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NEGATIVE DIFFERENTIAL CONDUCTIVITY IN THE CASE OF ELECTRON SCATTERING BY OPTICAL PHONONS AT LOW TEMPERATURES

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The differential conductivity in the streaming motion model is calculated in the resonance regions at microwave frequencies which are multiples of the "flight" frequency. The electron-bunching effect is shown to take place and because of that the negative defferential conductivity occurs.

1. Introduction

Recently substantial roles of the LO phonon emission in hot electron phenomena in silver halides have been pointed out [1]. The experiments have given a definite evidence that the electrical properties of the silver halides can be successfully interpreted in terms of streaming motion model developed in [2]. The streming motion model is discribed as follows. When the dominant electron scattering is emission of optical phonons of frequency ω_0 the momentum space consists of two regions as shown in Fig. (1). In the active region (electron energy $\varepsilon > \hbar\omega_0$) the emission of phonons with the characteristic time τ^+ takes place. In the passive region ($\varepsilon < \hbar\omega_0$) only the weak scattering by acoustical phonons, impurities etc. with the characteristic time τ^- is possible ($\tau^- \gg \tau^+$). At low temperatures ($kT \ll \hbar\omega_0$) all the electrons are in the passive region at the point $p=0$. When the steady electric field E_0 is swiched on, the electrons move towards the passive region boundary, reach it in the time of "flight" $\hat{\tau} = p_0/eE_0$ at the point K, emit optical phonon and return to the point $p=0$ and then periodically repeat the the same motion. If the electric field strength is such that inequalities $\tau^+ \ll \hat{\tau} \ll \tau^-$ are satisfied, all the electrons are concentrated on the main trajectory (solid line OK in Fig. (1)). As $\hat{\tau} \ll \tau^-$ the scattering in the passive region is weak and the background on other trajectories is low and because $\tau^+ \ll \hat{\tau}$ there is only a small penetration of electrons into the active region and the main trajectory is rather thin.

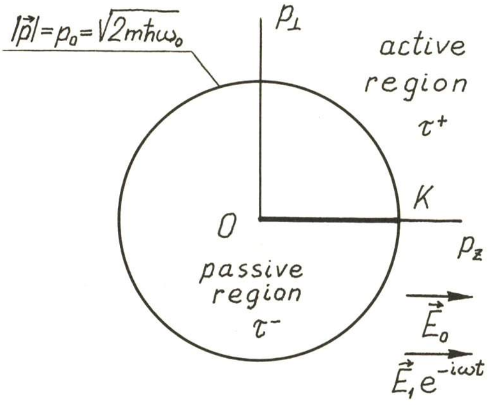

Fig. 1

In this streaming motion model there is a cyclic motion of electrons with frequency $\hat{\omega} = 2\pi/\hat{\tau}$. Consequently, in all high frequency characteristics the resonance phenomena are to take place. Actually, the spectral intensity of current fluctuations is shown [3] to be a superposition of peaks at frequencies $l\hat{\omega}$, where l is an integer.

In this report the results of the differential conductivity calculations in the resonance regions $\omega \cong l\hat{\omega}$ are presented.

2. Simple model

Assume the weak alternating electric field $\vec{E}_1 \exp(-i\omega t) \| \vec{E}_0$ to be swiched on ($E_1 \ll E_0$). Before writting down the exact kinetic equation solution consider a simple model obviously demonstrating the characteristic features of the exact model. Assume the momentum space to consist only of the main trajectory. The distribution of electrons in it is depicted in Fig. (2). Moreover, the main trajectory consists only of two parts, 1 and 2 respectively and the time the electrons spend in its every part is reciprocal to the resulting electric field. There exists additional weak scattering between

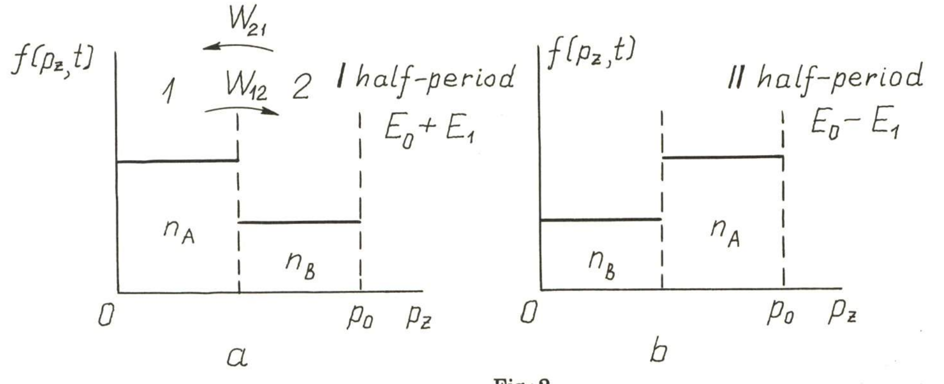

Fig. 2

the main trajectory parts characterized by probabilities W_{12} and W_{21}. Let us consider only the case of resonance $\omega = \hat{\omega}$. Taking into account the fact that the resulting electric field strength is greater during the first half-period (Fig. (2a)) then that in the second one (Fig. (2b)), the following simple balance equation for the electron distribution function $n_{A,B}$ can be obtained:

$$n_A \left\{ \frac{W_{12}}{E_0+E_1} + \frac{W_{21}}{E_0-E_1} \right\} = n_B \left\{ \frac{W_{21}}{E_0+E_1} + \frac{W_{12}}{E_0-E_1} \right\} . \tag{1}$$

Introducing the notations $n_A = n_o + \Delta n$, $n_B = n_o - \Delta n$ and considering the alternating current to be proportional to Δn we obtain the following expression for the differential conductivity:

$$\sigma(\omega) \sim \Delta n \cong \frac{n_o}{E_0} \cdot \frac{W_{12} - W_{21}}{W_{12} + W_{21}} . \tag{2}$$

On account of these simple considerations the following conclusions can be arrived at:
1. The alternating electrical field together with the weak scattering of electrons in the passive region leads to electron bunching ($\Delta n \neq 0$);
2. While the alternating current is caused by weak scattering it does not depend on the strength of that scattering;
3. The sign of $\sigma(\omega)$ is determined by nonhomogeneity of scattering ($W_{12} - W_{21}$). Consequently, the negative sign is possible, i. e. the negative differential conductivity (NDC) can take place.

3. The result of the kinetic equation solution

The linear kinetic equation for the alternating electron distribution function can be analytically solved due to the presence of two small parameters $\xi = \tau^+/\hat{\tau}$ and $\eta = \hat{\tau}/\tau^-$ and the following expression for the differential conductivity can be obtained [4]:

$$\sigma(\omega) = \frac{ine^2}{m\omega} \cdot \frac{2/3\, a_0 \xi^{2/3} \left[(\omega\hat{\tau})^2 - i\omega\hat{\tau} \right] + \eta B_0}{\exp(-i\omega\hat{\tau}) - 1 + a_0 \xi^{2/3} \left[(\omega\hat{\tau})^2 - i\omega\hat{\tau} \right] + \eta B_0} , \tag{3}$$

where
The following notations are used:

$$B_0 = \int\limits_{r<1} d^3 r \int\limits_0^1 dz\, W(0z', \vec{r}) \left\{ \exp\left[-i\omega\hat{\tau}\left(z - \sqrt{1-x^2-y^2}\right)\right] - \exp\left[-i\omega\hat{\tau}(z'-1)\right] \right\} , \tag{4}$$

$$B_1 = \int\limits_{r<1} d^3 r \int\limits_0^1 dz'\, W(0z', \vec{r}) \left\{ 1 - \exp\left[-i\omega\hat{\tau}\left(z - \sqrt{1-x^2-y^2} - z' + 1\right)\right] \right\} , \tag{5}$$

where $W(\vec{r}, \vec{r}\,') = p_0^3\, w(\vec{p}, \vec{p}\,') / \tau^-$ is dimensionless scattering probability and $\vec{r} = \vec{p}/p_0$ — dimensionless momentum, n — electron concentration.

Equation (3) is valid only in resonance regions, where $\omega \cong l\hat{\omega}$, $\exp(-i\omega\hat{\tau}) \sim 1$ and the fraction is of order 1. In other regions the $\xi^{2/3}$, η — corrections must be taken into account. Equation (3) appears to be of the similar form as eq. (2).

Making use of the notation $\omega = l\hat{\omega} + \Delta\omega$ and expanding the exponent in the denominator of eq. (3) we obtain

$$\sigma(\Delta\omega) = \frac{ne^2}{m\hat{\omega}} \cdot \frac{a + ib}{i\Delta\omega\hat{\tau} + c + id} , \tag{6}$$

where the coefficients a, b, c, d can be easily expressed in terms of B_0, B_1, furthermore, they are proportional to the small parameters η or $\xi^{2/3}$. It is convinient to depict the σ $\Delta\omega$-dependence in the complex plane as shown in Fig. (3). When $\Delta\omega$ alters from $-\infty$ to $+\infty$, $\sigma(\Delta\omega)$ runs the whole circle with its centre being at the point

$$\sigma^c = \frac{ne^2}{m\hat{\omega}} \cdot \frac{a + ib}{2c} . \tag{7}$$

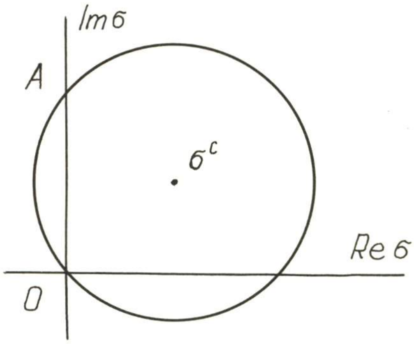

Fig. 3

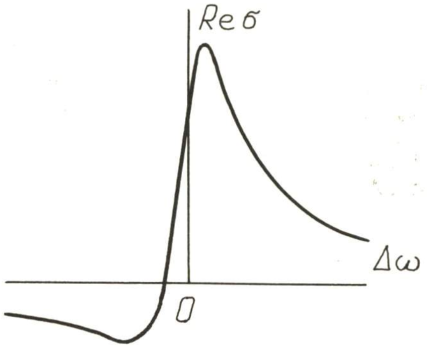

Fig. 4

As can be seen from Fig. (3) there is an interval OA, in which the dissipative component $\sigma(\omega)$ is negative. Thus, one of the resonance line wings is negative, as shown in Fig. (4). If $b^2 \gtrsim |ac| \cdot \max(\xi^{2/3}, \eta)$, the above mentioned NDC will not be smeared by unaccounted nonresonant terms in eq. (3). The numerical evaluation [4] of eq. (4), (5) shows the most favourable case for NDC to occur to be that when the scattering is more efficient at small momenta. For example, in the case of scattering by ionized impurities with screening radius of order $\hbar/4p_0$ the circle centre is on the imaginary axis and half the cirele is in the NDC region.

4. Steady magnetic field influence

Having switched on the steady magnetic field $\vec{H} \perp \vec{E}_0$ the main trajectory is curved as depicted in Fig. (5) by solid curve OK. Now the alternating electric field $\vec{E}_1 \exp(-i\omega t)$ swiched on in the plane perpendicular to $\vec{H}$ periodically changes not only the velocity of electrons moving along the main trajectory, but the position of the main trajectory as well (dotted lines in Fig. (5)) and consequently, changes its length. In the resonance regions when $\omega \approx l\hat{\omega}$ this synchronous changing of trajectory length leads to much greater electron bunching-effect than that caused by weak scattering in the passive region considered in the above presented case without H. In the case with H the scattering in the passive region and the electron penetration into the active region cause only the broadening of resonance peaks. Having taken into account merely the broadening caused by the scattering in the passive

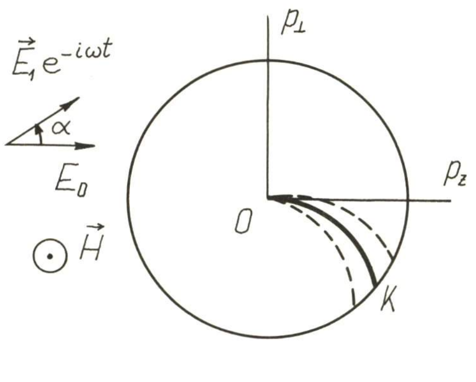

Fig. 5

region the differential conductivity is shown to be expressed in the following form [5]:

$$\sigma(\omega) = \frac{ne^2}{m}\left[\exp(-i\omega\hat{\tau}) - 1 + \eta B_1\right]^{-1} \frac{4\omega_c \sin(\omega_c\hat{\tau}/2)}{[\omega^2 - \omega_c^2]^2\,\hat{\tau}^2} \times \tag{8}$$

$$\times\left\{\frac{\omega}{\omega_c} - \frac{i}{2\cos(\omega_c\hat{\tau}/2)}\left[\left(\frac{\omega^2}{\omega_c^2}+1\right)\sin(\omega_c\hat{\tau}/2) - \left(\frac{\omega^2}{\omega_c^2}-1\right)\sin(2\alpha+\omega_c\hat{\tau}/2)\right]\right\},$$

where $\omega_c = eH/mc$, $\hat{\tau} = \frac{2}{\omega_c}\arcsin(p_0\omega_c/2eE_0)$.

The circle centre σ^c in the complex plane is proportional to the expression in the braces of eq. (8). It can be easily seen that $\mathrm{Re}\,\sigma^c > 0$, and $\mathrm{Im}\,\sigma^c$ is defined by the polarisation angle α . Thus the centre lies in the first or the fourth quadrant. If $\alpha = \alpha^\pm = -\omega_c\hat{\tau}/4 \pm \pi/4$ the centre is maximally remote from the real axis and the situation is most favourable for NDC to occur.

According to [1] at helium temperatures in AgBr $\tau^- \sim 5\cdot10^{-11}$ sec., $m = 0.3\ m_e$, $\hbar\omega_0 \sim 17$ meV. When $E_0 = 150$ V/cm, H = 550 Oe, the resonance frequency is of order $\nu \sim 30$ GHz. Making use of eq. (8) for the evaluation of σ we can expect NDC to occur of oder $ne^2/(20m\hat{\omega})$.

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