, the transition energy between the states I and $I-2$. In the MIDDLE of this spin interval where

$$I(I+1) = \frac{1}{2}[I(I+1) + (I-2)(I-1)] = I^2 - I + 1$$

one gets the following relation

$$\frac{\mathrm{d}E}{\mathrm{d}I(I+1)} = \frac{E_I - E_{I-2}}{4I-2} = \frac{\Delta E_I}{4I-2} = A_I \tag{5}$$

where A_I is the weighted transition energy introduced by Stephens *et al.*[6] Thus in terms of A_I the relevant quantities are

$$\frac{\mathscr{I}}{\hbar^2} = \frac{1}{2A_I} \tag{6}$$

and

$$(\hbar\omega)^2 = 4(I^2 - I + 1)A_I^2 \; . \tag{7}$$

These relations enable us to obtain $\mathscr{I} - \omega^2$ plots directly from the experimental data on transition energies. The behaviour expected from individual models is discussed below.

VMI Model:[7] In this model energy of a state with spin I depends on two parameters, $\mathscr{I}_0$, the ground-state moment-of-inertia and σ, the softness parameter. Energy equations are parametric and equivalent to the equations of Harris.[8] Relation between $\mathscr{I}$ and ω^2 is given as

$$\frac{\mathscr{I}}{\hbar^2} = \frac{\mathscr{I}_0}{\hbar^2}\left[1 + \left(\frac{\mathscr{I}_0}{\hbar^2}\right)^2\sigma(\hbar\omega)^2\right] \; . \tag{8}$$

Thus $\mathscr{I}$ is a linearly increasing function of ω^2 and no back-bending will appear in this model.

CS Model:[9] Expressions for energy in this model are also parametric and depend on two parameters, $\mathscr{I}_0$, the ground state moment of inertia and the adiabaticity parameter D depending on the relative magnitude of the vibrational and the rotational energies. $\mathscr{I}$ as a function of ω^2 is an infinite power series[10] and one has

$$\frac{\mathcal{J}}{\hbar^2}=\frac{\mathcal{J}_0}{\hbar^2}\left[1-\frac{1}{D}\left(\frac{\mathcal{J}_0}{\hbar^2}\right)^2(\hbar\omega)^2\right]^{-2} . \qquad (9)$$

All the terms in the expansion are positive and hence $\mathcal{J}$ increases with ω^2. Although the increase is more rapid than in the case of VMI model, no back-bending takes place.

Sood Model:[11] This is also a two parameter model, applicable to deformed nuclei whose energy ratio $R_4=E_4/E_2$ lies between 2.90 and 10/3. Evaluation of energy derivative is simple and one gets

$$\frac{\mathrm{d}E}{\mathrm{d}I(I+1)}=A\left(\frac{1+(N-2)\frac{B}{A}I(I+1)}{1+N\frac{B}{A}I(I+1)}\right)+A\left(\frac{\frac{B}{A}I(I+1)}{1+N\frac{B}{A}I(I+1)}\right)^2\times\left[N-0.05\frac{I(I+1)}{2I+1}\right] \qquad (10)$$

where A and B are the two parameters of the model and N is an empirically determined constant given by

$$N=2.85-0.05I . \qquad (11)$$

Because of the spin-dependent choice of N, the two terms in eq. (10) will change their sign at high spins. The first term becomes negative when $[1+(N-2)(B/A)I(I+)]<0$. This condition is satisfied only for $N<2$, that is, for $I\geqslant 18$. Exact value of course depends on the magnitude of B/A. The second term becomes negative for $I\geqslant 38$. Thus we find that ω^2, being proportional to the square of energy derivative, starts decreasing with increasing spin for $I\geqslant 18$. Correspondingly a rapid rise occurs in the moment-of-inertia which is inversely proportional to the energy derivative. Thus in this model back-bending behaviour will be observed for every nucleus around $I_c\sim 20$.

CP Model:[12] In this model the energy expression is a cubic polynomial (CP) in angular momentum.

$$E_I=aI+bI^2+cI^3 . \qquad (12)$$

This expression is equivalent to that adopted in several recent three-parameter models, *e.g.*, the interference model,[13] the anharmonic vibration model,[14] the shape fluctuation model[15] *etc.* The energy derivative is easily evaluated and its substitution in (3) gives

$$\hbar^2\omega^2=\frac{I^2+I}{I^2+I+\frac{1}{4}}[a+2bI+3cI^2]^2 . \qquad (13)$$

Through fits to experimental energies with eq. (12) we find that while coefficients a and b are both positive, c is negative and smaller than the other two parameters for almost all nuclei. As a result $\hbar^2\omega^2$ will attain a maximum for spin value

$$I_c=\frac{b}{3|c|} \qquad (14)$$

beyond which it will start decreasing with increasing spin. Thus $\mathcal{J}$-ω^2 plot will exhibit a back-bending behaviour.

Illustrative Examples: Now we discuss the results of numerical calculations for a few illustrative nuclei. For ^{160}Dy and ^{162}Er the results are presented in Fig. 1. For evaluation of model parameters we have

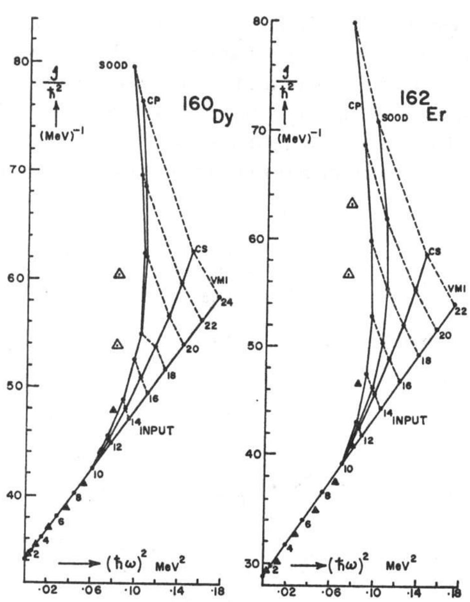

Fig. 1. $\mathcal{J}-\omega^2$ plots for well deformed nuclei ^{160}Dy and ^{162}Er. The experimental data is shown by triangles (closed for the input data and open for the levels not used in fitting) and the predictions of various models have been joined by smooth curves. Results for corresponding spin values for each model are joined by dotted lines for clarity.

included levels upto only 14^+. The parameters thus obtained were used to calculate the energies, moments of inertia and rotational frequencies for each of the levels. As regards the fits to energies, all these models are practically equally successful as demonstrated by comparable rms deviations for predictions of each model.[16] However in $\mathscr{I}$-ω^2 plots, as shown in Fig. 1, the results of various models start diverging even before the 'input' spin value $I = 14$. For higher spins, Sood and CP models show back-bending behaviour similar to that observed experimentally whereas VMI and CS models do not exhibit this feature. Thus numerical calculations bear out the conclusions drawn above on the basis of the mathematical structure of various formulations.

For nearly spherical nuclei ^{106}Cd and ^{194}Pt, shown in Fig. 2, neither CS nor VMI model results are found to be very satisfactory while Sood formula is not applicable. However, the general feature noted above, that is back-bending behaviour, appears (at much lower value of I_c) in CP model while it is absent in CS and VMI model predictions. Another interesting feature, though not directly related to our present problem, is also in evidence in Fig. 2, *i.e.*, whereas $\mathscr{I}_0$ (and also $\mathscr{I}_2$) are predicted to be widely different from each of the three models, the transition moment of inertia $\mathscr{I}_{02} = \frac{1}{2}(\mathscr{I}_0 + \mathscr{I}_2)$ is quite similar from all the three models.

Thus it is of interest to unambiguously identify not only very high angular momentum (with $I \gtrsim 20$) states in deformed nuclei but also to look for states with $I^\pi = 10^+$ and 12^+ in nearly spherical nuclei. The latter are expected to exhibit quite similar singularities and thus bring into evidence similar physical phenomenon at much lower angular momenta.

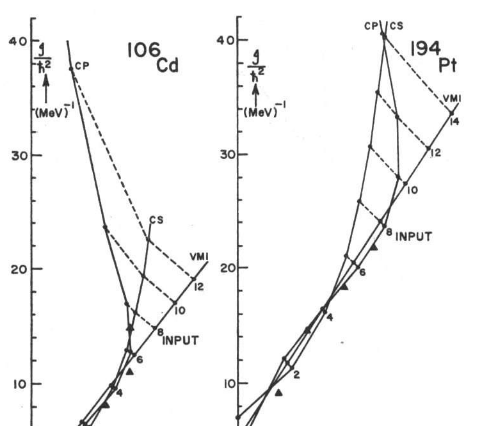

Fig. 2. Same as Fig. 1 but for nearly spherical nuclei ^{106}Cd and ^{194}Pt.

Finally we may also remark that, instead of dealing with the derived quantities like $\mathscr{I}$ and ω^2, one may study the variations with angular momentum I of directly measured experimental quantities, that is the gamma transition energies E_γ between successive members of the band and obtain similar information. A comparison of our plot[15] of E_γ versus I with the $\mathscr{I}$-ω^2 plots clearly points out the similarity and this conclusion can as well be deduced from eqs. (6–7) here.

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