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VII.e. Beta-Decay of Bound Nucleons

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Introduction

The β-decay of nucleons bound into complex nuclei differs from that of the free nucleon in several respects of which we here consider three:

(i) In complex nuclei the energy release is different from that in the free state and so the presence of momentum transfer-dependent terms, particularly the induced couplings, can become manifest;

(ii) In complex nuclei one can have contributions to the overall decay rate from the decay of mesons in flight *between* nucleons;

(iii) The intrinsic structure of the nucleon is modified by the nuclear binding so that the renormalization of its β decay by the strong interactions differs from that in the free state.

We note immediately that if CVC is respected these considerations have no effect for vector decay (other than through the well-understood weak magnetism) but that axial decay is a completely open question.

We here look into two aspects of these problems:

(a) Mirror Gamow-Teller β-decay reflects on (i) and (ii) in so far as a departure from identity of the ft-values. $(ft)^+$ and $(ft)^-$, from the two sides of the mirror, $(ft)^+$ referring to the decay of the proton-rich member, would indicate the presence of second-class induced terms in the Weinberg sense, unless that departure were due to lack of identity of the nuclear wavefunctions.

(b) The Gamow-Teller strength of β-decay of well defined $\langle\tau\sigma\rangle$ in complex nuclei would permit the definition of an effective axial coupling constant g_{Ae} renormalized from that, g_A, of the free neutron by the nuclear binding forces.

Mirror Gamow-Teller Decay

We divide the available cases into those of even-A and those of odd-A. We apply to the experimental asymmetry:

$$\delta_{\mathrm{exp}} = [(ft)^+/(ft)^-] - 1$$

a correction, δ_b, on account of the de facto binding energy differences to gain the "fundamental" asymmetry[1)]

$$\delta = \delta_{\mathrm{exp}} - \delta_b$$

as shown in Table I.

We now analyse the "fundamental" δ in terms of the expression:[2)]

$$\delta = -4\frac{\alpha g_v}{g_A}J + \frac{2g_v}{3g_A}(\alpha L - 2\zeta)(W_0^+ + W_0^-)$$

Table I.

A	δ_{exp}	δ_{b}	δ
8	0.107 ± 0.011	0.056	0.051
12	0.115 ± 0.009	0.106	0.009
12*	0.06 ± 0.04	0.056	0.004
18	-0.008 ± 0.015	0.002	-0.010
20	0.029 ± 0.012	0.013	0.016
24	-0.07 ± 0.06	0.011	-0.08
28	-0.04 ± 0.04	0.014	-0.054
30	0.035 ± 0.040	0.001	0.034

that takes into account second-class mesonic decays (*e.g.* $\omega \to \pi \nu e$) that figure in the constant α and also the off-shell single-nucleon decay which is taken to couple to the second-class current through:

$$g_{\text{T}}\sigma_{\lambda\mu}(p-p')_{\mu}\gamma_5 + g'_{\text{T}}i(p+p')_{\lambda}\gamma_5$$

where $\zeta = g_{\text{T}} + g'_{\text{T}}$. J and L are matrix elements of complicated 2-body operators and would not be expected to show systematic correlations in magnitude or sign from transition to transition. The trend of δ against $(W_0^+ + W_0^-)$ is then expected to reveal the magnitude of the fundamental single-nucleon parameter ζ. The analysis yields:

$$\zeta = -(2.0 \pm 3.3) \times 10^{-4},$$

$$\text{or} \quad |\zeta| < 1.0 \times 10^{-3}. \quad (99\,\% \text{ confidence})$$

The even-A systems therefore give no evidence for second-class effects in the hadronic weak current at this level. The odd A systems show a large and significant δ but for them the effects of departure from mirror symmetry in the *final* state nuclear wavefunctions have not been fully investigated and may be expected to be quite large.

g_{A} in Complex Nuclei

The chief problem in extracting g_{Ae} is lack of reliable knowledge of $\langle\tau\sigma\rangle$. This ignorance may be minimized in mirror nuclei for which the isoscalar magnetic moment μ_0 is known:

$$2\mu_0 = \langle l_3\rangle + (\mu_p + \mu_n)\langle\sigma_3\rangle + 2\mu_{x0}$$

where μ_{x0} is the small isoscalar exchange magnetic moment, because $\langle l_3\rangle$ may be e'iminated by the use of:

$$J = \langle l_3\rangle + 1/2\langle\sigma_3\rangle$$

while $\langle\sigma_3\rangle$ is related to the desired $\langle\tau_3\sigma_3\rangle$ by the small $\langle(1 \pm \tau_3)\sigma_3\rangle$.

This exercise is possible for the 6 mirror systems $A = 11, \ldots, 21$ for which the unknown μ_{x0} may be treated on a statistical basis. The result is:[3)]

$$g_{\text{Ae}}/g_{\text{A}} = 0.920 \pm 0.047$$

which is close to the theoretical sum-rule expectation of Ericson[4)] of $g_{\text{Ae}}/g_{\text{A}} \approx 0.93$ derived from a nuclear Adler-Weisberger-like approach.

References

1) D. H. Wilkinson: Phys. Rev. Letters **27** (1971) 1018.
2) K. Kubodera, J. Delorme and M. Rho: unpublished.
3) D. H. Wilkinson: unpublished.
4) M. Ericson: Ann. Phys. **63** (1971) 562.

Discussion

M. RHO (Saclay): I just would like to make a comment on your comparison of the effective axial coupling constant with that of Mrs. Ericson. The number which is obtained by the sum rule of Mrs. Ericson is actually the sum of all final states. Therefore, the comparison would be strictly valid if the actual strength is saturated by the ground to ground condition.

WILKINSON: I entirely agree. It is a conjecture that the same renormalization may apply effectively to all transitions. Again, the only reason I think for making the comparison directly is that we are here concerned with a collection of transitions and one might hope that if you compare sufficiently many transitions in different nuclei, it might add up to the same as many transitions in a single nucleus. But, it is a pure conjecture, and I do wish to stress that.

J. D. WALECKA (Stanford Univ.): Would you comment a little bit on the relation that $\langle(1\pm\tau_3)\sigma_3\rangle$ is 5% of $\langle\sigma_3\rangle$? How was that determined?

WILKINSON: That was calculated using for the mass 11 and 13 cases, Cohen-Kurath wave functions; for 15, 17, and 19, wave functions calculated by Drs. Stockman and Milliner at Oxford which are just about as good as one can do; they involve two particle and two hole excitations from the p to the (s,d) shell. They use standard matrix elements of the Kuo and Arima type matrix elements. For $A = 21$, it was calculated in the same way but without two-particle two-hole excitations. The average value of $\langle(1\pm\tau_3)\sigma_3\rangle$ was about 5% of $\langle\sigma_3\rangle$. But I should again remark that the core polarization of Arima is quite significant and will produce changes in that number. But as I remarked, so far as I can see, it has the same sign as the isoscalar exchange magnetic moment.

M. MORITA (Osaka Univ.): I would like to ask about your first equation of *ft*-value, *i.e.* $ft = 6152\pm10/\{1+R_e{}^2(J+1)\langle\tau_3\sigma_3\rangle^2/J\}$ where $R_e = |g_{\mathrm{Ae}}/g_{\mathrm{v}}|$. In the denominator, you have $\langle\tau_3\sigma_3\rangle^2$. This expression may be derived under the assumption of the impulse approximation and charge independent nuclear forces. So, there must be certaine errors introduced by the assumption. How much would you expect them?

WILKINSON: This I don't know. There will certainly be corrections to that expression which have an origin somewhat similar to the ones which I'm talking about in the magnetic moments. But, I have no possibility of quantifying this. I have regarded this rather as an empirical expression and I am putting all the blame on R_e. It is a parametrization, if you like. I am asking the question that if one takes the normal shell-model procedure, what conclusion do we draw about the R_e.

MORITA: The other thing I'd like to know is that if you assume the relativistic correction which Dr. Ohtsubo had a talk yesterday, then the effect is about 3 or 4% which increases the ratio R_e about the same size.

WILKINSON: Yes, I'd like to make two remarks, though one is that, as you say, there is a relativistic correction to g_A itself of 2 or 3% in the same sense as the 8 that I find. The other point is, of course, that one should also include the relativistic corrections in the isoscalar magnetic moment itself. When I was doing the sums a few weeks ago, I got Dr. Ohtsubo and your paper on the relativistic magnetic moments and, having no time, I could barely see the effect of putting them in here, but qualitatively the effect is to reduce the R_e, that is to say, to give an increased renormalization. I am afraid I can make no more than that semiquantitative remark.