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VI–2 Magnetic Moments of Single-Level Configurations

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The g factors of all states of identical nucleons in a single orbit j are constant and the M1 transitions between the states are forbidden, provided a M1 operator is a state-independent single-particle operator. If a two-body M1 operator acts between the nucleons in the orbit, these simple features are violated. Here, the general formulae are derived for the matrix elements of the two-body M1 operator in the j^n-configuration space of identical nucleons and some numerical results are presented assuming the first-order configuration mixing[1] as the origin of the operator. Calculations for proton-neutron system are given in a separate paper.

When the valence nucleons are in the $j=l-1/2$ orbit and the $j'=l+1/2$ orbit is closed, mixing of the configuration $j'^{-1}j^{n+1}$ into the j^n state gives rise to the important effects on M1 properties of the state.[1] The effects of such admixture can be expressed to the first-order of perturbation in terms of the following effective operator in j^n configuration space,[2] under the assumption that the unperturbed hamiltonian includes only single-particle energies.

$$\delta\mu=\Delta g_{l-1/2}J_{\mathrm{op}}-\frac{1}{2\sqrt{3}}\sum_J G(J)((a_j^\dagger\times a_j^\dagger)^{(J)}\times(\tilde{a}_j\times\tilde{a}_j)^{(J)})^{(1)}\,, \tag{1}$$

where J_{op} is a total angular momentum operator, $a_{jm}^\dagger$ is a creation operator of a nucleon in (jm) and $\tilde{a}_{jm}=(-)^{j-m}a_{j-m}$. $\Delta g_{l-1/2}$ is given as

$$\Delta g_{l-1/2}=\frac{2}{\sqrt{j(j+1)(2j+1)}}\langle jj|V'|j'j\rangle_{k=1}\times(j'\|\mu\|j)/\Delta E\,, \tag{2}$$

where

$$\langle j_1j_2|V'|j_1'j_2'\rangle_k=-\sum_J(2J+1)W(j_1j_2j_2'j_1';kJ)\times\langle j_2'j_1|V|j_2j_1'\rangle_J\,.$$

The first term on the right hand of eq. (1) corresponds to the so-called core polarization effect and is closely related to the effective moment. On the other hand, the second term is a two-body operator (denoted by $\delta\mu$(II)) and is not renormalized to the one-body M1 operator.

When the nucleons are in $j=l+1/2$ and the $j'=l-1/2$ orbit is vacant, mixing of the $j^{n-1}j'$ into the j^n configuration produces the effective operator $\delta\mu$(II) in eq. (1), while the one-body operator does not occur.

If the energy E in the energy denominator is approximated to the unperturbed one $E_0=n\varepsilon_j$, the operator $\delta\mu$ is independent of state and nucleon number. The matrix elements of $\delta\mu$(II) are given by

$$\langle j^2;J\|\delta\mu(\mathrm{II})\|j^2;J\rangle=G(J)=\pm2(2J+1)\times W(jJj'J;j1)\langle jj|V|jj'\rangle_J(j'\|\mu\|j)/\Delta E\,, \tag{3}$$

for $j=l\pm1/2$, where $j'=l\mp1/2$ and $\Delta E=\varepsilon_{l+1/2}-\varepsilon_{l-1/2}$. Other admixtures in single closed-shell nuclei lead only to the modification of the one-body operator.

To obtain the matrix element in n-body system, the operator $\delta\mu$(II) is decomposed into the irreducible tensors $\delta T_{S_0}^{(S)}$ in the quasispin space for j^n configurations,[3] s and s_0 being the rank and z-component of the tensor.

$$\begin{aligned}\delta\mu(\mathrm{II})&=\sum_s\delta T_0^{(s)}\,,\\ \delta T_0^{(0)}&=-F(1,0)J_{\mathrm{op}}\,,\\ \delta T_0^{(1)}&=F(1,0)J_{\mathrm{op}}+\delta\mu(\mathrm{II})\,,\\ \delta T_0^{(2)}&=0\,,\end{aligned} \tag{4}$$

where

$$F(1,0)=\sum_J(2J+1)W(jJjJ;j1)G(J)/\sqrt{j(j+1)(2j+1)}\,,$$

and $\Delta g_{l-1/2}=2F(1,0)$ for $j=l-1/2$. Vanishing of the tensor term $\delta T_0^{(2)}$ causes the results that the seniority change $\Delta v=4$ transitions are forbidden and that the g factors of the states with definite $(v\alpha J)$ but different n lie on a straight line. The linear change of g factors has been explicitly shown for the $v=1$ states.[1] In the half-closed-shell nucleus $n=\Omega=(2j+1)/2$, the contributions of $\delta\mu$(II) to the g factors of all $(v\alpha J)$ states become characteristically constant, because the quasispin vector term $\delta T_0^{(1)}$ does not contribute in this nucleus to $\Delta v=0$ matrix elements and $\delta T_0^{(0)}$ is proportional to J_{op}. The seniority mixing due to the two-body nuclear force does not alter the situation, since it can mix only $\Delta v=0$ and

4 states in the half-closed-shell nucleus. The above results hold for any two-body M1 operator which is state-independent.

The reduction formulae[4] are readily obtained from eq. (4). For example, a $\Delta v = 0$ matrix element is written

$$\begin{aligned}&\langle j^n v\alpha J\|\delta\mu(\mathrm{II})\|j^n v\alpha' J'\rangle\\ &\quad=\frac{n-v}{v-\Omega}F(1,0)\langle j^n v\alpha J\|J_{\mathrm{op}}\|j^n v\alpha J\rangle\delta\alpha\alpha'\delta JJ'\\ &\quad+\frac{n-\Omega}{v-\Omega}\langle j^v v\alpha J\|\delta\mu(\mathrm{II})\|j^v v\alpha' J'\rangle\,. \qquad (5)\end{aligned}$$

The contribution of $\delta\mu$(II) reasonably cancels the first term on the right hand of eq. (1) in the end of the shell j^{-1}. The operator is related to the blocking effect of the core polarization due to the presence of nucleons in the orbit. On the other hand, in the j^{2J} nucleus with $j = l + 1/2$, the contribution of $\delta\mu$(II) is reduced to the polarization effect of the j^{2J+1} core by amount $\Delta g_{l+1/2} \equiv -2F(1, 0)$. Thus, the effects of the two-body term are not minor corrections in the magnetic dipole case, but are the same order of magnitude as the core polarization effects.

The g factors of the states with $f_{7/2}^{Z-20}$ configurations in $N = 28$ isotones are calculated by using the above equations. The coupling interaction adopted is the Rosenfeld force of Gauss type with range $\lambda = 0.6$ and the ratio of the strength to the spin-orbit splitting is taken to be 5.6. The results are shown in Fig. 1, in which a straight line connects the g factors of $v = 1$ states (black circles). The excitation of $f_{7/2}^{8}$ neutrons renormalizes the g factor of single proton in the $f_{7/2}$ shell. The calculated g factors of all (vJ) states other than the $v = 1$ state lie in the shaded area.

The contributions from $\delta\mu$(II) are clearly dependent on the spin J in $j^{\pm 2}$ nuclei, while the nucleon number n getting closer to the half-closed shell, the dependence on state becomes weaker. This is ascribed to the factor $(n - \Omega)/(v - \Omega)$ appearing in the second term on the right of eq. (5). When one adopts the Serber force with the same range, the slope of the straight line becomes steeper and the g factor of ^{49}Sc shifts upwards from g_{sch}, though the amount of the shifts is very small. The differences are caused by the interaction in odd state. However the overall features mentioned above are not very different from those with the Rosenfeld force.

The relatively strong dependence on J in $j^{\pm 2}$ nuclei is also found in the calculation of the blocking effects in the $h_{9/2}$ nuclei. The matrix elements of $\delta\mu$(II) in $h_{9/2}^2$ configurations outside the ^{208}Pb core

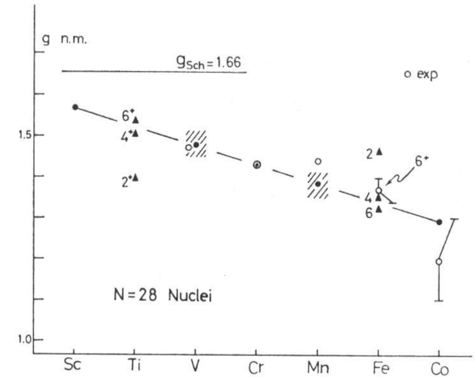

Fig. 1. The g factors of $N = 28$ isotones.

Table I. Blocking effect in ^{210}Po, $\delta g(h_{9/2}^2; J) = \langle h_{9/2}^2; J\|\delta\mu(\mathrm{II})\|h_{9/2}^2; J\rangle/\sqrt{J(J+1)(2J+1)}$.

J	2	4	6	8
$\delta g(h_{9/2}^2; J)$	−0.018	−0.020	−0.013	−0.008

Table II. Calculated g factor of high-spin state of $h_{9/2}$ shell.

	^{209}Bi	^{210}Po	^{211}At	^{212}Rn	^{213}Fr
J	9/2	8	21/2	8	all states
$g(h_{9/2}^n; J)$	0.752	0.744	0.734	0.722	0.710

are calculated using the Rosenfeld force with $\lambda = 0.5$ and $V_0 = -40$ MeV. (Table I). Both results in Table I[5] and Fig. 1 show that the highest-spin state receives the smallest effect in j^2 nucleus. Therefore the g factors of the highest spin states in $j^{\pm 2}$ nuclei are close to those of the neighbouring $j^{\pm 1}$ nuclei.

The results for some high-spin states of the $h_{9/2}^n$ configurations are listed in Table II. Two excitation modes contribute to those results, proton excitation ($h_{11/2} \rightarrow h_{9/2}$) and neutron excitation ($i_{13/2} \rightarrow i_{11/2}$), the former of which concerns the blocking effect. So the ratio of the blocking effect to the total deviation from the Schmidt value depends on how large the effect of the neutron excitation is in ^{209}Bi. Rather slow change of g factors with mass number predicted in Table II results from adopting the Rosenfeld force which works strongly repulsive in singlet-odd state and so enhances very much the neutron excitation effects. These effects do not occur with the delta-function force.[1]

References

1) H. Noya, A. Arima and H. Horie: Progr. theor. Phys. Suppl. **8** (1958) 33, and references cited therein.
2) B. R. Judd: *Second Quantization and Atomic Spectroscopy* (The Johns Hopkins Press, 1967); M. Harvey and F. C. Khanna: Nuclear Phys. **A152** (1970) 588; L. Zamick: Phys. Letters **34B** (1971) 472.
3) For example, M. Ichimura: Progr. in Nuclear Phys. **10** (1969) 307.
4) The reduction formulae are also derived in the preprint recently received; M. Nomura: Phys. Letters, to be published.
5) The numbers in the Table I are somewhat different from those contained in the paper of the Bulletin distributed on the Conference, which are obtained using the interaction CAL (V. Gillet *et al.*, Nuclear Phys. **88** (1969) 321).