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# VI–1 Magnetic Moments of $d_{3/2}$ Nuclei and K Isotopes

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The $g$ factors of proton-neutron system are investigated from viewpoint of configuration mixing. Two kinds of nuclei with different configurations are considered, the $d_{3/2}$ nuclei in which both nucleons occupy the same shell outside the $^{32}$S core and the K isotopes with configurations $d_{3/2}^{-1}f_{7/2}^{N-20}$. As well as the first-order effects of the configuration mixing[1] some other effects should be taken into account in these regions, since the $g$ factors of the (closed shell in $L$-$S$ coupling $\pm 1$) nuclei $^{39}$K and $^{41}$Ca deviate appreciably from the Schmidt lines. However, it may be probable that these other effects play for the most part a role to renormalize the single-particle $g$ factor and that the first-order effect gives the leading term of corrections responsible for the change of $g$ factors with mass number. We calculate the first-order effects assuming the renormalizability of other effects and examine the effective moment of single-particle as parameters.

The first-order effect is calculated by using the following magnetic moment operator,

$$\tilde{\mu} = \mu + \mu \frac{Q}{E_0 - H_0} V + V \frac{Q}{E_0 - H_0} \mu \,, \qquad (1)$$

where the operator $Q$ omitts the zeroth order configuration and $\mu$ is magnetic moment operator of free nucleons. The second and third terms correspond to the first order of perturbation and are decomposed into state-independent one-body and two-body operators under the approximation that only the energies of spin-orbit splitting of single-particle are taken into account in the energy denominator. $V$ is a coupling interaction in the perturbation and is assumed to be the phenomenological Serber force of Gauss type with $\lambda = 0.5$ and $V_0 = -50$ MeV throughout this paper.

## §1. $d_{3/2}$ Nuclei

Instead of the proton-neutron scheme, we employ the seniority scheme with isospin, application of which is essential in light nuclei. All states arising from the $d_{3/2}^n$ configurations are uniquely classified in the scheme and the binding energies of the nuclei from $^{33}$S to $^{40}$Ca are quite well reproduced under the configuration assignment of $d_{3/2}^n$.[2]

In calculating the matrix elements of $\tilde{\mu}$ between the $d_{3/2}^n$ configurations, the intermediate states of the perturbation are obtained by exciting a nucleon from $d_{5/2}$ to $d_{3/2}$ orbit. Four operators of different types occur, the one-body and two-body operators with isoscalar and isovector characters. The calculated $g$ factors $g_{\text{cal}}$ are shown in Table I together with the measured ones $g_{\text{exp}}$ and the zeroth order estimates in the isospin scheme $g_{\text{ind}}$. We adopt the energy denominator, $\langle E_0 - H_0 \rangle = \varepsilon_{d5/2} - \varepsilon_{d3/2} = -5.5$ MeV.

Overall agreement between $g_{\text{exp}}$ and $g_{\text{cal}}$ is satisfactory. In the zeroth order estimates, the $g$ factors of pairs of nuclei, $^{33}$S – $^{35}$S, $^{35}$Cl – $^{37}$K, $^{36}$Cl – $^{38}$K and $^{37}$Cl – $^{39}$K should be the same, respectively. The observed deviations from these relations are well reproduced. Further symmetric relation holds as noted by Lawson.[3] If the effective M1 operator for single-particle is state-independent and the two-body transition operator does not work, all states written

Table I. $g$ factors of $d_{3/2}$ nuclei in units of nuclear magneton.

| | $^{33}$S | $^{33}$Cl | $^{34}$Cl | $^{35}$S | $^{35}$Cl | $^{35}$Ar | $^{36}$Cl | $^{37}$Cl | $^{37}$Ar | $^{37}$K | $^{37}$Ca | $^{38}$K | $^{39}$K | $^{39}$Ca |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| J | 3/2 | 3/2 | 3 | 3/2 | 3/2 | 3/2 | 2 | 3/2 | 3/2 | 3/2 | 3/2 | 3 | 3/2 | 3/2 |
| T | 1/2 | 1/2 | 0 | 3/2 | 1/2 | 1/2 | 1 | 3/2 | 1/2 | 1/2 | 3/2 | 0 | 1/2 | 1/2 |
| $g_{\text{exp}}$ | 0.429 | | | 0.67(3) | 0.548 | 0.421 | 0.643 | 0.456 | 0.63(13) | 0.136 | | 0.459 | 0.261 | |
| $g_{\text{ind}}$ | 0.765 | 0.083 | 0.424 | 0.765 | 0.174 | 0.674 | 0.424 | 0.083 | 0.674 | 0.174 | 0.765 | 0.424 | 0.083 | 0.765 |
| $g_{\text{cal}}$ | 0.454 | 0.422 | 0.446 | 0.671 | 0.492 | 0.402 | 0.623 | 0.343 | 0.722 | 0.108 | 0.548 | 0.416 | 0.083 | 0.765 |
| $g^{*}_{\text{cal}}$ | 0.446 | 0.498 | 0.480 | 0.663 | 0.557 | 0.405 | 0.657 | 0.419 | 0.725 | 0.173 | 0.540 | 0.450 | 0.159 | 0.757 |

by $(j)^{n}_{\pi}(j)^{\pm n}_{\nu}$ configurations such as those in $^{36}$Cl and $^{38}$K should have constant $g$ factors. Moreover, the deviations from $g_{\mathrm{ind}}$ in these nuclei are expected to be small, because only the isoscalar part of the one-body operator which is not affected very much from various effects can contribute to $g$ factors of the configurations. Both points contradict the measured moments of $^{36}$Cl and $^{38}$K which the present calculation reproduces well. The fairly large deviations in $^{36}$Cl from the $g$ factor of $^{38}$K and from $g_{\mathrm{ind}}$ are predicted as the large effects of the isovector two-body operator which does not work in the $T=0$ state of $^{38}$K.

Furthermore, the effective moments for single-particle in the $d_{3/2}$ orbit are introduced. The magnetic moment operator $\mu$ is divided into the isoscalar and isovector components,

$$\mu = \sum_{i} (\mu_i^{(0)} + \mu_i^{(1)}(\tau_3)_i)\ ,$$

where $\tau_3$ is equal to $-1$ for proton and $+1$ for neutron. Instead of using the free nucleon values, the two matrix elements $(d_{3/2}\|\mu^{(0)}\|d_{3/2})$ and $(d_{3/2}\|\mu^{(1)}\|d_{3/2})$ are taken as parameters and determined so as to reproduce the experimental data minus the first-order corrections calculated above, *i.e.* $(g_{\mathrm{exp}} - \delta g_{\mathrm{1st\ order}})$. The double-barred matrix elements are the reduced matrix elements with respect to the total angular momentum. Using the effective moments thus obtained and $\delta g_{\mathrm{1st\ order}}$, the $g$ factors of the $d_{3/2}$ nuclei are calculated again and are also shown in Table I as $g^{*}_{\mathrm{cal}}$. The obtained effective moments are

$$(d_{3/2}\|\mu^{(0)}_{\mathrm{eff}}\|d_{3/2}) = 1.774$$

and

$$(d_{3/2}\|\mu^{(1)}_{\mathrm{eff}}\|d_{3/2}) = 1.159,$$

whereas the corresponding values of free nucleons are 1.643 and 1.321 respectively. It is to be noted that these effective moments are deduced after the first-order corrections are made.

In view of the good agreements with experimental data obtained in these two calculations, it may be concluded that the idea of the effective moment alone does not predict the whole features of the magnetic moments in the $d_{3/2}$ nuclei but that those terms of first-order corrections which are not renormalized to the effective moments play an important role.

## §2. K-Isotopes

The magnetic moments of the K isotopes have been extensively studied[4] and well reproduced assuming the $d^{-1}_{3/2}f^{N-20}_{7/2}$ configurations outside the $^{40}$Ca core and using the effective moments for the single $d_{3/2}$ proton and $f_{7/2}$ neutron, which are very close to the measured moments of $^{39}$K and $^{41}$Ca, respectively. Here we calculate the first-order corrections and examine how the situation is.

The first-order corrections come from both proton excitation ($d_{5/2} \to d_{3/2}$, $\varepsilon d_{5/2} - \varepsilon d_{3/2} = -5.5$ MeV) and neutron excitation ($f_{7/2} \to f_{5/2}$, $\varepsilon f_{7/2} - \varepsilon f_{5/2} = -5.9$), which produce the effective two-body operator working between $d^{-1}_{3/2}$ and $f_{7/2}$ and between $f_{7/2}$ neutrons. The wave functions obtained in ref. 4 are used for the unperturbed ones. The calculated first-order corrections are subtracted from the measured moments and the effective single-particle moments $\mu^{\mathrm{p}}_{d_{3/2}}$ and $\mu^{\mathrm{n}}_{f_{7/2}}$ are determined to fit the subtracted data. The results for the K isotopes and the effective moments are shown in Table II together with the experimental ones $\mu_{\mathrm{exp}}$, those in the simple *jj*-coupling scheme $\mu_{\mathrm{sp}}$ and those obtained in ref. 4.

The agreements between theory and experiment are quantitatively somewhat worse in $^{42}$K and $^{41}$Ca than the ones in ref. 4. Still it seems that the overall results are in good agreement with experiment within

Table II. Magnetic moments of K and Ca isotopes.[8]

| | $^{39}$K | $^{40}$K | $^{41}$K | $^{42}$K | $^{43}$K | $^{44}$K | $^{45}$K | $^{41}$Ca | $^{42}$Ca | $^{43}$Ca |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mu_{\mathrm{exp}}$ | 0.391 | −1.298 | 0.215 | −1.142 | 0.163(2) | | 0.173(1) | −1.595 | $-3.00^{+.12}_{-.18}$ [a]<br>−2.52(18) [b] | −1.317 |
| $\mu_{\mathrm{sp}}$ | 0.125 | −1.684 | 0.125 | −1.724 | 0.125 | −1.724 | 0.125 | −1.913 | −3.279 | −1.913 |
| $\mu_{\mathrm{TU}}$ | 0.335 | −1.256 | 0.208 | −1.199 | 0.187 | | | −1.569 | | |
| $\mu_{\mathrm{cal}}$ | 0.377 | −1.339 | 0.241 | −1.285 | 0.153 | −0.653 | 0.137 | −1.457 | −2.445 | −1.241 |

a) T. Nomura, T. Yamazaki, S. Nagamiya and T. Katou: Phys. Rev. Letters **27** (1971) 523.
b) M. Marmor, S. Cochavi and D. B. Fossan: Phys. Rev. Letters **25** (1970) 1033.
$\mu_{\mathrm{TU}}$ is the calculated moment in ref. 4.

the accuracy of this simple model and the first-order calculation does not lead to the destruction of the agreement so extremely as was stressed.[5] The first-order corrections are reproduced to be very small except in $^{44}$K, which receives appreciable correction and results in smaller value. A measurement on this quantity may help to see the situation concerning the first-order effect.

Using the effective moment $\mu^{n}_{f_{7/2}}$ and first-order corrections obtained, the magnetic moments of the $6^+$ state of $^{42}$Ca and the ground state of $^{43}$Ca are calculated. The moment of $^{43}$Ca is predicted to be smaller than that of $^{41}$Ca in accordance with the observation and the calculated $g$ factor of $^{42}$Ca is very close to that of $^{41}$Ca. This is a result of our assumption of renormalizability of other effects than the first-order corrections. But in view of the slightly smaller value of the obtained effective moment of $f_{7/2}$ neutron than the observed one in $^{41}$Ca, the assumption is not necessarily valid and other unrenormalized effects should also be studied.[6]

On the other hand, there is an evidence[7] that the MI transitions between the $d_{3/2}^{-1}f_{7/2}$ quartet in $^{40}$K can not be described in terms of state-independent one-body operator. The reduced matrix element of M1 transition between $(j_1j_2; J)$ states is given as

$$\begin{aligned}B(\mathrm{M1}; J'\to J) &= \frac{1}{2J'+1}|\langle j_1j_2; J\|\mathrm{M1}\|j_1j_2; J'\rangle|^2\\ &= \frac{3}{4\pi}\cdot\frac{j_1(j_1+1)(2j_1+1)}{j_2(j_2+1)}(2J+1)\\ &\quad\times[W(j_1Jj_1J'; j_21)]^2|M_{j_1j_2}|^2 .\end{aligned}$$

If the M1 operator includes only single-particle one, the quantity $|M_{j_1j_2}|^2$ defined in the second line can be written

$$|M^{(0)}_{j_1j_2}|^2 \equiv |M_{j_1j_2}|^2 = j_2(j_2+1)[g_{j_1}-g_{j_2}]^2.$$

Therefore if there are no configuration mixing in these states at all, $|M_{j_1j_2}|^2$ should be the same in all the transitions.[7] The data extracted from the $^{40}$K experiment fluctuate appreciably in each transition as shown in Fig. 1. The calculation by making use of the effective moments and first-order corrections obtained above can predict the fluctuations well as shown in the figure. (black circles)

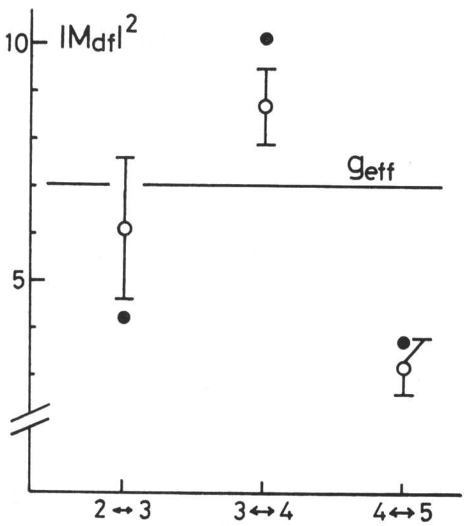


Fig. 1. MI transitions between the $(d_{3/2}^{-1}f_{7/2})_J$ states. The horizontal line is the $|M^{(0)}_{df}|^2$ obtained by the effective moments in the text.

#### References


1) H. Noya, A. Arima and H. Horie: Progr. theor. Phys. Suppl. **8** (1958) 33.
2) A. de-Shalit and I. Talmi: *Nuclear Shell Theory* (Academic Press. New York, 1963).
3) R. D. Lawson: Nuclear Phys. **A173** (1971) 17.
4) I. Talmi and I. Unna: Ann. Rev. Nucl. Sci. **10** (1960) 353.
5) A. de-Shalit: Nuclear Phys. **22** (1961) 677.
6) T. Nomura and T. Yamazaki: *Topical Conference on the Structure of the $f_{7/2}$-Shell Nuclei,* ed. Ricci (Editrice Compositori-Bologna) p. 437.
7) R. E. Segel *et al.*: Phys. Rev. Letters **25** (1970) 1352.
8) The numbers in the Table II are slightly different from those contained in the paper of the Bulletin distributed at the Conference, which are calculated using the same interaction but with $\lambda = 0.6$.


*Note added*—Measurement of the spin and magnetic moment of the ground state of $^{36}$K has been reported in this Conference. (H. Schweickert, J. Dietrich, R. Neugart and E. W. Otten, presented at this conference II-16. The present calculation predicts as $g_{\mathrm{cal}}$ ($^{36}$K; $2^+$) = 0.239 n.m. and $g^*_{\mathrm{cal}} = 0.273$, which are compared with the measured value $g_{\mathrm{exp}} = 0.274(1)$ and also with the zero-th order estimate $g_{\mathrm{ind}} = 0.424$.